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On invariant submanifolds of $\mathcal{S}$-manifolds

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Abstract. We consider invariant, pseudo-parallel and Ricci generalized pseudo-parallel submanifolds of $\mathcal{S}$-manifolds. We show that the submanifolds are totally geodesic under certain conditions. Also we study an invariant submanifold of $\mathcal{S}$-manifold satisfying $Q(\sigma, R) = 0$ and $Q(S, \sigma) = 0$, where S, R and σ are the Ricci tensor, curvature tensor and the second fundamental form respectively.

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1 Introduction

An n-dimensional submanifold M in an m-dimensional Riemannian manifold $\widetilde{M}$ is *pseudo-parallel* [1, 2] if its second fundamental form σ satisfies the following condition

$$\widetilde{R} \cdot \sigma = L\, Q(g, \sigma), \tag{1}$$

where $\widetilde{R}$ is the curvature operator with respect to the Van der Waerden–Bortolotti connection $\widetilde{\nabla}$ of $\widetilde{M}$, L is some smooth function on M and $Q(g, \sigma)$ is a $(0, 4)$-tensor on M determined by $Q(g, \sigma)(Z, W; X, Y) = ((X \wedge_g Y) \cdot \sigma)(Z, W)$.
Recall that the $(0, k + 2)$-tensor $Q(B, T)$ associated with any $(0, k)$-tensor field T, $k \geq 1$ and $(0, 2)$-tensor field B, is defined by

$$\begin{aligned} Q(B,T)(X_1, X_2, ..., X_k; X, Y) &= ((X \wedge_B Y) \cdot T)(X_1, X_2, ..., X_k) \\ &= -T((X \wedge_B Y)X_1, X_2, ..., X_k) - ... \\ &\quad -T(X_1, X_2, ..., X_{k-1}, (X \wedge_B Y)X_k), \end{aligned} \tag{2}$$

where $X \wedge_B Y$ is defined by

$$(X \wedge_B Y)Z = B(Y, Z)X - B(X, Z)Y. \tag{3}$$

In particular, if $L = 0$ in (1), M is called a *semi-parallel* submanifold [6].

Pseudo-parallel submanifolds were introduced in [1, 2] as naturel extension of semi-parallel submanifolds and as the extrinsic analogues of pseudo-symmetric Riemannian manifolds in the sense of Deszcz [7], which generalize semi-symmetric Riemannian manifolds.

On the other hand, Murathan et al. [13] defined submanifolds satisfying the condition

$$\widetilde{R}\,.\,\sigma = L\,Q(S,\sigma), \tag{4}$$

where S is the Ricci tensor of M.

The submanifolds satisfying the condition (4) are called *Ricci generalized pseudo-parallel*.

In [12] Kowalczyk studied semi-Riemannian manifold satisfying $Q(S,R) = 0$ and $Q(S,g) = 0$ where S and R are the Ricci tensor and curvature tensor respectively. Recently, many authors studied invariant submanifolds on various spaces, we refer, for example to [11, 14, 15, 18].

Motivated by the studies of the above authors, in this work we consider invariant, pseudo-parallel and Ricci generalized pseudo-parallel submanifolds of $\mathcal{S}$-manifolds. We show that these submanifolds are totally geodesic under certain conditions.

2 Basic concepts

We remember some necessary useful notions and results for our next considerations.

Let $\widetilde{M}^n$ be an n-dimensional Riemannian manifold and M^m an m-dimensional submanifold of $\widetilde{M}^n$. Let g be the metric tensor field on $\widetilde{M}^n$ as well as the metric induced on M^m. We denote by $\widetilde{\nabla}$ the covariant differentiation in $\widetilde{M}^n$ and by ∇ the covariant differentiation in M^m. Let $T\widetilde{M}$ (resp. TM) be the Lie algebra of vector fields on $\widetilde{M}^n$ (resp. on M^m) and $T^{\perp}M$ the set of all vector fields normal to M^m. The Gauss-Weingarten formulas are given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + \sigma(X,Y)\,,\;\; \widetilde{\nabla}_X V = -A_V X + \nabla^{\perp}_X V.$$

$X, Y \in TM$, $V \in T^{\perp}M$, where $\nabla^{\perp}$ is the connection in the normal bundle, σ is the second fundamental form of M^m and A_V is the Weingarten endomorphism associated with V. A_V and σ are related by $g(A_V X, Y) = g(\sigma(X,Y), V)$.
The submanifold M^m is said to be *totally geodesic* in $\widetilde{M}^n$ if its second fundamental form is identically zero and it is said to be *minimal* if $H \equiv 0$, where H is the mean curvature vector defined by $H = \frac{1}{m}\,trace(\sigma)$ [5]. From the definition it is clear that any totally geodesic submanifold is obviously a minimal submanifold.
We denote by $\widetilde{R}$, R and $R^{\perp}$ the curvature tensors associated with $\widetilde{\nabla}$, ∇ and $\nabla^{\perp}$ respectively.

