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The multiplicative Zagreb co-indices on two graph operators

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Abstract. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The first and second multiplicative Zagreb co-indices are defined as:

$$\overline{\prod}_1(G) = \prod_{uv \notin E(G)} [d_G(u) + d_G(v)] \ , \ \overline{\prod}_2(G) = \prod_{uv \notin E(G)} [d_G(u) d_G(v)],$$

respectively, where $d_G(u)$ is the degree of the vertex u of G. The aim of this paper is to investigate the multiplicative Zagreb co-indices of the subdivision graphs of tadpole graphs and wheel graphs

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1 Introduction

Throughout the paper, we consider connected finite graphs without any loops or multiple edges. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The degree of $v \in V(G)$, denoted by $d_G(v)$, is the number of vertices in G adjacent to v. A graphical invariant is a number related to a graph which is a structural invariant, in other words, it is a fixed number under graph automorphisms. In chemical graph theory, these invariants are also known as the topological indices. The Zagreb indices are among the oldest topological indices, and were introduced in 1972 [13]. Gutman and Trinajstic examined the dependence of total π-electron energy on molecular structure, elaborated in [12]. The first and second Zagreb indices of G are denoted by $M_1(G)$ and $M_2(G)$, respectively, and defined as follows:

$$M_1(G) = \sum_{v \in V(G)} d_G^2(v) \ \text{ and } \ M_2(G) = \sum_{uv \in V(G)} d_G(u) d_G(v).$$

The first Zagreb index can be also expressed as a sum over edges of G:

$$M_1(G) = \sum_{uv \in E(G)} [d_G(u) + d_G(v)].$$

The main properties of the Zagreb indices were summarized in [4, 5, 10]. In particular, Deng [5] gave a unified approach to determine extremal values of Zagreb indices for trees, unicyclic graphs and bicyclic graphs. Other recent results on ordinary Zagreb indices can be found in [15]. Note that the contribution of non-adjacent vertex pairs should be taken into account when computing the weighted Wiener polynomials of certain composite graphs [4]. The first and second Zagreb co-indices, as the sums involved run over the edges of the complement of G, are denoted by $\overline{M}_1(G)$ and $\overline{M}_2(G)$ and were defined in 2010 [1] as follows:

$$\overline{M}_1(G) = \sum_{uv \notin E(G)} [d_G(u) + d_G(v)] \quad \text{and} \quad \overline{M}_2 = \sum_{uv \notin E(G)} [d_G(u) d_G(v)].$$

The multiplicative versions of Zagreb indices were introduced by Gutman in 2012 [9]. The first and second multiplicative Zagreb indices of G are denoted by $\prod_1(G)$ and $\prod_2(G)$, respectively, and are defined as:

$$\prod\nolimits_1(G) = \prod\nolimits_{u \in V(G)} d_G(u)^2 \quad \text{and} \quad \prod\nolimits_2(G) = \prod\nolimits_{uv \in E(G)} [d_G(u) d_G(v)].$$

The first and second multiplicative Zagreb indices were extensively studied in [9, 18, 19]. In particular, Gutman have determined the extremal tree with respect to multiplicative Zagreb indices. In 2012 Xu and Hua [19] provided a unified approach to extremal trees, unicyclic and bicyclic graphs with respect to this multiplicative version of Zagreb indices. Xu et al. introduced the first and second multiplicative Zagreb co-indices of G [14]. The first and second multiplicative Zagreb co-indices of G are denoted by $\overline{\prod}_1(G)$ and $\overline{\prod}_2(G)$, respectively, and defined as:

$$\overline{\prod}_1(G) = \prod\nolimits_{uv \notin E(G)} [d_G(u) + d_G(v)] \quad \text{and} \quad \overline{\prod}_2(G) = \prod\nolimits_{uv \notin E(G)} [d_G(u) d_G(v)].$$

The subdivision graph $S(G)$ is the graph obtained from G by replacing each of its edges by a path of length 2, or equivalently, by inserting an additional vertex into each edge of G, and the operator $R(G)$ is the graph obtained from G by adding a new vertex corresponding to each edge of G and by joining each new vertex to the end vertices of the edge corresponding to it [16]. The tadpole graph, $T_{n,k}$, is the graph obtained by joining a cycle graph C_n to a path of length k [17]. The wheel graph W_{n+1} is defined as the graph $K_1 + C_n$, where K_1 is the singleton graph and C_n is the cycle graph. In this paper we will calculate the multiplicative Zagreb co-indices of $T_{n,k}$, W_{n+1} and the subdivision $S(G)$ and $R(G)$ on these graphs.

