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Unified Approach to Starlike and Convex Functions Involving Poisson Distribution Series

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Abstract. The motivation behind present paper is to establish connection between analytic univalent functions $\mathcal{T}S_p(\zeta,\gamma,\delta)$ and $UC\mathcal{T}(\zeta,\gamma,\delta)$ by applying Hadamard product involving Poisson distribution series. We likewise consider an integral operator connection with this series.

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1 Introduction

We letting $\mathfrak{A}$ denote the class of functions $\mathfrak{f}$ of the form:

$$\mathfrak{f}(z) = z + a_2 z^2 + a_3 z^3 + ... \tag{1}$$

which are analytic in $\mathbb{U}$ and $\mathfrak{S}$ the subclass of $\mathfrak{A}$ which includes univalent functions normalized by conditions $\mathfrak{f}(0) = 0 = \mathfrak{f}'(0) - 1$. Let $\mathcal{T}$ be the subclass of $\mathfrak{A}$ consisting of functions whose non zero coefficient of the form second on, given by (see [19])

$$\mathfrak{f}(z) = z - \sum_{n=2}^{\infty} a_n z^n. \tag{2}$$

Kanas and Wisniowska [11] introduced the class $\delta - UCV$ which includes geometric aspect in connection with conic domains. The family $\delta - UCV$ is of extraordinary enthusiasm for it contains some notable, just as new, classes of univalent functions. The class $\delta - UCV$ map each circular arc contained in the unit disk $\mathbb{U}$ with a center ξ, $|\xi| \leq \delta\,(0 \leq \delta < 1)$, onto a convex arc. The notion of δ-uniformly convex function is straightforward expansion of classical convexity. In 2011, Murugusundaramoorthy and Magesh [13] unified the classes $S_p(\gamma,\delta)$ and $UCV(\gamma,\delta)$ into the classes $S_p(\zeta,\gamma,\delta)$ and $UCV(\zeta,\gamma,\delta)$ which is defined as, a function $f \in \mathcal{A}$ is said to in the class δ-uniformly starlike functions of order γ, denoted by $S_p(\zeta,\gamma,\delta)$ if it satisfies analytic criterion

$$Re\left\{\frac{zf'(z)}{(1-\zeta)f(z)+\zeta zf'(z)} - \gamma\right\} > \delta\left|\frac{zf'(z)}{(1-\zeta)f(z)+\zeta zf'(z)} - 1\right|,\ z \in \mathbb{U} \tag{3}$$

and the $f \in \mathcal{A}$ is said to in the class δ-uniformly convex functions of order γ, denoted by $UCV(\zeta, \gamma, \delta)$ if it satisfies analytic criterion

$$Re\left\{\frac{f'(z)+zf''(z)}{f'(z)+\zeta zf''(z)}-\gamma\right\} > \delta\left|\frac{f'(z)+zf''(z)}{f'(z)+\zeta zf''(z)}-1\right|, \ z\in\mathbb{U}. \tag{4}$$

We note that $\mathcal{T}S_p(\zeta,\gamma,\delta) = S_p(\zeta,\gamma,\delta)\cap\mathcal{T}$ and $UC\mathcal{T} = UCV\cap\mathcal{T}$.

Remark 1. From among the many choices of ζ, γ, δ which would provide the following known subclasses:
1) $\mathcal{T}S_p(0,\gamma,\delta) = \mathcal{T}S_p(\gamma,\delta)$ (see [4]),
2) $\mathcal{T}S_p(0,0,\delta) = \mathcal{T}S_p(\delta)$ (see [20]),
3) $\mathcal{T}S_p(0,\gamma,1) = \mathcal{T}S_p(\gamma)$ (see [4]),
4) $\mathcal{T}S_p(\zeta,\gamma,0) = \mathcal{T}(\zeta,\gamma)$ (see [2],[16]),
5) $\mathcal{T}S_p(0,\gamma,0) = \mathcal{T}^*(\gamma)$ (see [19]),
6) $UC\mathcal{T}(0,\gamma,\delta) = UC\mathcal{T}(\gamma,\delta)$ (see [4]),
7) $UC\mathcal{T}(0,0,\delta) = UC\mathcal{T}(\delta)$ (see [21]),
8) $UC\mathcal{T}(0,\gamma,1) = UC\mathcal{T}(\gamma)$ (see [4]),
9) $UC\mathcal{T}(\zeta,\gamma,0) = \mathcal{C}(\zeta,\gamma)$ (see [2]),
10) $UC\mathcal{T}(0,\gamma,0) = \mathcal{C}(\gamma)$ (see [19]).