The basic equations of Gauss and Ricci are

$$\begin{aligned} g(\widetilde{R}(X,Y)Z,W) &= g(R(X,Y)Z,W) + g(\sigma(X,Z),\sigma(Y,W)) \\ &\quad - g(\sigma(X,W),\sigma(Y,Z))\,, \end{aligned}$$

$$g(\widetilde{R}(X,Y)N,V) = g(R^{\perp}(X,Y)N,V) - g([A_N,A_V]X,Y)\,,$$

respectively, $X, Y, Z, W \in TM$, $N, V \in T^{\perp}M$.

The covariant derivative $\widetilde{\nabla}\sigma$ of the second fundamental form σ is given by

$$\widetilde{\nabla}_X \sigma(Y, Z) = \nabla^{\perp}_X(\sigma(Y, Z)) - \sigma(\nabla_X Y, Z) - \sigma(Y, \nabla_X Z),$$

where $\widetilde{\nabla}\sigma$ is a normal bundle-valued tensor of type $(0,3)$. If $\widetilde{\nabla}\sigma = 0$, then M is called *parallel* [8].

The operators $\widetilde{R}(X, Y)$ from the curvature of $\widetilde{\nabla}$ and $X \wedge Y$ can be extended as derivations of tensor fields in the usual way, so

$$\begin{aligned} (\widetilde{R}(X, Y) \,.\, \sigma)(Z, W) &= R^{\perp}(X, Y)(\sigma(Z, W)) - \sigma(R(X, Y)Z, W) \\ &\quad -\sigma(Z, R(X, Y)W), \end{aligned} \tag{5}$$

Putting in formulas (2) and (3) $T = \sigma$ and $B = g$ or $B = S$ respectively, we obtain the tensors $Q(g, \sigma)$ and $Q(S, \sigma)$

$$\begin{aligned} Q(g, \sigma)(Z, W; X, Y) &= ((X \wedge_g Y) \,.\, \sigma)(Z, W) \\ &= -\sigma((X \wedge_g Y)Z, W) - \sigma(Z, (X \wedge_g Y)W) \\ &= -g(Y, Z)\sigma(X, W) + g(X, Z)\sigma(Y, W) \\ &\quad -g(Y, W)\sigma(Z, X) + g(X, W)\sigma(Z, Y), \end{aligned} \tag{6}$$

and

$$\begin{aligned} Q(S, \sigma)(Z, W; X, Y) &= ((X \wedge_S Y) \,.\, \sigma)(Z, W) \\ &= -\sigma((X \wedge_S Y)Z, W) - \sigma(Z, (X \wedge_S Y)W) \\ &= -S(Y, Z)\sigma(X, W) + S(X, Z)\sigma(Y, W) \\ &\quad -S(Y, W)\sigma(Z, X) + S(X, W)\sigma(Z, Y). \end{aligned} \tag{7}$$

Now, let $\widetilde{M}^{2n+s}$ be a $(2n+s)$-dimensional Riemannian manifold endowed with an φ-structure (that is a tensor field of type $(1,1)$ and rank $2n$ satisfying $\varphi^3 + \varphi = 0$). If moreover there exist on $\widetilde{M}^{2n+s}$ global vector fields $\xi_1, ..., \xi_s$ (called structure vector fields), and their duals 1-forms $\eta_1, ..., \eta_s$ such that for all $X, Y \in T\widetilde{M}$ and $\alpha, \beta \in \{1, ..., s\}$ (see [3, 10])

$$\eta_\alpha(\xi_\beta) = \delta_{\alpha\beta}, \;\; \varphi\xi_\alpha = 0, \;\; \eta_\alpha(\varphi X) = 0, \;\; \varphi^2 X = -X + \sum_{\alpha=1}^{s} \eta_\alpha(X)\xi_\alpha\,, \tag{8}$$

then there exists on $\widetilde{M}$ a Riemannian metric g satisfying

$$g(X, Y) = g(\varphi X, \varphi Y) + \sum_{\alpha=1}^{s} \eta_\alpha(X)\eta_\alpha(Y), \tag{9}$$

and

$$\eta_\alpha(X) = g(X, \xi_\alpha), \;\; g(\varphi X, Y) = -g(X, \varphi Y), \tag{10}$$

