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Early History of the Theory of Rings in Novosibirsk

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Abstract. It is a note on early history of ring theory in Novosibirsk. We mostly cover the first 10-15 years of the existence of the A. I.Malcev department of algebra and mathematical logic and A. I.Shirshov (1921–1981) laboratory of ring theory at the Sobolev Institute of Mathematics. By all means, this note is far from being complete, see also a survey by L. A. Bokut, I. P. Shestakov [16]. This article is written in a cooperation with E. N. Kuzmin (1938–2011) who was the active participant of events discussed below.

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1 Introduction

These notes are written thanks to an initiative of Dr. Larissa Sbitneva. At the opening ceremony of the 5th International conference on Nonassociative Algebra and its Applications, Oaxtepec, Morelos, Mexico, 2003, she asked me to say a few words on the history of ring theory in Novosibirsk. Some other participants of the conference supported this idea. I will restrict myself mostly to the first 10–15 years of the existence of the department of algebra and mathematical logic at the Sobolev Institute of Mathematics, Novosibirsk. By all means, these notes are far from being complete, see also a survey L. A. Bokut, I. P. Shestakov [16]. This article is written in cooperation with E. N. Kuzmin who was an active participant of events discussed below.

2 A. I. Malcev (1909–1967) and A. I. Shirshov (1921–1981) are the founders of ring theory in Novosibirsk

Let me recall that in 1957 prominent Russian mathematicians and mechanicians S. A. Khristianovich (1908–2000), M. A. Lavrentev (1900–1980), and S. L. Sobolev (1908–1986) came up with the idea of organizing a Siberian branch of the Soviet Academy of Sciences. Their idea was supported by the Russian leader at that time, N. S. Khruschev. As a result, the Russian government decided to create some 20 academic research institutes together with Novosibirsk State University and build a special town, now known as Akademgorodok, near Novosibirsk.

Thus, (now Sobolev) Institute of Mathematics was founded in 1957 by S. L. Sobolev, who had then been its director until 1983. He invited A. I. Malcev from Ivanovo (near Moscow) Pedagogical Institute to organize a department of algebra and mathematical logic.

A. I. Malcev was a graduate student (1934–1937) of a great Russian mathematician A. N. Kolmogorov (1903–1987), who recognized his very first result, the locality (compactness) theorem in mathematical logic, as the beginning of a new branch of mathematics [53]. This prediction had been fully established. Later Malcev was recognized as "a man who showed a road from logic to algebra" (A. Robinson). By the way, Malcev graduated from the Moscow State University, the "mehmat", in 1931 and began to work at Ivanovo in the same year. It should be mentioned that students of the MSU had to spend 4 years for undergraduate studies, had no diploma works and had no scientific advisors at that time. Malcev studied himself mathematical logic and philosophy at the MSU. He had proved the locality theorem of mathematical logic in 1934 and had sent a manuscript with the proof to Kolmogorov. As the result, Kolmogorov invited him immediately for graduate study in ... algebra (it was a surprise for Malcev) at the MSU. Malcev had defended Candidate of Science Thesis at the MSU in 1937 (on the theory of abelian groups) and Doctor of Science Thesis at the Steklov Mathematical Institute, Kazan, December, 1941 (on the theory of representations of infinite dimensional algebras and infinite groups), with N. G. Chebotarev (1894–1947) (Kazan) and V. A. Tartakovskii (1901–1973) (Leningrad) as official experts. By the way, S. L. Sobolev was the director of the Steklov Mathematical Institute during the war in 1941–42 (the Institute had to move from Moscow to Kazan; since 1943, Sobolev was the first deputy-director of the Laboratory N2 of the Academy of Sciences of the USSR, now the Kurchatov Institute for Nuclear Research).

In nonassociative algebra, Malcev is known as an author of the Levy-Malcev theorem for Lie algebras, as the originator of the theory of Malcev algebras and binary-Lie algebras. He made profound contributions to the theory of Lie groups. Speaking about associative algebras, he was an author of the Malcev-Wedderburn theorem on finite dimensional associative algebras, a founder with O. Ore of the theory of imbedding of rings into skew fields (and semigroups into groups), an author of the Malcev-Neumann division ring construction, a founder of the representation theory of infinite algebras (and infinite groups) by matrices over fields. His collected papers have been published in two volumes [71, 72].

Also S. L. Sobolev invited A. I. Shirshov, a pupil of A. G. Kurosh (1908–1971), from Moscow State University to be the first deputy-director of the new institute. No doubt, the invitation was supported by Malcev who knew Shirshov's results very well. Malcev was an official expert on Shirshov's Doctor of Science Thesis, MSU, 1958, and admired it very much; as it happened, we with E. N. Kuzmin were at the defence meeting and remember that Malcev called Shirshov's Thesis "brilliant" (the other expert was V. M. Glushkov (1923–1982) (Kiev), a prominent specialist in algebra and cybernetics; by the way, his colleagues were trying to check some of Shirshov's calculations by computer). It worth to be mentioned, that A. I. Shirshov was the first deputy-dean of the faculty of mechanics and mathematics (the “mehmat”) of the MSU at that time (the dean was A. N. Kolmogorov).

