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A Note on 2-Hypersurfaces of the Nearly Kählerian Six-Sphere

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Abstract. It is proved that hypersurfaces with type number two in a nearly Kählerian sphere S^6 admit almost contact metric structures of cosymplectic type that are non-cosymplectic.

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1 Introduction

The six-dimensional sphere S^6 with a canonical nearly Kählerian structure was the first example of non-Kählerian almost Hermitian manifold. That is why it presents a special interest for researchers in the area of Hermitian geometry. Such outstanding geometers as A. Gray, V. F. Kirichenko, K. Sekigawa and N. Ejiri have studied diverse aspects of the geometry of nearly Kählerian six-dimensional sphere. Of course, the geometry of nearly Kählerian manifolds (or W_1-manifolds, after Gray–Hervella classification [14]) is a spacious and important part of Hermitian geometry.

It is known that almost contact metric structures are induced on oriented hypersurfaces of almost Hermitian manifolds. Many specialists observe that this fact determines the profound connection between the contact and Hermitian geometries. Almost contact metric structures on hypersurfaces of almost Hermitian manifolds were studied by some remarkable geometers. The work of D. E. Blair, S. Goldberg, S. Ishihara, S. Sasaki, H. Yanamoto and K. Yano are assumed classical. In the present note, almost contact metric structures on 2-hypersurfaces (i.e. on hypersurfaces with type number 2) of nearly Kählerian six-dimensional sphere are considered.

In [3] and [11], it was proved that if t is the type number of an oriented hypersurface of the nearly Kählerian six-sphere S^6, then the condition $t \leq 1$ holds if and only if the induced almost contact metric structure on this hypersurface is nearly cosymplectic.

In this paper, an additional result on almost contact metric hypersurfaces of nearly Kählerian six-dimensional sphere is given. Namely, we shall show that 2-type hypersurfaces of a nearly Kählerian six-sphere S^6 admit almost contact metric structures of cosymplectic type.

2 Preliminaries

The almost Hermitian manifold is an even-dimensional manifold M^{2n} with a Riemannian metric $g = \langle\cdot,\,\cdot\rangle$ and an almost complex structure J. These objects must satisfy the following condition

$$\langle JX,\ JY\rangle = \langle X,\ Y\rangle\,, \quad X, Y \in \aleph(M^{2n}),$$

where $\aleph(M^{2n})$ is the module of smooth vector fields on M^{2n}. All considered manifolds, tensor fields and similar objects are assumed to be of the class C^{∞}.

The specification of an almost Hermitian structure on a manifold is equivalent to the setting of a G-structure, where G is the unitary group $U(n)$ [5], [15]. Its elements are the frames adapted to the structure (A-frames). They look as follows:

$$(p,\,\varepsilon_1,\,\ldots,\varepsilon_n,\,\varepsilon_{\hat{1}},\,\ldots,\,\varepsilon_{\hat{n}}\,),$$

where ε_a are the eigenvectors corresponded to the eigenvalue $i = \sqrt{-1}$, and $\varepsilon_{\hat{a}}$ are the eigenvectors corresponded to the eigenvalue $-i$. Here the index a ranges from 1 to n, and we state $\hat{a} = a + n$.

The matrixes of the operator of the almost complex structure and of the Riemannian metric written in an A-frame look as follows, respectively:

$$\left(J_j^k\right) = \left(\begin{array}{c|c} iI_n & 0 \\ \hline 0 & -iI_n \end{array}\right), \quad (g_{kj}) = \left(\begin{array}{c|c} 0 & I_n \\ \hline I_n & 0 \end{array}\right),$$

where I_n is the identity matrix; $k, j = 1, ..., 2n$.

We recall [16] that the fundamental form of an almost Hermitian manifold is determined by the relation

$$F(X,\ Y) = \langle X,\ JY\rangle\,, \quad X, Y \in \aleph(M^{2n}).$$

By direct computing it is easy to obtain that in an A-frame the fundamental form matrix looks as follows:

$$(F_{kj}) = \left(\begin{array}{c|c} 0 & iI_n \\ \hline -iI_n & 0 \end{array}\right).$$