2 Preliminary Results

A remarkably large number of special functions (series) have been presented in geometric function theory. Among those special functions, due mainly to greater abstruseness of their properties, Bieberbach conjecture have found special attention in various problems of geometric function theory. Recently, a large number of special functions involving hypergeometric functions and their various extension (or generalizations) have been investigated, see also ([3],[5],[6],[8],[9],[15],[18],[22],[23]).

Recently, Porwal [16] introduced a power series as

$$\chi(p,z) = z + \sum_{n=2}^{\infty}\frac{e^{-p}p^{n-1}}{(n-1)!}z^n, \quad z\in\mathbb{U}, \tag{5}$$

where $p>0$. Further Porwal [16] defined a series

$$\varphi(p,z) = 2z - \chi(p,z) = z - \sum_{n=2}^{\infty}\frac{e^{-p}p^{n-1}}{(n-1)!}z^n, \quad z\in\mathbb{U}. \tag{6}$$

The convolution (or Hadamard product) of two series

$$(\mathfrak{f}*\mathfrak{g})(z) = (\mathfrak{g}*\mathfrak{f})(z) = \sum_{n=2}^{\infty}a_nb_nz^n.$$

Porwal and Kumar [17] introduced the linear operator $\mathfrak{I}(p)\mathfrak{f} : \mathfrak{A} \to \mathfrak{A}$ defined by using the Hadamard product as

$$\mathfrak{I}(p)\mathfrak{f} = \chi(p,z) * f(z) = z + \sum_{n=2}^{\infty} \frac{e^{-p}p^{n-1}}{(n-1)!} a_n z^n, \quad z \in \mathbb{U}. \tag{7}$$

Altinkaya and Yalcin [1] gave obligatory conditions for the Poisson distribution series belonging to the class $\mathcal{T}(\gamma,\delta)$. Murugusundaramoorthy et al.[14] investigated some characterization for Poisson distribution series. In recent times, the univalent function theorists have shown good affinity towards Possion distribution series by relating it with the area of geometric function theory (see also,[10] [12],[16],[17]). To prove our results, we will need the following results.

Theorem 1. *[13] A function $\mathfrak{f}(z)$ of the form (1) is in $\mathcal{T}S_p(\zeta,\gamma,\delta)$ if and only if*

$$\sum_{n=2}^{\infty} \Big[n(1+\delta) - (\gamma+\delta)(1+n\zeta-\zeta)\Big]|a_n| \le 1-\gamma. \tag{8}$$

Theorem 2. *[13] A function $\mathfrak{f}(z)$ of the form (1) is in $UC\mathcal{T}(\zeta,\gamma,\delta)$ if and only if*

$$\sum_{n=2}^{\infty} n\Big[n(1+\delta) - (\gamma+\delta)(1+n\zeta-\zeta)\Big]|a_n| \le 1-\gamma. \tag{9}$$

Inspired by results between various subclasses of analytic univalent functions by utilizing hypergeometric functions ([9],[15],[22]), Bessel functions ([3],[5],[6],[8]) and Struve functions ([23]), we established connections between the classes $UC\mathcal{T}(\zeta,\gamma,\delta)$ and $\mathcal{T}S_p(\zeta,\gamma,\delta)$ by applying the above mentioned results (8), (9) and convolution operator given by (7).

3 Main Results

Theorem 3. *The function $\chi(p,z)$ is in $\mathcal{T}S_p(\zeta,\gamma,\delta)$ if*

$$pe^p[(1+\delta) - \zeta(\gamma+\delta)] \le 1-\gamma \tag{10}$$

holds for $p > 0$. Moreover $\varphi(p,z)$ belongs to $\mathcal{T}S_p(\zeta,\gamma,\delta)$ if and only if (10) holds.