2 The multiplicative Zagreb co-indices on $S(G)$ and $R(G)$ for tadpole graph

In this section, we compute the multiplicative Zagreb co-indices on two graph operators $S(G)$ and $R(G)$ for tadpole graph $T_{n,k}$. At first we prove the following lemma, which plays an important role in the proofs.

Proposition 1. *For a connected graph G, we have*

$$\overline{\prod}_2(G) = \prod_{v \in V(G)} d_G(v)^{(n-1-d_G(v))}.$$

Proof. By definition of complement graph of G we find that for each vertex $v \in V(G)$, the factor $d_G(v)$ occurs $n-1-d_G(v)$ times in $\overline{\prod}_2(G)$. Thus this theorem follows immediately. $\square$

Theorem 1. *For the tadpole graph, the multiplicative Zagreb co-indices satisfy the following equations:*

$$\overline{\prod}_1(T_{n,k}) = (2^{n^2+k^2+2nk-7n-7k+16})(5^{n+k-5})(3^{k+n-4})$$

and

$$\overline{\prod}_2(T_{n,k}) = (2^{n^2+k^2-5n-5k+2nk+6})(3^{n+k-4}).$$

Proof. The tadpole graph $T_{n,k}$ contains $n+k-2$ vertices of degree 2, one vertex of degree 3 and a pendent vertex. The subdivision graph $S(T_{n,k})$ contains $n+k$ additional vertices of degree 2. In $T_{n,k}$, let v_l be a vertex of degree 3 and $v_{1'}$ and $v_{2'}$ be the neighbors of v_l in the cycle C_n and v_j be the neighbor of v_l in the path P_{k+1}. Let v_1 be the pendent vertex in $T_{n,k}$. We calculate $\overline{\prod}_1[d_G(u)+d_G(v)]$:
1. Among the vertices in C_n.
2. From cycle C_n to the path P_{k+1}.
3. Among the vertices in the path P_{k+1}.

Case I. In C_n, $v_{1'}$ and $v_{2'}$ are non-adjacent with $n-3$ vertices of degree 2. Remaining $n-3$ vertices in C_n are non-adjacent with $n-4$ vertices of degree 2 and one vertex of degree 3. Also v_l is non-adjacent with $n-3$ vertices of degree 2. Hence in C_n, $\overline{\prod}_1[d_G(u)+d_G(v)] = (4^{n^2-5n+6})(5^{2n-6})$. Since one edge is shared between a pair of vertices, $\overline{\prod}_1[d_G(u)+d_G(v)]$ in C_n is

$$\overline{\prod}_1[d_G(u)+d_G(v)] = (4^{n^2-5n+6}5^{2n-6})^{\frac{1}{2}}. \tag{2.1}$$

Case II. From cycle C_n to path P_{k+1}, all the $n-1$ vertices other than v_l in C_n are non-adjacent with v_1. Also all of $n-1$ vertices except v_l in C_n are non-adjacent with $k-1$ vertices of degree 2 and one vertex of degree 1. Hence

$$\overline{\prod}_1[d_G(u)+d_G(v)] = (4^{(k-1)(n-1)})(3^{(n-1)}). \tag{2.2}$$

Case III: In the path P_{k+1}, the vertex v_l is non-adjacent with $k-2$ vertices of degree 2 and one vertex of degree 1. The neighbor of v_l in P_{k+1} is non-adjacent with $k-3$ vertices of degree 2 and one vertex of degree 1. The vertex v_j is non-adjacent with $k-4$ vertices of degree 2 and one vertex of degree 1 and one vertex of degree 3 for $3 \le j \le k-1$. Also the vertex v_2 has $k-3$ non-adjacent vertices of degree 2 and one vertex of degree 3. The vertex v_2 has $k-2$ non-adjacent vertices of degree 2 and one vertex of degree 3. Thus $\overline{\prod}_1[d_G(u)+d_G(v)] = (5^{2k-4})(4^{k^2-5k+8})(3^{2k-6})$. Since one edge is shared between a pair of vertices,

$$\overline{\prod}_1[d_G(u)+d_G(v)] = 5^{k-2}2^{k^2-5k+8}3^{k-3}. \tag{2.3}$$

The product of equations (2.1), (2.2) and (2.3) implies that

$$\overline{\prod}_1(T_{n,k}) = (2^{n^2+k^2+2nk-7n-7k+16})(5^{n+k-5})(3^{k+n-4}).$$