The first group of the Cartan structural equations of an almost Hermitian manifold written in an A-frame looks as follows [9], [16]:

$$d\,\omega^a = \omega_b^a \wedge \omega^b + B^{ab}{}_c\,\omega^c \wedge \omega_b + B^{abc}\,\omega_b \wedge \omega_c\,; \tag{1}$$

$$d\,\omega_a = -\omega_a^b \wedge \omega_b + B_{ab}{}^c\,\omega_c \wedge \omega^b + B_{abc}\,\omega^b \wedge \omega^c\,,$$

where

$$B^{ab}{}_c = -\frac{i}{2}\,J^a_{\hat{b},\,c};\ B_{ab}{}^c = \frac{i}{2}\,J^{\hat{a}}_{b,\,\hat{c}};$$

$$B^{abc} = \frac{i}{2}\,J^a_{[\hat{b},\,\hat{c}]};\ B_{abc} = -\frac{i}{2}\,J^{\hat{a}}_{[b,\,c]}.$$

The systems of functions $\{\ B^{ab}{}_{c}\ \}$, $\{\ B_{ab}{}^{c}\ \}$, $\{\ B^{abc}\ \}$, $\{\ B_{abc}\ \}$, are the components of the Kirichenko tensors of the almost Hermitian manifold [2], [9], $a, b, c = 1, ..., n;\ ,\ \hat{a} = a + n$.

An almost Hermitian manifold is called nearly Kählerian [14], [16] if

$$\nabla_X\ (F)\ (X, Y)\ =\ 0, \quad X, Y \in \aleph(M^{2n}).$$

The almost contact metric structure on an odd-dimensional manifold N is defined by the system of tensor fields $\{\Phi,\ \xi,\ \eta,\ g\}$ on this manifold, where ξ is a vector field, η is a covector field, Φ is a tensor of the type (1, 1) and $g = \langle\cdot,\ \cdot\rangle$ is the Riemannian metric [12],[16]. Moreover, the following conditions are fulfilled:

$$\eta(\xi) = 1,\ \Phi(\xi) = 0,\ \eta\circ\Phi = 0,\ \Phi^2 = -id + \xi\otimes\eta,$$

$$\langle\Phi X, \Phi Y\rangle = \langle\Phi X, \Phi Y\rangle - \eta\,(X)\,\eta\,(Y)\,,\ X, Y \in \aleph(N),$$

where $\aleph(N)$ is the module of smooth vector fields on N. As examples of almost contact metric structures we can consider the cosymplectic structure, the nearly cosymplectic structure, the Sasakian structure and the Kenmotsu structure.

The cosymplectic structure that is characterized by the following condition:

$$\nabla\eta = 0, \quad \nabla\Phi = 0,$$

where ∇ is the Levi-Civita connection of the metric. It has been proved that the manifold, admitting the cosymplectic structure, is locally equivalent to the product $M\times R$, where M is a Kählerian manifold [15].

An almost contact metric structure $\{\Phi,\ \xi,\ \eta,\ g\}$ is called nearly cosymplectic if the following condition is fulfilled [16], [19]:

$$\nabla_X(\Phi)\,Y\ + \nabla_Y(\Phi)\,X = 0,\ X, Y \in \aleph(N).$$

We note that the nearly cosymplectic structures have many remarkable properties and play an important role in contact geometry. We mark out a number of articles by H. Endo on the geometry of nearly cosymplectic manifolds as well as the fundamental research by E. V. Kusova on this subject [19].

Is it known if $(N, \{\Phi,\ \xi,\ \eta,\ g\})$ is an almost contact metric manifold, then an almost Hermitian structure is induced on the product $N\times R$ [12], [21]. If this almost Hermitian structure is integrable, then the input almost contact metric structure is called normal. A normal contact metric structure is called Sasakian [16]. On the other hand, we can characterize the Sasakian structure by the following condition:

$$\nabla_X(\Phi)Y = \langle X, Y\rangle\,\xi - \eta(Y)X,\ \ X, Y \in \aleph(N).$$

For example, Sasakian structures are induced on totally umbilical hypersurfaces in a Kählerian manifold [21]. As it is well known, the Sasakian structures have many remarkable properties and play a fundamental role in contact geometry.

In 1972 Katsuei Kenmotsu introduced a new class of almost contact metric structures, defined by the condition:

$$\nabla_X(\Phi)Y = \langle \Phi X, Y\rangle\, \xi - \eta(Y)\Phi X,\ X, Y \in \aleph(N).$$

The Kenmotsu manifolds are normal and integrable, but they are not contact manifolds. We mark out that the fundamental monograph by Gh. Pitiş [20] contains a detailed description of Kenmotsu manifolds and their generalizations and a set of important results on this subject.