Novosibirsk was the home region for Shirshov, he had been born at Kolyvan and grown up at Aleisk, small towns near (by the Siberian scale) Novosibirsk [110]. What is more, he studied for one year (1939–1940) at Tomsk State University, that is also near Novosibirsk, and he had begun his high school teacher career at Aleisk. By the way, Shirshov was a high school teacher for 7 years during 1940–1950, with three years interruption, 1942–1945, for the Second World War. Shirshov graduated from Voroshilovograd (Lugansk) Pedagogical Institute in Ukraine by the distance education in 1949. He had started his graduate study at the MSU in 1950, had defended his Candidate of Science Thesis in 1953, and his Doctor of Science Thesis in 1958.

A. I. Shirshov is known for his contributions to the theories of free Lie algebras (Shirshov-Witt theorem on subalgebras, Lyndon-Shirshov words, the Composition-Diamond lemma, Gröbner-Shirshov bases), of PI-algebras (the Shirshov height theorem), of Jordan and alternative algebras (solution of the Kurosh problem, the Shirshov theorem on special Jordan algebras). His collected papers had been published in the book [107].

Shirshov had five students at the MSU: L. A. Bokut, G. V. Dorofeev, E. N. Kuzmin, V. N. Latyshev, and K. A. Zhevlakov (we graduated from the MSU in 1958–1961). Three of us (Kuzmin, Zhevlakov, and me) left Moscow for Novosibirsk with Shirshov, two others remained in Moscow. We had a number of students at Institute of Mathematics, Novosibirsk State University, Moscow State University and Moscow Pedagogical Institute: I. P. Shestakov, A. M. Slinko, A. A. Nikitin, I. M. Miheev, R. E. Roomeldi (1949–1999), A. S. Markovichev (students of Zhevlakov, and after his death, students of Shirshov); V. T. Filippov (1948–2001), F. S. Kerdman, Sh. M. Kasymov, O. Saudi (Syria) (students of Kuzmin, the first one, Filippov, joint with Shirshov); S. V. Pchelintsev (student of Dorofeev); V. E. Barbaumov, S. A. Pikhtilkov, Mekei Abish (Mongolia), I. L. Guseva, T. Gateva (Bulgaria), V. V. Borisenko, N. A. Iyudu, V. V. Schigolev (students of Latyshev); I. V. L’vov (1947–2003), G. P. Kukin (1948–2004), Yu. N. Maltsev, A. V. Yagzhev (1950–2001), V. K. Kharchenko, A. Z. Ananin, E. M. Zjabko (he had been excluded from the NSU after two years of education, see below), V. N. Gerasimov, Ts. Dashdorzh (Mongolia), R. Gonchigdorzh (Mongolia), A. N. Grishkov, A. A. Urman, V. V. Talapov, B. V. Tarasov, G. V. Kryazhovskikh, A. I. Valitskas, O. K. Bobkov, V. V. Vdovin, A. V. Chehonadskikh, A.Ṡ. Stern, A. Ya. Vais, N. G. Nesterenko, A. V. Sidorov, E. P. Petrov, A. T. Kolotov, A. R. Kemer, E. I. Zelmanov (my students, last three joint with Shirshov). Next generation of Shirshov’s school include V. N. Zhelyabin (student of Shestakov and Slinko); Yu. A. Medvedev, A. V. Iltyakov, O. N. Smirnov, U. U. Umirbaev, I. M. Isaev, S. R. Sverchkov, V. G. Skosyrskii (1956–1995), S. V. Polikarpov, N. A. Pisarenko, S. Yu. Vasilovskii (students of Shestakov); A. P. Pozhidaev (student of Filippov); A. Ya. Belov (undergraduate student of Pchelintsev, Belov participated A. V. Mikhalev and V. N. Latyshev’s seminar on ring theory at the MSU for many years); A. N. Koryukin (student of Kharchenko), and many others. My students, V. B. Kulchinovskii (joint with S. N. Vasilev, Irkutsk), E. N. Poroshenko, P. S. Kolesnikov (joint with I. V. L'vov and E. I. Zelmanov), E. S. Chibrikov, I. A. Firdman, I. A. Dolguntseva (joint with P. S. Kolesnikov) have got Candidate of Science Degrees at the Sobolev IM and the NSU. P. S. Kolesnikov has got P. Deligne grant (2006–2008) for his study of associative conformal algebras.