By Proposition 1, $\overline{\prod}_2(T_{n,k})$ can be easily obtained,

$$\overline{\prod}_2(T_{n,k}) = (2^{n^2+k^2-5n-5k+2nk+6})(3^{n+k-4}).$$

□

Theorem 2. *For the subdivision graph $S(G)$ of a tadpole graph, the multiplicative Zagreb co-indices are:*

$$\overline{\prod}_1(S(T_{n,k})) = (2^{4n^2+4k^2-14n-14k+8nk+16})(5^{2n+2k-5})(3^{2k+2n-5})$$

and

$$\overline{\prod}_2(S(T_{n,k})) = (2^{4n^2+4k^2-10n-10k+8nk+6})(3^{2n+2k-4}).$$

Proof. $S(T_{n,k})$ contains $2(n+k-1)$ vertices of degree 2, one vertex of degree 3 and a pendent vertex. In $S(T_{n,k})$, let v_l be the vertex of degree 3 and $v_{1'}$ and $v_{2'}$ be the neighbors of v_l in the cycle $S(C_n)$ and v_j be the neighbor of v_1 in the path $S(P_{k+1})$. Let v_1 be the pendent vertex in $S(T_{n,k})$. We calculate $\overline{\prod}_1[d_G(u)+d_G(v)]$:
1. Among the vertices in $S(C_n)$.
2. From cycle $S(C_n)$ to the path $S(P_{k+1})$.
3. Among the vertices in the path $S(P_{k+1})$.
In $S(C_n)$, $v_{1'}$ and $v_{2'}$ are non-adjacent with $2n-3$ vertices of degree 2. Remaining $2n-3$ vertices in $S(C_n)$ are non-adjacent with $2n-4$ vertices of degree 2 and one vertex of degree 3. Also v_1 is non-adjacent with $2n-3$ vertices of degree 2. Hence in $S(C_n)$, $\overline{\prod}_1[d_G(u)+d_G(v)] = (4^{(4n^2-10n+6)})(5^{4n-6})$. Since one edge is shared between a pair of vertices, $\overline{\prod}_1[d_G(u)+d_G(v)]$ in $S(C_n)$ is

$$\overline{\prod}_1[d_G(u)+d_G(v)] = 2^{4n^2-10n+6}5^{2n-3}. \tag{2.4}$$

From cycle $S(C_n)$ to path $S(P_{k+1})$, all the $2n-1$ vertices other than v_l in $S(C_n)$ are non-adjacent with v_1. Also all of $2n-1$ vertices except v_l in $S(C_n)$ are non-adjacent with $2k-1$ vertices of degree 2 and one vertex of degree 1. In the $S(P_{k+1})$, the vertex v_l is non-adjacent with $2k-2$ vertices of degree 2 and pendent vertex. Hence

$$\overline{\prod}_1[d_G(u)+d_G(v)] = (4^{4nk-2n-2k+2})(3^{(2n-1)})(5^{2k-2}). \tag{2.5}$$

In the path $S(P_{k+1})$, the neighbor of v_1 in $S(P_{k+1})$ is non-adjacent with $2k-3$ vertices of degree 2 and one vertex of degree 1. The vertex v_j is non-adjacent with $2k-4$ vertices of degree 2 and one vertex of degree 1 for $3 \le j \le 2k-1$. Also the vertex v_2 has $2k-3$ non-adjacent vertices of degree 2. Thus $\overline{\prod}_1[d_G(u)+d_G(v)] = (4^{4k^2-18k++18})(3^{4k-4})$. Since one edge is shared between a pair of vertices,

$$\overline{\prod}_1[d_G(u)+d_G(v)] = (2^{4k^2-18k+18})(3^{2k-2}). \tag{2.6}$$

By multiplying equations (2.4), (2.5) and (2.6) we have:

$$\overline{\prod}_1(S(T_{n,k})) = (2^{4n^2+4k^2-14n-14k+8nk+16})(5^{2n+2k-5})(3^{2k+2n-5}).$$