Proof. In view of Theorem 1, it is sufficient to show that

$$\sum_{n=2}^{\infty} \Big[n(1+\delta) - (\gamma+\delta)(1+n\zeta-\zeta)\Big] \frac{p^{n-1}}{(n-1)!} e^{-p} \le 1-\gamma.$$

Let

$$\Omega_1(p,\zeta,\gamma,\delta) = \sum_{n=2}^{\infty} \Big[n(1+\delta) - (\gamma+\delta)(1+n\zeta-\zeta)\Big] \frac{p^{n-1}}{(n-1)!} e^{-p}$$

$$
\begin{aligned}
&= e^{-p}\sum_{n=2}^{\infty}\Big[n(1+\delta)-(\gamma+\delta)(1+n\zeta-\zeta)\Big]\frac{p^{n-1}}{(n-1)!}\\
&= e^{-p}\left[\{(1+\delta)-\zeta(\gamma+\delta)\}\sum_{n=2}^{\infty}\frac{p^{n-1}}{(n-2)!}+(1-\gamma)\sum_{n=2}^{\infty}\frac{p^{n-1}}{(n-1)!}\right]\\
&= e^{-p}\Big[\{(1+\delta)-\zeta(\gamma+\delta)\}pe^{p}+(1-\gamma)(e^{p}-1)\Big]\\
&= [(1+\delta)-\zeta(\gamma+\delta)]p+(1-\gamma)(1-e^{-p}).
\end{aligned}
$$

But the last expression is bounded above by $1-\gamma$, if (10) holds. Since

$$
\varphi(p,z)=2z-\chi(p,z)=z-\sum_{n=2}^{\infty}\frac{e^{-p}p^{n-1}}{(n-1)!}z^{n} \tag{11}
$$

the necessary of (10) for $2z-\chi(p,z)$ to be in $\mathcal{TS}_p(\zeta,\gamma,\delta)$ follows from Theorem 1. □

Remark 2. Putting $\delta=0$ in Theorem 3, we obtain the result investigated by Porwal [16] Theorem 3.

Corollary 1. *The function $\chi(p,z)$ is in $\mathcal{TS}_p(\gamma,\delta)$ if*

$$
pe^{p}(1+\delta)\le 1-\gamma \tag{12}
$$

holds for $p>0$.

Corollary 2. *The function $\chi(p,z)$ is in $\mathcal{TS}_p(\gamma)$ if*

$$
pe^{p}\le 1-\gamma \tag{13}
$$

holds for $p>0$.

Corollary 3. *The function $\chi(p,z)$ is in $\mathcal{TS}_p(\zeta,\gamma,\delta)$ if*

$$
e^{p}\Big[\{(1+\delta)-\zeta(\gamma+\delta)\}p\Big]\le 1-\gamma \tag{14}
$$

holds for $p>0$.

Theorem 4. *The function $\chi(p,z)$ is in $UC\mathcal{T}(\zeta,\gamma,\delta)$ if*

$$
e^{p}\Big(\{(1+\delta)-\zeta(\gamma+\delta)\}p^{2}+\{3(1+\delta)-(1+2\zeta)(\gamma+\delta)\}p\Big)\le 1-\gamma \tag{15}
$$

holds for $p>0$.

Proof. In view of Theorem 2, it is sufficient to show that

$$\sum_{n=2}^{\infty} n\Big[n(1+\delta)-(\gamma+\delta)(1+n\zeta-\zeta)\Big]\frac{p^{n-1}}{(n-1)!}e^{-p} \le 1-\gamma.$$