There were a lot of activities in algebra and logic at Novosibirsk and the USSR in 1960th. Some well known algebraists and logicians had visited Malcev and his group at Novosibirsk in the 1960s: P. S. Novikov (Moscow), A. Tarsky (Berkeley), B. Neumann (Canberra), P. G. Kontorovich (Sverdlovsk), L. A. Kaluzhnin (Kiev), D. A. Suprunenko (Minsk), V. M. Glushkov (Kiev), B. I. Plotkin (Riga), V. A. Andrunakievich (Kishinev), L. A. Skornyakov (Moscow), S. I. Adyan (Moscow), A. I. Kostrikin (Moscow), V. P. Platonov (Minsk), V. D. Belousov (Kishinev), A. L. Shmelkin (Moscow), L. N. Shevrin (Sverdlovsk), Yu. M. Ryabuhin (Kishinev), V. I. Arnautov (Kishinev). There was 5-th All-Union Algebra Colloquium at Novosibirsk in 1963 headed by A. I. Malcev. All leading specialists in Algebra and Logic of the USSR came to it, including A. G. Kurosh (Moscow). The preceding All-Union Algebra Colloquiums were: Moscow, 1958, 1959, A. G. Kurosh (Chair); Sverdlovsk, 1960, P. G. Kontorovich (Chair); Kiev, Ukraine, 1962, L. A. Kaluzhnin (Chair). Further All-Union Colloquiums were: Minsk, Belorussia, 1964, D. A. Suprunenko (Chair); Kishinev, Moldavia, 1965, V. A. Andrunakievich (Chair); Riga, Latvia, 1967, B. I. Plotkin (Chair); Gomel, Belorussia, 1968, V. A. Chunikhin (Chair); Novosibirsk, 1969, A. I. Shirshov (Chair). The last All-Union Mathematical Congress held in Leningrad at 1961 with algebra section headed by D. K. Faddeev. N. Jacobson (Yale) visited this Congress. A. I. Malcev was the head of Algebra Section of the Moscow International Mathematical Congress (1966). S. Amitsur (Jerusalem) and P. M. Cohn (London) came to this Congress. There was an All-Union Topological Conference at Novosibirsk in 1967 headed by A. I. Malcev. All leading specialists in topology from the USSR came to it, including P. S. Aleksandrov (Moscow). Also K. Kuratovsky and A. Mostovsky from Poland and M. Katetov from Czech-Slovakia had participated in the Topological Conference.

All of this stimulated the Novosibirsk Ring Theory group in a great respect.

Last but not least, N. Jacobson's profound books on Ring Theory [41]–[45] influenced all members of Shirshov's school very much.

3 Alternative algebras. K.A.Zhevlakov (1939–1972)

K. A. Zhevlakov came to Novosibirsk after graduating from the MSU in 1961. In his master degree work, Zhevlakov [129] proved a result all of us liked very much. He proved an analogue for alternative algebras of the Nagata–Higman (–Dubnov–Ivanov [24]) theorem: the solvability of any alternative algebra with an identity $x^n = 0$ of characteristic $p > n$ (or $p = 0$). After moving to Novosibirsk at 1961, he was trying to solve the analogous problem for Jordan algebras. Time was not ripe for this problem; it was solved for characteristic 0 by Efim Zelmanov 30 years later ([121], 1991); for characteristic $> 2n$, it was solved by V. Skosyrskii and E. Zelmanov

([102], 1983) only in the case of special Jordan algebras. We with E. N. Kuzmin remember that Zhevlakov had spent about two years trying to solve this problem (actually, Kuzmin and Zhevlakov had shared a room at an apartment at that time). Sometimes he thought that he had found a positive solution, other times he believed that he constructed a counter-example to the problem. But each time, he was able to find a mistake in his reasonings. At last, A. I. Malcev and A. I. Shirshov convinced him to abandon this problem. I remember how Malcev was once telling to Zhevlakov that the structure theory of rings, for example, alternative, is a good and respectable issue. It should be mentioned the first among Shirshov's students adored combinatorial problems of ring theory more then structural problems. Probably it was due to the influence of Shirshov's beautiful combinatorial papers. Malcev was trying to change this one-sided point of view. I should say also that N. Jacobson's book "Structure of rings" was very important for all members of Novosibirsk ring theory group. As the result, one can see a harmonious combination of both theories in papers by K. A. Zhevlakov and E. N. Kuzmin on the structure theory of alternative and Malcev algebras, later on in papers by I. P. Shestakov, V. T. Filippov, A. N. Grishkov, S. V. Pchelintsev on the same classes of algebras and on binary-Lie and $(-1,1)$-algebras, and at last in works by E. I. Zelmanov on the structure theory of Jordan and Lie algebras with brilliant applications to group theory.