At the end of this section, note that when we give a Riemannian manifold and its submanifold (in particular, its hypersurface), the rank of determined second fundamental form is called the type number [18].

3 The main result

Let us use the first group of Cartan structural equations of an almost contact metric structure on an oriented hypersurface N^{2n-1} of an almost Hermitian manifold M^{2n} [6], [21]:

$$\begin{aligned}
d\omega^a &= \omega^a_b \wedge \omega^b + B^{ab}{}_c\, \omega^c \wedge \omega_b + B^{abc}\, \omega_b \wedge \omega_c + \\
&+ \left(\sqrt{2}\, B^{an}{}_b + i\sigma^a_b\right) \omega^b \wedge \omega + \left(-\sqrt{2}\, \tilde{B}^{nab} - \frac{1}{\sqrt{2}} B^{ab}{}_n - \frac{1}{\sqrt{2}} \tilde{B}^{abn} + i\, \sigma^{ab}\right) \omega_b \wedge \omega; \\
d\omega_a &= -\omega^b_a \wedge \omega_b + B_{ab}{}^c\, \omega_c \wedge \omega^b + B_{abc}\, \omega^b \wedge \omega^c + \\
&+ \left(\sqrt{2}\, B_{an}{}^b - i\sigma^b_a\right) \omega_b \wedge \omega + \left(-\sqrt{2}\, \tilde{B}_{nab} - \frac{1}{\sqrt{2}} \tilde{B}_{abn} - \frac{1}{\sqrt{2}} B_{ab}{}^n - i\, \sigma_{ab}\right) \omega^b \wedge \omega; \\
d\omega &= \sqrt{2}\, B_{nab}\, \omega^a \wedge \omega^b + \sqrt{2}\, B^{nab}\, \omega_a \wedge \omega_b + \\
&+ \left(\sqrt{2}\, B^{na}{}_b - \sqrt{2}\, B_{nb}{}^a - 2i\, \sigma^a_b\right) \omega^b \wedge \omega_a + \\
&+ \left(\tilde{B}_{nbn} + B_{nb}{}^n + i\, \sigma_{nb}\right) \omega \wedge \omega^b + \left(\tilde{B}^{nbn} + B^{nb}{}_n - i\, \sigma^b_n\right) \omega \wedge \omega_b,
\end{aligned} \tag{2}$$

where

$$\tilde{B}^{abc} = \frac{i}{2}\, J^a_{\hat{b},\, \hat{c}};\ \tilde{B}_{abc} = -\frac{i}{2}\, J^{\hat{a}}_{b,\, c}$$

and σ is the second fundamental form of the immersion of N into M^{2n}. We also use the detailed structural equations (1) of a six-dimensional almost Hermitian submanifold of Cayley algebra [5], [6], [7]:

$$\begin{aligned}
d\omega^a &= \omega^a_b \wedge \omega^b + \frac{1}{\sqrt{2}} \varepsilon^{abh} D_{hc} \omega^c \wedge \omega_b + \frac{1}{\sqrt{2}} \varepsilon^{ah[b} D_h{}^{c]} \omega_b \wedge \omega_c; \\
d\omega_a &= -\omega^b_a \wedge \omega_b + \frac{1}{\sqrt{2}} \varepsilon_{abh} D^{hc} \omega_c \wedge \omega^b + \frac{1}{\sqrt{2}} \varepsilon_{ah[b} D^h{}_{c]} \omega^b \wedge \omega^c;
\end{aligned} \tag{3}$$

Here $\varepsilon_{abc} = \varepsilon^{123}_{abc}$, $\varepsilon^{abc} = \varepsilon^{abc}_{123}$ are the components of the third-order Kronecher tensor;

$$D^{hc} = D_{\hat{h}\hat{c}}, \quad D_h{}^c = D_{h\hat{c}}, \quad D^h{}_c = D_{\hat{h}c};$$

$$D_{cj} = \mp T^8_{cj} + iT^7_{cj}, \quad D_{\hat{c}j} = \mp T^8_{\hat{c}j} - iT^7_{\hat{c}j},$$

where $\left\{T^{\varphi}_{kj}\right\}$ are the components of the configuration tensor (in Gray–Kirichenko notation [13], [15]); $\varphi = 7, 8$ $a, b, c, d, g, h = 1, 2, 3$; $\hat{a} = a + 3$; $k, j = 1, 2, 3, 4, 5, 6$. Comparing these equations with (1), we get the expressions for the Kirichenko tensors of six-dimensional almost Hermitian submanifolds of Cayley algebra (in particular, for the nearly Kählerian six-dimensional sphere S^6):