Let

$$\begin{aligned}
&\Omega_2(p,\zeta,\gamma,\delta)\\
&= \sum_{n=2}^{\infty} n\Big[n(1+\delta)-(\gamma+\delta)(1+n\zeta-\zeta)\Big]\frac{p^{n-1}}{(n-1)!}e^{-p}\\
&= e^{-p}\sum_{n=2}^{\infty} n\Big[n(1+\delta)-(\gamma+\delta)(1+n\zeta-\zeta)\Big]\frac{p^{n-1}}{(n-1)!}\\
&= e^{-p}\Bigg[\{(1+\delta)-\zeta(\gamma+\delta)\}\left(\sum_{n=2}^{\infty}\frac{p^{n-1}}{(n-3)!}+3\sum_{n=2}^{\infty}\frac{p^{n-1}}{(n-2)!}+\sum_{n=2}^{\infty}\frac{p^{n-1}}{(n-1)!}\right)\\
&\qquad +\{\zeta(\gamma+\delta)-(\gamma+\delta)\}\left(\sum_{n=2}^{\infty}\frac{p^{n-1}}{(n-2)!}+\sum_{n=2}^{\infty}\frac{p^{n-1}}{(n-1)!}\right)\Bigg]\\
&= e^{-p}\Big(\{(1+\delta)-\zeta(\gamma+\delta)\}p^2e^p+\{3(1+\delta)-(1+2\zeta)(\gamma+\delta)\}pe^p\\
&\qquad\qquad +(1-\gamma)(e^p-1)\Big)\\
&= \Big(\{(1+\delta)-\zeta(\gamma+\delta)\}p^2+\{3(1+\delta)-(1+2\zeta)(\gamma+\delta)\}p\\
&\qquad\qquad +(1-\gamma)(1-e^{-p})\Big).
\end{aligned}$$

But the last expression is bounded above by $1-\gamma$, if (15) holds. □

Remark 3. Putting $\delta = 0$ in Theorem 4, we obtain the result investigated by Porwal [16] Theorem 4.

Corollary 4. *The function $\chi(p,z)$ is in $UC\mathcal{T}(\gamma,\delta)$ if*

$$pe^p\Big[(1+\delta)p+2\delta-\gamma+3\Big]\le 1-\gamma \tag{16}$$

holds for $p>0$.

Corollary 5. *The function $\chi(p,z)$ is in $UC\mathcal{T}(\gamma)$ if*

$$pe^p(p-\gamma+3)\le 1-\gamma \tag{17}$$

holds for $p>0$.

Corollary 6. *The function $\chi(p,z)$ is in $UC\mathcal{T}(\zeta,\gamma,\delta)$ if*

$$e^p\Big(\{(1+\delta)-\zeta(\gamma+\delta)\}p^2+\{3(1+\delta)-(1+2\zeta)(\gamma+\delta)\}p\Big)\le 1-\gamma \tag{18}$$

holds for $p>0$.

4 Inclusion Properties

A function $f \in \mathcal{A}$ is said to in the class $\mathcal{R}_\nu^\tau(\delta)$, if it satisfies the inequality

$$\left| \frac{(1-\delta)\frac{f(z)}{z} + \nu f'(z) - 1}{2\tau(1-\delta) + (1-\nu)\frac{f(z)}{z} + \nu f'(z) - 1} \right| < 1, \quad (z \in \mathbb{U})$$

where $\tau \in \mathbb{C} \setminus \{0\}, \delta < 1, 0 < \nu \leq 1$.

The class was introduced by Swaminathan [18]. for $\nu = 1$ the class is reduces to familiar class introduced by Dixit and Pal [7]. Making use of following lemma, we will prove inclusion result on the class $UC\mathcal{T}(\zeta, \gamma, \delta)$.

Lemma. *If $f \in \mathcal{R}_\nu^\tau(\delta)$ is of the form (1) then*

$$|a_n| = \frac{2|\tau|(1-\delta)}{1+\nu(n-1)}, \quad n \in \mathbb{N} \setminus \{1\}. \tag{19}$$

The bounds given in (4) is sharp.