K. A. Zhevlakov made fast progress in the structure theory of alternative algebras, including the structure of alternative Artinian algebras [130], the existence of Jacobson radical in the class of alternative algebras [131], and so on (see [132]). He defended his Candidate of Science Thesis in 1965 and Doctor of Science Thesis in 1967, soon after Malcev's death. His work had been supported by S. P. Novikov, a 1970 Fields Laureate, and he had got a prestigious Lenin Komsomol Prize in 1970. K. A. Zhevlakov attracted to ring theory a group of undergraduate students including I. V. L'vov, Yu. N. Maltsev, G. P. Kukin, A. M. Slinko, A. A. Nikitin, I. P. Shestakov. The first three became soon my students and participated in my seminar "Associative rings and Lie algebras", and the other three participated in Zhevlakov's seminar on nonassociative rings. It should be mentioned that at the time we are speaking about (1960s) we had a hierarchy of seminars. At the top was "Algebra and Logic" seminar directed by A. I. Malcev before his death, then "Ring theory" seminar directed by A. I. Shirshov, and two student seminars in ring theory. The same was in the group theory (M. I. Kargapolov (1928–1976), Yu. I. Merzlyakov (1940–1995), V. N. Remeslennikov, A. I. Kokorin (1929–1987), V. M. Koputov, V. D. Mazurov), in model theory and mathematical logic (A. D. Taimanov (1917–1990), Yu. L. Ershov, A. V. Gladkii, D. M. Smirnov (1918–2005), M. I. Taizlin, D. A. Zakharov (1925–1996), L. L. Maksimova, I. A. Lavrov). I have to mention also Boris Abramovich Trakhtenbrot (born 1921) who was a student of the prominent mathematician P. S. Novikov (1900–1976), he headed a seminar in logic and computer science and was a chair of the automata theory department at the IM.

K. A. Zhevlakov has left a strong scientific trace in Novosibirsk school of ring theory. A well known book [132] (English translation [133]) had been based on lectures by Shirshov at the MSU and Zhevlakov at the NSU.

I. P. Shestakov made a great progress in the theory of alternative algebras, especially for free alternative algebras [99, 100] (the latter publication is a summary of his Doctor of Science Thesis, 1978). He had proved that the basis rank of the variety of alternative algebras is infinite (it was a solution of Shirshov's problem, see Ch. 7 below for some details) [101]. In a joint paper ([102], 1990), I. P. Shestakov and E. I. Zelmanov had described prime alternative super algebras over a field of characteristic not 2, 3, and had applied this result to a proof of nilpotency of the Jacobson radical of any free alternative algebra over a field of characteristic 0. The latter result was a solution of a Zhevlakov problem. A description of prime alternative algebras had been done earlier by M. Slater, a student of I. Herstein ([98], 1972).

Yu. A. Medvedev, a student of Shestakov, had proved that a periodic loop is locally finite if it is embeddable into an alternative PI-algebra [80].

Many results for alternative algebras had been done also by G. V. Dorofeev (see Ch. 7), S. V. Pchelintsev, V. T. Filippov, A. V. Iltyakov, S. R. Sverchkov, Yu. A. Medvedev, and others.

Recently I. P. Shestakov and U. U. Umirbaev [103]–[105] has solved one of the fundamental problems for polynomial automorphisms. In 1942 H. W. E. Jung had proved that any automorphism of an algebra $k[x, y]$ of polynomials over a field of characteristic 0 is tame (a product of elementary automorphisms). In 1972 M. Nagata had conjectured that the following polynomial automorphism over complex numbers

$$(x, y, z) \to (x - 2(xz + y^2)y - (xz + y^2)^2 z, y + (xz + y^2)z, z)$$

is not tame. At last in 2003 Shestakov and Umirbaev have proved that the Nagata's conjecture is true!

4 Jordan algebras

Some radicals in the class of (special) Jordan algebras had been studied by A. M. Slinko [114, 115]. He had proved that the Baer (lower) radical is locally nilpotent in special Jordan algebras, and the Levitzki (local nilpotent) radical is ideal-hereditary in the class of Jordan algebras.

The class of special Jordan algebras is not a variety, P. M. Cohn [18], but it is a quasi-variety. S. R. Sverchkov [116] had proved that this quasi-variety can not be defined by a finite number of quasi-identities. It is an analogue of a well known Malcev's result (1940) for the class of semigroups embeddable into groups.

V. N. Zhelyabin [127, 128] had proved theorems on splitting of the Jacobson radical for Jordan and alternative algebras over a Hensel ring that are analogous to the ones obtained for associative algebras by G. Azumaya (1951).

"Russian revolution in Jordan algebras" (these are K. McCrimmon's words) had been made by Efim Isaakovich Zelmanov at the end of 1970s-beginning of 1980s. His firsts of these results had been done before Shirshov's death. He had settled a long standing gap in the theory of Jordan algebras with minimal condition proving that the Jacobson radical is nilpotent in such algebras ([119], 1978). Zelmanov had proved local nilpotency of Jordan nil algebras of bounded index ([120], 1979). Previously it was proved by Shirshov (1957) for special Jordan algebras. Then he had described Jordan division algebras giving a positive response to a longstanding problem of Jacobson ([121], 1979). Also he had described prime Jordan algebras without nonzero nil ideals ([121], 1979). Some of these results of Zelmanov's I had announced in my talk at a Conference on Division Rings, Oberwolfach, 1978. P. M. Cohn and G. Bergman were among the participants. P. M. Cohn was very astonished by Zelmanov's results. I had given a manuscript of my talk to G. Bergman and he had sent it to N. Jacobson. I believe it was the first information about Zelmanov's results to the West mathematicians. Later Jacobson [42] had lectured Zelmanov's first results on structure theory of Jordan algebras with a great enthusiasm. He had also lectured on Skosyrskii's theorem [112] that the Levitzki radical of a special Jordan algebra J is the intersection of J with the Levitzki radical of an envelope.