for all $\alpha \in \{1, ..., s\}$. $\widetilde{M}$ is then said to be a metric φ-manifold. The φ-structure is normal if $N_\varphi + 2\sum_{\alpha=1}^{s} \xi_\alpha \otimes d\eta_\alpha = 0$ where N_φ is the Nijenhuis torsion of φ.
Let ϕ be the fundamental 2-form on M defined for all vector fields X, Y on $\widetilde{M}$ by

$$\phi(X, Y) = g(X, \varphi Y).$$

A normal metric φ-structure with closed fundamental 2-form will be called K-structure and $\widetilde{M}^{2n+s}$ called K-manifold. Finally if $d\eta_1 = ... = d\eta_s = \phi$, the K-structure is called $\mathcal{S}$*-structure* and $\widetilde{M}$ is called $\mathcal{S}$*-manifold*.

The Riemannian connection $\widetilde{\nabla}$ of an $\mathcal{S}$-manifold satisfies

$$\widetilde{\nabla}_X \xi_\alpha = -\varphi X,\ \alpha \in \{1, ..., s\},$$

$$(\widetilde{\nabla}_X \varphi)Y = \sum\nolimits_{\alpha=1}^{s} (g(\varphi X, \varphi Y)\xi_\alpha + \eta_\alpha(Y)\varphi^2 X), \quad X, Y \in T(\widetilde{M}).$$

Also in an $\mathcal{S}$-manifold the following relations hold [9]:

$$\widetilde{R}(X, Y)\xi_\alpha = (\sum_{\beta=1}^{s} \eta_\beta(X))\varphi^2 Y - (\sum_{\beta=1}^{s} \eta_\beta(Y))\varphi^2 X\,, \tag{11}$$

$$S(X, \xi_\alpha) = 2n \sum_{\beta=1}^{s} \eta_\beta(X), \tag{12}$$

for all $\alpha \in \{1, ..., s\}$.

When $s = 1$, an $\mathcal{S}$-manifold reduces to a Sasakian manifold.

Let $\widetilde{M}$ be a $(2n+s)$-dimensional $\mathcal{S}$-manifold with structure tensors $(\varphi, \xi_\alpha, \eta_\alpha, g)$, $\alpha \in \{1, ..., s\}$. A $(2m + s)$-dimensional submanifold M of $\widetilde{M}$ is said to be *invariant* if all of ξ_α $(\alpha = 1, ..., s)$ are always tangent to M and $\varphi X \in TM$, for any $X \in TM$. It is easy to show that an invariant submanifold of an $\mathcal{S}$-manifold is an $\mathcal{S}$-manifold too.

Lemma 1 (see [4])**.** *Let M be an invariant submanifold of an $\mathcal{S}$-manifold $\widetilde{M}$. Then, for any $X, Y \in TM$, $\alpha \in \{1, ..., s\}$*

$$\sigma(X, \xi_\alpha) = 0\,, \tag{13}$$

$$\sigma(X, \varphi Y) = \varphi\sigma(X, Y) = \sigma(\varphi X, Y)\,. \tag{14}$$

Proposition 1 (see [10])**.** *Any invariant submanifold M of an $\mathcal{S}$-manifold $\widetilde{M}$ is minimal.*

Theorem 1 (see [16])**.** *Let M be an invariant submanifold of an $\mathcal{S}$-manifold $\widetilde{M}$. Then M is parallel if and only if M is totally geodesic.*

3 Pseudo-parallel invariant submanifolds

In this section we study pseudo-parallel (resp. Ricci generalized pseudo-parallel) invariant submanifolds of $\mathcal{S}$-manifolds.

Theorem 2. *Let M be an invariant submanifold of an $\mathcal{S}$-manifold. Then, M is pseudo-parallel if and only if M is totally geodesic, provided $L \neq 1$.*

Proof. Suppose that M is pseudo-parallel, then $\widetilde{R}\,.\,\sigma = L\,Q(g,\sigma)$ holds on M. So, from (5) and (6) we get

$$\begin{aligned} R^{\perp}(X,Y)\sigma(Z,W) \;&-\; \sigma(R(X,Y)Z,W) - \sigma(Z,R(X,Y)W) \\ &=\; L\,\{-g(Y,Z)\sigma(X,W) + g(X,Z)\sigma(Y,W) \\ &\qquad -g(Y,W)\sigma(X,Z) + g(X,W)\sigma(Y,Z)\}\,, \end{aligned} \tag{15}$$

for all $X,Y,Z,W \in TM$.