Theorem 5. *Let $p > 0$, $\tau \in \mathbb{C} \setminus \{0\}, \delta < 1$ and $0 < \nu \leq 1$. If $f \in \mathcal{R}_\nu^\tau(\delta)$, then $\mathfrak{I}(p,z)f \in UC\mathcal{T}(\zeta, \gamma, \delta)$ if and only if*

$$\Big[\{(1+\delta) - \zeta(\gamma+\delta)\}p + (1-\gamma)(1-e^{-p})\Big] \leq \frac{\nu(1-\gamma)}{2|\tau|(1-\delta)}. \tag{20}$$

Proof. In view of Lemma 4 it is sufficient to show that

$$\sum_{n=2}^{\infty} n\Big[n(1+\delta) - (\gamma+\delta)(1+n\zeta-\zeta)\Big]\frac{p^{n-1}}{(n-1)!}e^{-p}|a_n| \leq 1-\gamma.$$

Since $f \in \mathcal{R}_\nu^\tau(\delta)$, then by Lemma 4, we have

$$|a_n| = \frac{2|\tau|(1-\delta)}{1+\nu(n-1)}.$$

Let

$$\begin{aligned} &\Omega_3(p,\zeta,\gamma,\delta) \\ &= \sum_{n=2}^{\infty} n\Big[n(1+\delta) - (\gamma+\delta)(1+n\zeta-\zeta)\Big]\frac{p^{n-1}}{(n-1)!}e^{-p}|a_n| \leq 1-\gamma \\ &= \sum_{n=2}^{\infty} n\Big[n(1+\delta) - (\gamma+\delta)(1+n\zeta-\zeta)\Big]\frac{p^{n-1}}{(n-1)!}e^{-p}\frac{2|\tau|(1-\delta)}{1+\nu(n-1)} \end{aligned}$$

Since $1+\nu(n-1) \geq \nu n$

$$\Omega_3(p,\zeta,\gamma,\delta)$$

$$\leq \frac{2|\tau|(1-\delta)}{\nu}\sum_{n=2}^{\infty}\Big[n(1+\delta)-(\gamma+\delta)(1+n\zeta-\zeta)\Big]\frac{p^{n-1}}{(n-1)!}e^{-p}$$
$$\leq \frac{2|\tau|(1-\delta)}{\nu}\Big[\{(1+\delta)-\zeta(\gamma+\delta)\}p+(1-\gamma)(1-e^{-p})\Big].$$

But the last expression is bounded by $1-\gamma$, if (20) holds. □

Corollary 7. *Let $p > 0$, $\tau \in \mathbb{C}\setminus\{0\}, \delta < 1$. If $f \in \mathcal{R}_1^{\tau}(\delta)$, then $\Im(p,z)f \in UC\mathcal{T}(\zeta,\gamma,\delta)$ if and only if*

$$\Big[\{(1+\delta)-\zeta(\gamma+\delta)\}p+(1-\gamma)(1-e^{-p})\Big] \leq \frac{(1-\gamma)}{2|\tau|(1-\delta)}. \tag{21}$$

Corollary 8. *Let $p > 0$, $\tau \in \mathbb{C}\setminus\{0\}, \delta < 1$. If $f \in \mathcal{R}_1^{\tau}(\delta)$, then $\Im(p,z)f \in UC\mathcal{T}(\gamma,\delta)$ if and only if*

$$\Big[(1+\delta)p+(1-\gamma)(1-e^{-p})\Big] \leq \frac{(1-\gamma)}{2|\tau|(1-\delta)}. \tag{22}$$

Theorem 6. *Let $p > 0$, $\tau \in \mathbb{C}\setminus\{0\}, \delta < 1$ and $0 < \nu \leq 1$. If $f \in \mathcal{R}_\nu^{\tau}(\delta)$, then $\Im(p,z)f \in \mathcal{T}S_p(\zeta,\gamma,\delta)$ if and only if*

$$\Big[\{(1+\delta)+(1-\zeta)(\gamma+\delta)\}(1-e^{-p})-\frac{(\gamma+\delta)}{p}(1-e^{-p}-pe^{-p})\Big] \leq \frac{\nu(1-\gamma)}{2|\tau|(1-\delta)}. \tag{23}$$

Proof. The proof of Theorem 6 is similar to the proof of Theorem 5, therefore we omit the details involved. □