A. I. Shirshov was very proud of Zelmanov's results. It was long before Zelmanov had obtained a solution of the Restricted Burnside Problem and long before he had got a Fields Medal. But Shirshov had understand a phenomenon of Zelmanov very well. He had told me once: "People will remember us for we save Zelmanov for science". By the way, I must say that Shirshov was very unhappy that Zelmanov had failed (!) to defend his Candidate of Science Thesis "Jordan Division Algebras" at a Science Counsel at the Institute of Mathematics on 25 of October, 1980. On this very day Shirshov's mother had died (they were living together for many years) and this very day was the last day that Shirshov had visited his dear Institute of Mathematics, when he was the first deputy-director since 1958 to 1973. Later on Zelmanov was successful in this business due to the help of S. L. Sobolev in May 1981, after Shirshov's death on 28 of February, 1981. Shirshov's Ring Theory Department had been divided into two laboratories: my laboratory "Associative and Lie rings" (with Ananin, Gerasimov, Kharchenko, Lvov, Zelmanov) and Shestakov's laboratory "Nonassociative rings" (with Filippov, Gainov, Kuzmin, Medvedev, Skosyrskii, together with two specialists in group theory, N. S. Romanovskii and S. A. Syskin). I would like to say my thanks to the first deputy-director of the IM at that time, a prominent specialist in Riemanian Geometry Viktor Andreevich Toponogov (1930–2004) for his help to establish my laboratory. In three years, Zelmanov had finished his "Jordan revolution" and had written his Doctor of Science Thesis "Jordan Systems and Graded Simple Lie algebras". He had successfully defend this Thesis at a Science Counsel headed by D. K. Faddeev, deputy-head was Z. I. Borevich, at Leningrad State University in 1985 (with some supports from A. I. Kostrikin, V. N. Latyshev, V. P. Platonov). The last Chapter 5 of his Dr.Sc. Thesis was "Burnside Type Problems: Algebraic Algebras" (algebraic Jordan algebras and algebraic Lie algebras). It was a beginning of his thoughts on Lie nil (Engel) algebras and finally on the Restricted Burnside Problem for finite groups, that was successfully finished in another 4 years [125, 126].

A lot of results for Jordan algebras had been proved by Yu. A. Medvedev [82–

84] at the end of 1980th. The results in the paper [82] continue the researches of I. P. Shestakov [Mat. Sb., Nov. Ser. 122 (164), No. 1 (9), 31–40 (1983)] concerning polynomial identities in finitely generated Jordan and alternative algebras. Let J be a finitely generated Jordan PI-algebra over a commutative ring R with $\frac{1}{2}$. Then:

1) The universal multiplicative enveloping algebra of J is a PI-algebra as well.

2) If the ring R is Noetherian then the nil radical of the algebra J is nilpotent.

3) The algebra of the multiplications of J is an associative PI-algebra.

In the paper [83], Medvedev proved that an absolute zero divisor in a finitely generated Jordan algebra generates a nilpotent ideal.

Medvedev's work [84] was based on the results and methods of his earlier study of Jordan A-algebras [Algebra Logika 26, No. 6, 731–755 (1987)]. In particular, he proved: The free Jordan algebra from more than two generators is not prime and has a nonzero center.

5 Malcev algebras and binary-Lie algebras

In 1955, A. I. Malcev [70] invented two classes of nonassociative algebras: Moufang–Lie algebras and binary-Lie algebras. A. A. Sagle [97] changed name "Moufang-Lie algebras" to "Malcev algebras". A great contribution to the theory of Malcev algebras had been made by Evgenii Nikiforovich Kuzmin (born in 1938). In the middle of the 1960s-beginning of 1970s, he proved some fundamental results on structure theory of Malcev algebras and on connections of Malcev algebras and local analytic Moufang loops [59–61], see also [63]. His results included a description of central simple finite dimensional (f.d.) Malcev algebras over a field of characteristic > 3. He had also proved the existence of local analytic Moufang loop with any given tangent f.d. Malcev algebra over the real field. Some of these results had been presented in a joint talk with A. I. Malcev at the All-Union Topological Conference in Novosibirsk a few days before Malcev's death. Kuzmin had defended his Doctor of Science Thesis on the subject in 1972. F. S. Kerdman, a student of Kuznim, had studied global analytic Moufang loops and their connections with Malcev algebras [48]. Later on Kuzmin's student Valerii Terentevich Filippov (1952–2001) was very successful in his study of Malcev algebras and alternative algebras. He had described central simple infinite dimensional Malcev algebras in [27]: all of them are Lie algebras. Also he invented a new class of algebras, the n-Lie algebras [28], which are now called Filippov algebras. Later Sh. M. Kasymov, a student of Kuzmin from Uzbekistan, had proved that Cartan subalgebras of any f.d. n-Lie algebra are conjugated in a case of algebraically closed field of characteristic 0 [46].