Putting $X = \xi_\alpha$ in (15) and from (13), for any $\alpha \in \{1,...,s\}$ we have

$$\begin{aligned} &R^{\perp}(\xi_\alpha,Y)\sigma(Z,W) - \sigma(R(\xi_\alpha,Y)Z,W) - \sigma(Z,R(\xi_\alpha,Y)W) \\ &= L\{g(\xi_\alpha,Z)\sigma(Y,W) + g(\xi_\alpha,W)\sigma(Y,Z)\}\,. \end{aligned} \tag{16}$$

Again putting $W = \xi_\alpha$ in (16), we have

$$-\sigma(Z,R(\xi_\alpha,Y)\xi_\alpha) = L\sigma(Y,Z). \tag{17}$$

Since M is an $\mathcal{S}$-manifold, then from (11) we get

$$\widetilde{R}(\xi_\alpha,Y)\xi_\alpha = \varphi^2 Y\,. \tag{18}$$

Substituting (18) in (17) and from (14), (8) we have

$$(1-L)\sigma(Z,Y) = 0\,,$$

i. e. $\sigma(Z,Y) = 0$, which gives M is totally geodesic, provided $L \neq 1$. If $\sigma = 0$, then it can be trivially proved that M is pseudo-parallel. $\square$

As a direct consequence of Theorem 2, we get the following

Corollary 1. *Let M be an invariant submanifold of an $\mathcal{S}$-manifold. Then, M is semi-parallel if and only if M is totally geodesic.*

Theorem 3. *Let M be an invariant submanifold of an $\mathcal{S}$-manifold. Then, M is Ricci generalized pseudo-parallel if and only if M is totally geodesic, provided $L \neq \frac{1}{2m}$.*

Proof. Let M be a $(2m+s)$-dimensional invariant Ricci generalized pseudo-parallel submanifold of an $\mathcal{S}$-manifold. Therefore

$$\widetilde{R}\,.\,\sigma = L\,Q(S,\sigma)\,,$$

for all vector fields X, Y, Z, W tangent to M, where L denotes the real-valued function on M. The above equation can written as

$$\begin{aligned} R^{\perp}(X,Y)\sigma(Z,W) &- \sigma(R(X,Y)Z,W) - \sigma(Z,R(X,Y)W) \\ &= L\{-S(Y,Z)\sigma(X,W) + S(X,Z)\sigma(Y,W) \\ &\quad -S(Y,W)\sigma(X,Z) + S(X,W)\sigma(Y,Z)\}\,. \end{aligned} \tag{19}$$

Putting $W = X = \xi_\alpha$, for any $\alpha \in \{1, ..., s\}$ in (19), we obtain

$$-\sigma(Z, \sum_{\beta=1}^{s} \eta_\beta(\xi_\alpha)\varphi^2 Y - \sum_{\beta=1}^{s} \eta_\beta(Y)\varphi^2\xi_\alpha) = L\,S(\xi_\alpha,\xi_\alpha)\sigma(Y,Z)\,.$$

This implies

$$-\sigma(Z,\varphi^2 Y) = L\,S(\xi_\alpha,\xi_\alpha)\sigma(Y,Z)\,. \tag{20}$$

Since M is an $\mathcal{S}$-manifold, by the use of (12), (8) and (14), we have

$$\sigma(Y,Z) = 2m\,L\,\sigma(Y,Z)\,.$$

So,

$$(1-2m\,L)\,\sigma(Y,Z) = 0\,.$$

Conversely, let M be totally geodesic, i.e. $\sigma = 0$, then from (19) we get M is Ricci generalized pseudo-parallel. □

4 Invariant submanifolds satisfying some conditions

Recall that the $(0,4)$ Riemann-Christoffel curvature tensor R of (M,g) is related to the $(1,3)$-tensor R by $R(X,Y,Z,W) = g(R(X,Y)Z,W)$, it possesses the following property [17]:

$$R(X,Y,Z,Z) = 0,$$

for any vector fields X, Y, Z and W.

Let us consider that M be an invariant $(2m+s)$-dimensional submanifold of an $(2n+s)$-dimensional $\mathcal{S}$-manifold $\widetilde{M}$.