Corollary 9. *Let $p > 0$, $\tau \in \mathbb{C}\setminus\{0\}, \delta < 1$. If $f \in \mathcal{R}_1^{\tau}(\delta)$, then $\Im(p,z)f \in \mathcal{T}S_p(\zeta,\gamma,\delta)$ if and only if*

$$\Big[\{(1+\delta)+(1-\zeta)(\gamma+\delta)\}(1-e^{-p})-\frac{(\gamma+\delta)}{p}(1-e^{-p}-pe^{-p})\Big] \leq \frac{(1-\gamma)}{2|\tau|(1-\delta)}. \tag{24}$$

Corollary 10. *Let $p > 0$, $\tau \in \mathbb{C}\setminus\{0\}, \delta < 1$. If $f \in \mathcal{R}_1^{\tau}(\delta)$, then $\Im(p,z)f \in \mathcal{T}S_p(\gamma,\delta)$ if and only if*

$$\Big[\{(1+\delta)+(\gamma+\delta)\}(1-e^{-p})-\frac{(\gamma+\delta)}{p}(1-e^{-p}-pe^{-p})\Big] \leq \frac{(1-\gamma)}{2|\tau|(1-\delta)}. \tag{25}$$

5 An Integral Operator

In this section, we define a particular integral operator $\mathcal{I}(p,z)$ as follows:

$$\mathcal{I}(p,z) = \int_0^z \frac{\chi(p,s)}{s}ds. \tag{26}$$

Theorem 7. *If $p > 0$, then $\mathcal{I}(p,z)$ defined by (26) is in $UC\mathcal{T}(\zeta,\gamma,\delta)$ if and only if*

$$pe^p[(1+\delta)+(1-\zeta)(\gamma+\delta)] \leq 1-\gamma. \tag{27}$$

Proof. It is easy to see that

$$\mathcal{I}(p,z) = z - \sum_{n=2}^{\infty} \frac{e^{-p}p^{n-1}}{n!} z^n, \tag{28}$$

In view of Theorem 1 it is sufficient to show that

$$\sum_{n=2}^{\infty} \Big[n(1+\delta) - (\gamma+\delta)(1+n\zeta-\zeta)\Big] \frac{p^{n-1}}{n!} e^{-p} \leq 1-\gamma.$$

Let

$$\begin{aligned}
\Omega_4(p,\zeta,\gamma,\delta) &= \sum_{n=2}^{\infty} \Big[n(1+\delta) - (\gamma+\delta)(1+n\zeta-\zeta)\Big] \frac{p^{n-1}}{(n-1)!} e^{-p} \\
&= e^{-p} \sum_{n=2}^{\infty} \Big[n(1+\delta) - (\gamma+\delta)(1+n\zeta-\zeta)\Big] \frac{p^{n-1}}{(n-1)!} \\
&= e^{-p} \left[\{(1+\delta) - \zeta(\gamma+\delta)\} \sum_{n=2}^{\infty} \frac{p^{n-1}}{(n-2)!} + (1-\gamma) \sum_{n=2}^{\infty} \frac{p^{n-1}}{(n-1)!}\right] \\
&= e^{-p} \Big[\{(1+\delta) - \zeta(\gamma+\delta)\} p e^{p} + (1-\gamma)(e^{p}-1)\Big] \\
&= [(1+\delta) - \zeta(\gamma+\delta)]p + (1-\gamma)(1-e^{-p}).
\end{aligned}$$

But the last expression is bounded by $1-\gamma$, if (27) holds. ☐

Theorem 8. *If $p > 0$, then $\mathcal{I}(p,z)$ defined by (26) is in $\mathcal{T}S_p(\zeta,\gamma,\delta)$ if and only if*

$$\left[\{(1+\delta) + (1-\zeta)(\gamma+\delta)\}(1-e^{-p}) - \frac{(\gamma+\delta)}{p}(1-e^{-p} - pe^{-p})\right] \leq (1-\gamma). \tag{29}$$

Proof. The proof of Theorem 8 is similar to the proof of Theorem 7, therefore we omit the details involved. ☐

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