$$B^{ab}{}_c = \frac{1}{\sqrt{2}}\varepsilon^{abh} D_{hc}\,; \;\; B_{ab}{}^c = \frac{1}{\sqrt{2}}\varepsilon_{abh} D^{hc}\,;$$

$$B^{abc} = \frac{1}{\sqrt{2}}\varepsilon^{ah[b} D_h{}^{c]}\,; \;\; B_{abc} = \frac{1}{\sqrt{2}}\varepsilon_{ah[b} D^h{}_{c]}\,.$$

Knowing that the Kirichenko tensors $B^{ab}{}_c$ and $B_{ab}{}^c$ of the nearly Kählerian six-sphere vanish [8], we rewrite these structural equations as follows:

$$\begin{gathered}
d\omega^{\alpha} = \omega^{\alpha}_{\beta} \wedge \omega^{\beta} \; + B^{\alpha\beta\gamma}\,\omega_{\beta} \wedge \omega_{\gamma} + \\
+ i\sigma^{\alpha}_{\beta}\,\omega^{\beta} \wedge \omega \,+\, (-\sqrt{2}\,\tilde{B}^{n\alpha\beta} \,-\, \frac{1}{\sqrt{2}}\tilde{B}^{\alpha\beta n} + i\sigma^{\alpha\beta})\,\omega_{\beta} \wedge \omega; \\
d\omega_{\alpha} \;=\; -\omega^{\beta}_{\alpha} \wedge \omega_{\beta} \;\; + B_{\alpha\beta\gamma}\omega^{\beta} \wedge \omega^{\gamma} - \\
- i\sigma^{\beta}_{\alpha}\,\omega_{\beta} \wedge \omega + \left(-\sqrt{2}\,\tilde{B}_{n\alpha\beta} - \frac{1}{\sqrt{2}}\tilde{B}_{\alpha\beta n} - \, i\,\sigma_{\alpha\beta}\right)\,\omega^{\beta} \wedge \omega; \\
d\omega \;=\; \sqrt{2}\,B_{n\alpha\beta}\,\omega^{\alpha} \wedge \omega^{\beta} \,+\, \sqrt{2}\,B^{n\alpha\beta}\,\omega_{\alpha} \wedge \omega_{\beta} - \\
- 2i\,\sigma^{\alpha}_{\beta}\,\omega^{\beta} \,\wedge\, \omega_{\alpha} \,+\, \left(\tilde{B}_{n\beta n} \,+\, i\,\sigma_{n\beta}\right)\,\omega \wedge \omega^{\beta} \,+\, \left(\tilde{B}^{n\beta n} \,-\, i\,\sigma^{\beta}_{n}\right)\,\omega \wedge \omega_{\beta}.
\end{gathered} \tag{4}$$

On the other hand, we obtain the more precise structural equations of the nearly Kählerian structure on the six-sphere [9]:

$$d\omega^a = \omega^a_b \wedge \omega^b + \mu\,\varepsilon^{acb}\omega_b \wedge \omega_c;$$

$$d\omega_a = -\omega^b_a \wedge \omega_b + \bar{\mu}\,\varepsilon_{acb}\omega^b \wedge \omega^c.$$

If an almost contact metric hypersurface of a nearly Kählerian manifold is of type number two, then we get the simplest matrix of its second fundamental form:

$$(\sigma_{ps}) = \left(\begin{array}{c|c|c}
 & 0 & \\
(\sigma_{\alpha\beta}) & \ldots & 0 \\
 & 0 & \\
\hline
0\ldots 0 & 0 & 0\ldots 0 \\
\hline
 & 0 & \\
0 & \ldots & (\sigma_{\bar{\alpha}\hat{\beta}}) \\
 & 0 &
\end{array}\right), \quad p, s = 1,\,2,\,3,\,4,\,5\;,$$

and what is more

$$rank\,(\sigma_{\alpha\beta}) = rank\,(\sigma_{\hat{\alpha}\hat{\beta}}) = 1.$$

That is why from (4) and (5) we obtain the following Cartan structural equations of an almost contact metric structure on an oriented 2-hypersurface of nearly Kählerian six-sphere:

$$d\,\omega^{\alpha} = \omega^{\alpha}_{\beta} \wedge \omega^{\beta} + B^{\alpha\beta\gamma}\,\omega_{\beta} \wedge \omega_{\gamma} + (-\sqrt{2}\,\tilde{B}^{n\alpha\beta} - \frac{1}{\sqrt{2}}\tilde{B}^{\alpha\beta n} + i\,\sigma^{\alpha\beta})\,\omega_{\beta} \wedge \omega;$$

$$d\,\omega_{\alpha} = -\omega^{\beta}_{\alpha} \wedge \omega_{\beta} + B_{\alpha\beta\gamma}\,\omega^{\beta} \wedge \omega^{\gamma} + (-\sqrt{2}\,\tilde{B}_{n\alpha\beta} - \frac{1}{\sqrt{2}}\tilde{B}_{\alpha\beta n} - i\,\sigma_{a\beta})\,\omega^{\beta} \wedge \omega; \quad (6)$$

$$d\,\omega = 0\,.$$

In [17], V. F. Kirichenko and I. V. Uskorev have introduced a new class of almost contact metric structure. Namely, they have defined the almost contact metric structure with the close contact form as the structures of cosymplectic type. As they have established, the condition

$$d\,\omega = 0$$

is necessary and sufficient for an almost contact metric structure to be of cosymplectic type. V. F. Kirichenko and I. V. Uskorev have also proved that the structure of cosymplectic type is invariant under canonical conformal transformations [17]. We recall also that a conformal transformation of an almost contact metric structure $\{\Phi, \xi, \eta, g\}$ on the manifold N is a transition to the almost contact metric structure $\left\{\tilde{\Phi}, \tilde{\xi}, \tilde{\eta}, \tilde{g}\right\}$, where $\tilde{\Phi} = \Phi$, $\tilde{\xi} = e^{f}\xi$, $\tilde{\eta} = e^{-f}\,\eta$ and $\tilde{g} = e^{-2f}\,g$. Here f is a smooth function on the manifold N [16], [21].

Evidently, a trivial example of structure of cosymplectic type is the cosymplectic structure with well-known Cartan structural equations [7], [16]:

$$d\omega^{\alpha} = \omega^{\alpha}_{\beta} \wedge \omega^{\beta},$$

$$d\omega_{\alpha} = -\omega^{\beta}_{\alpha} \wedge \omega_{\beta},$$

$$d\omega = 0\,.$$

Another important example of the almost contact metric structure of cosymplectic type is the Kenmotsu structure with following Cartan structural equations [1], [16]:

$$d\omega^{\alpha} = \omega^{\alpha}_{\beta} \wedge \omega^{\beta} + \omega \wedge \omega^{\alpha},$$

$$d\omega_{\alpha} = -\omega^{\beta}_{\alpha} \wedge \omega_{\beta} + \omega \wedge \omega_{\alpha},$$

$$d\omega = 0\,.$$

It is easy to see that the structural equations (6) perfectly correspond to the structure of cosymplectic type, but this almost contact metric structure is not cosymplectic or Kenmotsu. So, we have proved the following result.

Theorem 1. *Hypersurfaces with type number two in a nearly Kählerian six-sphere admit non-cosymplectic and non-Kenmotsu almost contact metric structures of cosymplectic type.*

4 Some comments

As we have mentioned, in [3] and [11] it was proved that the almost contact metric structure on a totally geodesic or on 1-type hypersurface in a nearly Kählerian six-sphere must be nearly cosymplectic. We remark that the nearly cosymplectic structure is not of cosymplectic type because its Cartan structural equations look as follows [4],[19]:

$$d\,\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta + H^{\alpha\beta\gamma}\,\omega_\beta \wedge \omega_\gamma + H^{\alpha\beta}\,\omega_\beta \wedge \omega\,;$$

$$d\,\omega_\alpha = -\omega^\beta_\alpha \wedge \omega_\beta + H_{\alpha\beta\gamma}\,\omega^\beta \wedge \omega^\gamma + H_{\alpha\beta}\,\omega^\beta \wedge \omega\,;$$

$$d\,\omega = -\frac{2}{3}G_{\alpha\beta}\,\omega^\alpha \wedge \omega^\beta - \frac{2}{3}\,G^{\alpha\beta}\,\omega_\alpha \wedge \omega_\beta\,.$$

On the other hand, in [10], it has been proved that 2-hypersurfaces in an arbitrary Kählerian manifold also admit non-cosymplectic and non-Kenmotsu almost contact metric structures of cosymplectic type. Taking into account that the class of nearly Kählerian manifolds is situated "between" the classes of Kählerian and quasi-Kählerian manifolds [14], we can pose an open problem.