By Proposition 1, it can be easily obtained:

$$\overline{\prod}_2(S(T_{n,k})) = (2^{4n^2+4k^2-10n-10k+8nk+6})(3^{2n+2k-4}).$$

□

Theorem 3. *For the tadpole graph $T_{n,k}$ we have:*

$$\overline{\prod}_1(R(T_{n,k})) = (2^{\frac{1}{2}(7n^2+7k^2-23k-17n+23)})(3^{k^2+n^2-4k-3n+3})(5^{k+n-5})$$

and

$$\overline{\prod}_2(R(T_{n,k})) = (2^{6(n+k)^2-17(n+k)+17})(3^{2(n+k)-7}).$$

Proof. The vertices which are of degree l in $S(T_{n,k})$ are of degree $2l$ in $R(T_{n,k})$. All the subdivision vertices are of the same degree in both $S(T_{n,k})$ and in $R(T_{n,k})$.
In the cycle $R(C_n)$, the vertices which are adjacent to v_1 make the sum 8 with remaining $n-3$ vertices in the cycle and the remaining $n-3$ vertices make the sum 8 with $n-4$ vertices in the cycle. Also v_1 makes the sum 10 with the $n-3$ vertices. All the n subdivision vertices make the sum 4 with the remaining $n-1$ subdivision vertices. The vertex v_1 makes the sum 8 with $n-2$ subdivision vertices. The $n-1$ vertices other than v_1 make the sum 6 with the $n-2$ subdivision vertices. Therefore in $R(C_n)$,

$$\overline{\prod}_1[d_G(u)+d_G(v)] = [(2^{7n^2-15n+4})(3^{n^2-3n+2})(5^{2n-6})]^{\frac{1}{2}}. \tag{2.7}$$

To calculate $\overline{\prod}_1[d_G(u)+d_G(v)]$ from $R(C_n)$ to $R(P_{k+1})$, all the $n-1$ vertices in the cycle other than v_1 make the sum 6 with v_l and k subdivision vertices in the path. All the n subdivision vertices in the cycle make the sum 4 with v_l and k subdivision vertices in the path. Also all n subdivision vertices in $R(C_n)$ make the sum 6 with $k-1$ vertices in the path. So from cycle to path,

$$\overline{\prod}_1[d_G(u)+d_G(v)] = (2^{7nk-4k-n+2})(3^{2nk-k-1}). \tag{2.8}$$

In the path $R(P_{k+1})$, vertex v_1 makes the sum 8 with $k-1$ subdivision vertices in the path as well as with v_l. Also v_1 makes the sum 10 with $k-2$ vertices in the path. The subdivision vertex v_j in the path makes the sum 4 with the remaining $k-1$ subdivision vertices as well as with v_l. It also makes the sum 6 with $k-2$ vertices in the path. The neighbors of v_j in the path make the sum 8 with $k-3$ vertices and 6 with $k-2$ vertices and so on. Thus in the path,

$$\overline{\prod}_1 [d_G(u)+d_G(v)] = [(2^{7k^2-15k+15})(3^{2k^2-6k+4})(5^{2k-4})]^{\frac{1}{2}}. \tag{2.9}$$

By multiplying equations (2.7), (2.8) and (2.9) we have:

$$\overline{\prod}_1 (R(T_{n,k})) = (2^{\frac{1}{2}(7n^2+7k^2-23k-17n+23)})(3^{k^2+n^2-4k-3n+3})(5^{k+n-5}).$$

By Proposition 1, $\overline{\prod}_2 (R(T_{n,k})) = (2^{6(n+k)^2-17(n+k)+17})(3^{2(n+k)-7})$. □

3 The multiplicative Zagreb co-indices on $S(G)$ and $R(G)$ for wheel graph

In this section we compute the multiplicative Zagreb co-indices on two graph operators $S(G)$ and $R(G)$ for wheel graph W_{n+1}.

Theorem 4. *The multiplicative Zagreb co-indices for the wheel graph W_{n+1} are*

$$\overline{\prod}_1 (W_{n+1}) = 6^{\frac{n^2-3n}{2}}, \quad \overline{\prod}_2 (W_{n+1}) = 3^{n(n-3)}.$$

Proof. In W_{n+1}, the hub of the wheel is of degree n and the remaining vertices are of degree 3. Each vertex on C_n has $n-3$ non-adjacent vertices of degree 3. Hence $\overline{\prod}_1 [d_G(u)+d_G(v)] = 6^{n^2-3n}$. Since one edge is shared between a pair of vertices, then

$$\overline{\prod}_1 (W_{n+1}) = 6^{\frac{n^2-3n}{2}}.$$

Proposition 1 implies that

$$\overline{\prod}_2 (W_{n+1}) = 3^{n(n-3)}.$$

□

Theorem 5. *For the subdivision graph $S(G)$ of a wheel graph, the multiplicative Zagreb co-indices are*

$$\overline{\prod}_1 S(W_{n+1}) = [5^{4n-6}\ 4^{4n-2}\ 6^{n-1}\ (2+n)^2\ (n+3)^2]^{\frac{n}{2}}$$