A. N. Grishkov [38] and E. N. Kuzmin [62] had independently proved an analogue of Levi's theorem for Malcev algebras.

Malcev algebras became a popular subject in Novosibirsk. Some important results on the subject have been made by I. P. Shestakov, A. N. Grishkov, S. V. Pchelintsev.

A lot of papers for binary-Lie algebras have been published by E. N. Kuzmin, A. N. Grishkov, V. T. Filippov, I. P. Shestakov. Kuzmin [58] had proved an analogue of Engel theorem for binary-Lie algebras. Grishkov [39] had established that any simple finite dimensional binary-Lie algebra over an algebraically closed field of characteristic 0 is Malcev algebra.

6 Other classes of non-associative algebras

The class of mono-composition algebras was invented by Alexei Timofeevich Gainov (born 1929). He was an Ivanovo student of A. I. Malcev. His first result [31] was a characterization of binary-Lie algebras by two identities. Gainov moved to Novosibirsk in 1960. He introduced mono-composition algebras as a generalization of the composition algebras [32].

Raul Roomeldi (1949–1999) had graduated from Tartu University (Estonia). He was a graduate student of Zhevlakov at the NSU and after his death a student of Shirshov. He had proved an analogue of the Nagata–Higman (–Dubnov–Ivanov) theorem for $(-1,1)$-algebras [94].

I. M. Miheev, a student of Zhevlakov, had proved an analogue of the Wedderburn principal theorem for $(-1,1)$-algebras [85]. He had resolved a long-standing question of A. A. Albert that there exists a simple, right alternative (infinite dimensional) algebra that is not alternative [86]. Later V. G. Skosyrskii [113] had proved that any simple, right alternative algebra either alternative or nil.

S. V. Pchelincev, a student of Dorofeev, had proved that the associator ideal of a free finitely generated $(-1,1)$-algebra is nilpotent [89].

A. A. Nikitin had proved an analogue of Wedderburn's principal theorem for (γ,δ)-algebras over a field of characteristic > 5 [87].

A. S. Markovichev had proved that radicals in (γ,δ)-algebras are hereditary [79].

7 Varieties of non-associative algebras

Georgii Vladimirovich Dorofeev (1938–2008) was as it was mentioned above a student of Shirshov at the MSU. His first result was an example of a solvable alternative algebra that is not nilpotent [21]. He constructed an identity that is valid on any 3-generated alternative algebras of characteristic 0 but not valid in the class of all alternative algebras [22]. This identity leads naturally to the question of whether the basis rank of the class of alternative algebras is finite or infinite. The question was known in Novosibirsk as a problem of Shirshov.

Later on I. P. Shestakov [100] proved that the basis rank is infinite. It means that the series

$$\mathrm{Alt}_1 \subseteq \mathrm{Alt}_2 \subseteq \cdots \subseteq \mathrm{Alt}_n \subseteq \mathrm{Alt}_{n+1} \subseteq \dots$$

does not stabilize at a finite step, where $\mathrm{Alt_n}$ is the variety of alternative algebras generated by he free alternative algebra of the rank n. V. T. Filippov [28, 29] had proved that the above series (over an associative-commutative ring) is strictly increasing at any step but possibly $n = 3$. Actually, both Shestakov's and Filippov's papers contain analogous results for the Malcev algebras.

At the end of 1970th, Dorofeev found identities that characterize the join of some important varieties of non-associative algebras [23].

Valerii Anatolievich Parfenov (1944–2016) was a student of Shirshov in Novosibirsk. He proved [90] that varieties of Lie algebras over a field of characteristic zero consist of a free semigroup under the Malcev-Neumann multiplication. It is a Lie algebra analogue of the Neumann-Shmelkin theorem for groups. The same kind of results have been obtained by my student Alexandr Aronovich Urman (born 1944) [117] for commutative varieties of (anticommutative) non-associative algebras.

8 Varieties of associative algebras

Viktor Nikolaevich Latyshev (born 1934) was a student of Shirshov at the MSU. He is a specialist in PI-algebras. Since his university years, he had been working on the Specht problem, whether every associative algebra over a field of characteristic zero is finitely based in the sense of identities. It should be noted that the Specht problem had been one of the most appreciated problems in Shirshov's school. A. I. Malcev also knew this problem and certainly recognized it as a central problem of the theory of varieties of associative algebras. Latyshev had been working on the problem for many years, doing more and more cases of varieties that are finitely based [65,66]. Among other aspects, his works kept the Specht problem alive not only in the USSR, but also in Bulgaria, (see M. B. Gavrilov (1940–1998) [36], G. K. Genov [33], A. P. Popov [92, 93], V. S. Drensky [25, 26]). There were close relations of Novosibirsk and Moscow algebraists with algebraists of this country. The Specht problem was solved positively by Alexandr Robertovich Kemer in 1986 (see [47]). Recently A. Ya. Belov, A. V. Grishin and V. V. Shchigolev published important results on the analogue of the Specht problem for associative algebras in finite characteristic. In general, the last problem has negative solution even for finitely generated algebras over finite fields, but for finitely generated algebras over an infinite field of finite characteristic the solution is still positive (see [6]).