Problem. Find a characterization of the almost contact metric structure on a 2-type hypersurface in a quasi-Kählerian manifold. In particular, can the almost contact metric structure on such a hypersurface be of cosymplectic type?

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References

[1] Abu-Saleem A., Banaru M. *Two theorems on Kenmotsu hypersurfaces in a W_3-manifold* Studia Univ. Babeş–Bolyai. Math., 2005, **51**, No. 3, 3–11.

[2] Abu-Saleem A., Banaru M. *Some applications of Kirichenko tensors*, Analele Univ. Oradea, Fasc. Mat., 2010, **17**, No. 2, 201–208.

[3] Abu-Saleem A., Banaru M. *On almost contact metric hypersurfaces of nearly Kählerian 6-sphere*, Malaysian Journal of Mathematical Sciences, 2014, **8**, No. 1, 35–46.

[4] Banaru M. *On nearly-cosymplectic hypersurfaces in nearly-Kählerian manifolds*, Studia Univ. Babeş–Bolyai. Math., 2002, **47**, No. 3, 3–11.

[5] Banaru M. *Hermitian geometry of 6-dimensional submanifolds of the Cayley algebra*, Sbornik Mathematics, 2002, **193**, No. 5, 635–648.

[6] Banaru M. *Sasakian hypersurfaces of six-dimensional Hermitian manifolds of the Cayley algebra*, Sbornik Mathematics, 2003, **194**, No. 8, 1125–1136.

[7] Banaru M. *The type number of the cosymplectic hypersurfaces of 6-dimensional Hermitian submanifolds of the Cayley algebra*, Siberian Math. J., 2003, **44**, No. 5, 765–773.

[8] Banaru M. *On the Gray-Hervella classes of AH-structures on six-dimensional submanifolds of Cayley algebra*, Annuaire de l'universite de Sofia "St. Kl. Ohridski", 2004, **95**, 125–131.

[9] Banaru M. *Geometry of 6-dimensional Hermitian manifolds of the octave algebra*, Journal of Mathematical Sciences (New York), 2015, **207**, No. 3, 354–388.

[10] Banaru M. *On almost contact metric 2-hypersurfaces in Kählerian manifolds*, Bulletin of the Transilvania University of Braşov. Series III. Mathematics, Informatics, Physics, 2016, **9(58)**, No. 1, 1–10.

[11] Banaru M., Banaru G. *A note on almost contact metric hypersurfaces of nearly Kählerian 6-sphere*, Bulletin of the Transilvania University of Braşov. Series III. Mathematics, Informatics, Physics, 2015, **8(57)**, No. 2, 21–28.

[12] Blair D. E. *Riemannian geometry of contact and symplectic manifolds*, Progress in Math. Birkhauser. Boston-Basel-Berlin, 2002.

[13] Gray A. *Six-dimensional almost complex manifolds defined by means of three-fold vector cross products*, Tôhoku Math. J., 1969, **21**, No. 4, 614–620.

[14] Gray A., Hervella L. M. *The sixteen classes of almost Hermitian manifolds and their linear invariants*, Ann. Mat. Pura Appl., 1980, **123**, No. 4, 35–58.

[15] Kirichenko V. F. *Classification of Kahlerian structures, defined by means of three-fold vector cross products on six-dimensional submanifolds of Cayley algebra*, Izvestia Vuzov. Math., 1980, **8**, 32–38.

[16] Kirichenko V. F. *Differential-geometrical structures on manifolds*, MSPU, Moscow, 2003 (in Russian).

[17] Kirichenko V. F., Uskorev I. V. Invariants of conformal transformations of almost contact metric structures, Mathematical Notes, 2008, **84**, No. 5, 783–794.

[18] Kurihara H. *The type number on real hypersurfaces in a quaternionic space form*, Tsukuba J. Math., 2000, **24**, 127–132.

[19] Kusova E. V. *On geometry of nearly cosymplectic structures (PhD thesis)*, Moscow State Pedagogical University, 2013 (in Russian).

[20] Pitiş G. *Geometry of Kenmotsu manifolds*, Publ. House Transilvania Univ., Braşov, 2007.

[21] Stepanova L. V. *Contact geometry of hypersurfaces in quasi-Kählerian manifolds (PhD thesis)*, MSPU, Moscow, 1995 (in Russian).

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