and

$$\overline{\prod}_2 S(W_{n+1}) = (3^{3n-3}4^{3n-2}n^3)^n.$$

Proof. $S(W_{n+1})$ contains n vertices of degree 3, $2n$ vertices of degree 2 and one vertex of degree n. Each vertex of degree 3 has $n-1$ non-adjacent vertices of degree 3, $2n-3$ non-adjacent vertices of degree 2 and one vertex of degree n. So,

$$\overline{\prod}_1[d_G(u)+d_G(v)] = [6^{n-1}\ 5^{2n-3}\ (3+n)]^n. \tag{3.1}$$

The subdivision vertices of degree 2 on $S(C_n)$ are non-adjacent with $n-2$ vertices of degree 3, $2n-1$ vertices of degree 2 and one vertex of degree n. Hence

$$\overline{\prod}_1[d_G(u)+d_G(v)] = [5^{n-2}\ 4^{2n-1}\ (2+n)]^n. \tag{3.2}$$

The remaining subdivision vertices of degree 2 are non-adjacent with $n-1$ vertices of degree 3 and $2n-1$ vertices of degree 2. So,

$$\overline{\prod}_1[d_G(u)+d_G(v)] = [5^{n-1}\ 4^{2n-1}]^n. \tag{3.3}$$

The hub of the wheel has n non-adjacent vertices of degree 3 and n non-adjacent vertices of degree 2. Hence

$$\overline{\prod}_1[d(u)+d(v)] = [n+2(3+n)]^n. \tag{3.4}$$

The equations (3.1), (3.2), (3.3) and (3.4) make the product

$$\overline{\prod}_1[d(u)+d(v)] = [5^{4n-6}\ 4^{4n-2}\ 6^{n-1}\ (2+n)^2\ (n+3)^2]^n.$$

Since one edge is shared between a pair of vertices, then

$$\overline{\prod}_1 S(W_{n+1}) = [5^{4n-6}\ 4^{4n-2}\ 6^{n-1}\ (2+n)^2\ (n+3)^2]^{\frac{n}{2}}.$$

Proposition 1 implies that

$$\overline{\prod}_2 S(W_{n+1}) = (3^{3n-3}4^{3n-2}n^3)^n.$$

□

Theorem 6. *For the subdivision graph $R(G)$ of a wheel graph, the multiplicative Zagreb co-indices are*

$$\overline{\prod}_1 R(W_{n+1}) = [(2n+2)^2\ 2^{24n-28}\ 3^{n-3}]^{\frac{n}{2}}$$

and

$$\overline{\prod}_2 R(W_{n+1}) = 2^{6n^2+6k^2+12nk-17n-17k+9}3^{2n+2k-7}.$$

Proof. In $R(W_{n+1})$, n vertices are of degree 6, hub of the wheel is of degree $2n$ and all subdivision vertices are of degree 2. Hence, $\overline{\prod}_1[d(u)+d(v)]$ with respect to the hub of the wheel is

$$\overline{\prod}_1[d(u)+d(v)] = (2n+2)^n. \tag{3.5}$$

The product of [d(u)+d(v)] degrees with respect to all the n vertices of C_n is given by

$$\overline{\prod}_1[d(u)+d(v)] = [8^{2n-3}\ 12^{n-3}]^n. \tag{3.6}$$

With respect to the n subdivision vertices on the spokes of the wheel, $\overline{\prod}_1[d(u)+d(v)]$ is

$$\overline{\prod}_1[d(u)+d(v)] = [4^{2n-1}\ 8^{n-1}]^n. \tag{3.7}$$

The calculation with respect to n subdivision vertices on the edge of the cycle C_n of $R(W_{n+1})$ is

$$\overline{\prod}_1[d(u)+d(v)] = [8^{n-2}\ 4^{2n-1}\ (2n+2)]^n. \tag{3.8}$$

The equations (3.5), (3.6), (3.7) and (3.8) make the product

$$\overline{\prod}_1[d(u)+d(v)] = [(2n+2)^2\ 2^{22n-28}\ 3^{n-3}]^n.$$

Since one edge is shared by a pair of vertices, then

$$\overline{\prod}_1 R(W_{n+1}) = [(2n+2)^2\ 2^{24n-28}\ 3^{n-3}]^{\frac{n}{2}}.$$

Proposition 1 implies that

$$\overline{\prod}_2 R(W_{n+1}) = 2^{6n^2+6k^2+12nk-17n-17k+9}3^{2n+2k-7}.$$

□

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