Igor Vladimorovich Lvov (1947–2003) was my student. His main results belong to the theory of PI-algebras. He proved that any finite associative ring is finitely based, the Lvov-Kruse theorem. Lvov proved it in 1969, and published in 1973 [67]. The last result is also valid for finite alternative rings (I. V. Lvov [69]), finite Lie rings (Yu. A. Bahturin, A. Yu. Olshanskii, students of A. L. Shmelkin [4]), finite Jordan rings (Yu. A. Medvedev [81]), but not valid in general for finite (nonassociative) rings (S. V. Polin, a student of A. G. Kurosh [91]). All these positive results are analogues of the Oates-Powell theorem for finite groups [88] (Sheila Oates-Macdonald and M. B. Powel were students of G. Higman). Later I. V. Lvov, A. Z. Ananin, Yu. N. Maltsev, and V. T. Markov (a student of A. V. Mikhalev from the MSU) proved the following result in the middle of 1970th: Let M be a variety of associative algebras over an infinite field k. Then the following properties are equivalent: (1)

All finitely generated (f.g.) algebras from M are representable by matrices over commutative algebras; (2) All f.g. algebras from M are weakly Noetherian (i.e., any two-sided ideal is finitely generated); (3) All f.g. algebras from M are residually finite; (4) All f.g. algebras from M are Hopfian; (5) M has an identity $xy^nx = \sum_{i+j>0} \alpha_{ij} y^i x y^{n-i-j} x y^j$ ($\alpha_{ij} \in k$) (see a survey by Bokut, Kharchenko, Lvov [13] translated in [54]). A variety M with these properties is sometimes called a Hilbert-Malcev variety. Just before his death, Lvov published [69] a detailed proof of A. Smoktunovich's result on the existence of simple nil associative algebra.

Yu. N. Maltsev in his Candidate of Science Thesis, 1973, had proved the following interesting results. All identities of an algebra of all upper triangular $n \times n$ matrices over a field of characteristic zero are the consequences of only one, $[x_1, x_2][x_3, x_4] \dots [x_{2n-1}, x_{2n}] = 0$ [73]. If R is an algebra that is nil over a right (algebra) ideal A satisfying an identity of degree d, then R satisfies a standard identity of degree d provided R has no nonzero nil ideals [74]. Actually Zelmanov [118] had later proved that if an algebra has no nonzero nil ideals and is nil over a PI-subalgebra then it is a PI-algebra. A ring R is said to be an H-extension of its subring A if, for every $x \in R$, there is a natural $n > 1$ such that $x^n - x \in A$. If A is commutative, then the ring R satisfies the identity $[[x_1, x_2], [x_3, x_4]] = 0$; if R is an algebra and A a right ideal of R satisfying an identity, then R satisfies an identity [75]. Also Maltsev had described varieties of associative algebras with the commutative product of subvarieties [76], just non-commutative varieties of rings (*Sib. Mat. J.*, 17, 803–810 (1976)) and found a basis of identities of the second order matrices over a finite field (*Algebra Logika*, 17, 18–20 (1978)). Later (1986) he had defended Doctor of Science Thesis at the LSU, Yu. N. Maltsev, *Critical rings and varieties of associative rings* (see [77, 78]).

9 Lie algebras, associative algebras, and groups

In 1958, published in [7], I found a basis of a free Lie algebra that is compatible with the derived series (see also [95]). It gives a basis of a free solvable Lie algebra. In the same paper, a basis of any free polynilpotent Lie algebra had been found. These results are based on a Shirshov's result from his Candidate of Science Thesis [106], published in [107], on series of bases of free Lie algebras (see also [96]). Some applications of my basis had been found by V. N. Latyshev [65], A. L. Shmelkin [111], and Yu. M. Gorchakov [37]. In 1959, published in [8], I generalized a result by J. Dixmier [20] on nilpotent Lie algebras. Those were my master degree results. In my Candidate of Science Thesis, 1963, I proved that any Lie algebra can be imbedded into an algebraically closed Lie algebra (in the sense that any equation over the algebra has a solution in this algebra) [9]. It was initiated by P. M. Cohn's result [19] that any Lie algebra is embeddable into a division Lie algebra. The proof used Shirshov's method [108] on what is now called the Gröbner-Shirshov bases for ideals of free Lie algebras. In [10], I had actually found Gröbner-Shirshov bases for P. S. Novikov's groups, and based on it, I had fully analyzed the conjugacy problem for these groups. As a result, I proved that for any Turing degree of insolvability there exists a Novikov's group with this degree of the conjugacy problem.