Theorem 4. *An invariant submanifold of an $\mathcal{S}$-manifold satisfies $Q(\sigma,R) = 0$ if and only if it is totally geodesic.*

Proof. If M satisfies $Q(\sigma,R) = 0$, then we have

$$0 = Q(\sigma,R)(U_1,U_2,U_3,U_4;X,Y)$$

$$\begin{aligned} &= ((X \wedge_\sigma Y) \,.\, R)(U_1, U_2, U_3, U_4) \\ &= -R((X \wedge_\sigma Y)U_1, U_2, U_3, U_4) - R(U_1, (X \wedge_\sigma Y)U_2, U_3, U_4) \\ &\quad -R(U_1, U_2, (X \wedge_\sigma Y)U_3, U_4) - R(U_1, U_2, U_3, (X \wedge_\sigma Y)U_4), \end{aligned}$$

for all vector fields X, Y, U_i $(i = 1, ..., 4)$ tangent to M.

Putting in formula (2) $B = \sigma$ and $T = R$, the above equation turns into

$$\begin{aligned} 0 \;=\; & -\sigma(Y, U_1)R(X, U_2, U_3, U_4) + \sigma(X, U_1)R(Y, U_2, U_3, U_4) \\ & -\sigma(Y, U_2)R(U_1, X, U_3, U_4) + \sigma(X, U_2)R(U_1, Y, U_3, U_4) \\ & -\sigma(Y, U_3)R(U_1, U_2, X, U_4) + \sigma(X, U_3)R(U_1, U_2, Y, U_4) \\ & -\sigma(Y, U_4)R(U_1, U_2, U_3, X) + \sigma(X, U_4)R(U_1, U_2, U_3, Y). \end{aligned} \tag{21}$$

Putting $U_3 = Y = \xi_\alpha$, $\alpha \in \{1, ..., s\}$ in (21) we have

$$\sigma(X, U_1)R(\xi_\alpha, U_2, \xi_\alpha, U_4) + \sigma(X, U_2)R(U_1, \xi_\alpha, \xi_\alpha, U_4) = 0, \tag{22}$$

by the use of (11), (22) becomes

$$\sigma(X, U_1)g(\varphi^2 U_2, U_4) - \sigma(X, U_2)g(\varphi^2 U_1, U_4) = 0.$$

This implies

$$\begin{aligned} & -\sigma(X, U_1)g(U_2, U_4) + \sigma(X, U_1)\sum_{\beta=1}^{s} \eta_\beta(U_2)\eta_\beta(U_4) \\ & +\sigma(X, U_2)g(U_1, U_4) - \sigma(X, U_2)\sum_{\beta=1}^{s} \eta_\beta(U_1)\eta_\beta(U_4) = 0, \end{aligned} \tag{23}$$

We consider $\{e_1, ..., e_m, e_{m+1} = \varphi e_1, ..., e_{2m} = \varphi e_m, e_{2m+1} = \xi_1, ..., e_{2m+s} = \xi_s\}$ a local orthonormal frame of TM. We insert $U_4 = U_2 = e_k$ in (23) and taking summation over $k = 1, ..., 2m + s$, we get

$$-(2m + s)\sigma(X, U_1) + s\sigma(X, U_1) + \sigma(X, U_1) = 0,$$

which implies

$$(-2m + 1)\sigma(X, U_1) = 0.$$

Hence, M is totally geodesic. Conversely, if $\sigma = 0$, then from (21) it follows that $Q(\sigma, R) = 0$. □

Theorem 5. *An invariant submanifold of an $\mathcal{S}$-manifold satisfies $Q(S, \sigma) = 0$ if and only if it is totally geodesic.*

Proof. If $Q(S, \sigma) = 0$, and from (7) we get

$$\begin{aligned} 0 &= -S(Y, Z)\sigma(X, W) + S(X, Z)\sigma(Y, W) \\ &= -S(Y, W)\sigma(Z, X) + S(X, W)\sigma(Z, Y). \end{aligned}$$

Putting $X = W = \xi_\alpha$ in the above equation, for any $\alpha \in \{1, ..., s\}$ we have

$$S(\xi_\alpha, \xi_\alpha)\sigma(Y, Z) = 0.$$

By (12), we have

$$2m\,\sigma(Y, Z) = 0.$$

Conversely, let M be totally geodesic, taking account of (7), we get $Q(S, \sigma) = 0$. $\square$

In view of our results and Theorem 1 , we can state the following Corollary:

Corollary 2. *Let M be an invariant $(2m + s)$-dimensional submanifold of an $\mathcal{S}$-manifold. Then the following assertions are equivalent:*

1. *M is parallel.*
2. *M is semi-parallel.*
3. *M is pseudo-parallel, provided $L \neq 1$.*
4. *M is Ricci generalized pseudo-parallel, provided $L \neq \frac{1}{2m}$.*
5. *M satisfies the condition $Q(\sigma, R) = 0$.*
6. *M satisfies the condition $Q(S, \sigma) = 0$.*
7. *M is totally geodesic.*

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