In [11], I had found an example of a semigroup S such that the multiplicative semigroup of a semigroup algebra of S (namely, $GF(2)\langle S\rangle$) can be imbedded into a group but the algebra can not be imbedded into any division ring. Up to now, this is the only known example of a semigroup with the property. The proof is based on a (relative) Gröbner-Shirshov basis of the universal group of the multiplicative semigroup of the algebra $GF(2)\langle\langle S\rangle\rangle$, of infinite power series over S with coefficients in $G(2)$. In particular, it gave a solution of a Malcev's problem (see [71], p. 6).

Last two results consist of my Doctor of Science Thesis, 1969.

In [12], I had proved that some recursively presented Lie algebras can be imbedded into finitely presented Lie algebras. It gave the existence of a finitely presented Lie algebra with the unsolvable word (equality) problem (solution of a problem of Shirshov [106]). A proof is based on Gröobner-Shirshov bases for Lie algebras.

Explicit examples of finitely presented Lie algebras with the unsolvable word problem had been found by my student Georgii Petrovich Kukin (1948–2004) [55]. In [56], he proved that the Cartesian subalgebra of the free product of Lie algebras is free. Also he had found a description of any subalgebra of the free (amalgamated) product of Lie algebras by means of generators and defining relations [57]. Recently E. S. Chibrikov [17] has solved Kukin's problem of an explicit construction of a left normed basis of a free Lie algebra.

Our joint book with G. P. Kukin [15] contains some of results mentioned above in this chapter, see also my survey [14].

My student since 1970 Vladislav Kirillovich Kharchenko in his Master of Science Diploma, 1974, had proved that if the ring of invariants R^G of an associative ring R with a finite group G of automorphisms is a PI-ring, then R is also a PI-ring, provided R has no additive $|G|$-torsion [49]. He had described [50] the structure of prime rings satisfying a generalized identity with automorphisms. This generalized a theorem of W. S. Martindale (1969) and was in the same spirit as a theorem of S. A. Amitsur on rings with involution (1969). In the same paper he had answered in the affirmative a question studied by G. M. Bergman and I. M. Isaacs (1973): Let G be a finite group of automorphisms of a ring R without nilpotent elements; then $R^G \neq (0)$. There were main results of his Candidate of Science Thesis, 1976. Kharchenko had published a survey "Groups and Lie algebras acting on noncommutative rings" [51], 1980, and had got his Doctor of Science Degree on the subject in 1984 at the Leningrad State University. Later Kharchenko published his results on non-commutative Galois theory in his well known book [52].

Victor Nikolaevich Gerasimov was also my student since 1970 (in fact, Gerasimov and Kharchenko were classmates). His 1974 Master of Science Diploma [34] contains a deep study of one-relator associative algebras. From his results it follows that the Hilbert series of any one-relator homogeneous associative algebra is rational [5]. His Candidate of Science Thesis, V. N. Gerasimov, *Free associative algebras and inverting homomorphisms of rings*, had been translated by the AMS, together with ones by N. G. Nesterenko, *Representations of algebras by triangular matrices*, and

A. I. Valitskas, *Embedding rings in (Jacobson) radical rings and rational identities of (Jacobson) radical algebras* [35].

Last but not least, Aleksandr Zigfridovich Ananin was my student since 1971. His first paper with my other student Evgenii Mikhailovich Zjabko [1], 1974 had contained a solution of a well known C. Faith problem. Let me give a review by W. G. Leavitt, see (MR0360721 (50 13168)) of this paper that shows the real significance of it (remember that the authors were 2nd year undergraduate students): "For a ring R, consider the property: $(*)$ For an arbitrary pair $x, y \in R$ there exist positive integers $m(x,y)$, $n(x,y)$ such that $x^{m(x,y)}$ commutes with $y^{n(x,y)}$. The authors show in a very ingenious way that if R has property $(*)$ and no nil ideals then R is commutative. Even more, it is shown that if R is arbitrary with $(*)$ then the set I of all nilpotent elements of R is an ideal of R, with R/I commutative. This paper is the last in a long sequence of commutativity theorems by various authors, the previous best result being that of A. I. Lihtman [Mat. Sb. (N. S.) 83 (125) (1970), 513–523; MR 42 6023] who proved the same two theorems for the special case of $(*)$ in which m and n are functions, respectively, of x and y alone." Later this theorem had been reproved by I. Herstein [40]. Zjabko was a very promising mathematician. It was a big tragedy for him and for us, that Zjabko had been excluded (1973) from the NSU for "dirtiness in his dormitory room" despite our efforts with Shirshov to save him (he was an excellent student but "worse luck", he was a Jew (!?)). Later Ananin had proved important results on (triangular) matrix representable varieties of associative algebras [2, 3].

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