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A note on six-dimensional planar Hermitian submanifolds of Cayley algebra

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Abstract. Six-dimensional planar Hermitian submanifolds of Cayley algebra are considered. It is proved that if such a submanifold of the octave algebta satisfies the U-Kenmotsu hypersurfaces axiom, then it is Kählerian. It is also proved that a symmetric non-Kählerian Hermitian six-dimensional submanifold of the Ricci type does not admit totally umbilical Kenmotsu hypersurfaces.

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1 Introduction

The almost Hermitian structures (AH-structures) belong to the most important and meaningful differential-geometrical structures. The existence of 3-vector cross products on Cayley algebra gives a lot of substantive examples of almost Hermitian manifolds. As it is well known, every 3-vector cross product on Cayley algebra induces a 1-vector cross product (or, what is the same in this case, an almost Hermitian structure) on its six-dimensional oriented submanifold (see [10–12]). Such almost Hermitian structures (in particular, Hermitian, special Hermitian, nearly-Kählerian, Kählerian etc) were studied by a number of remarkable geometers: E. Calabi, J.-T. Cho, R. Deszcs, F. Dillen, N. Ejiri, S. Funabashi, A. Gray, Guoxin Wei, Haizhong Li, H. Hashimoto, V. F. Kirichenko, J. S. Pak, K. Sekigawa, L. Verstraelen, L. Vranchen and others. For example, a complete classification of nearly-Kählerian [15], Kählerian [16] and locally symmetric Hermitian structures [17] on six-dimensional submanifolds of the octave algebra has been obtained.

The almost contact metric structures are also remarkable and very important differential-geometrical structures. These structures are studied from the point of view of differential geometry as well as of modern theoretical physics. We mark out the close connection of almost contact metric and almost Hermitian structures. For instance, an almost contact metric structure is induced on an oriented hypersurface of an almost Hermitian manifold [22].

In the present paper, we consider six-dimensional Hermitian planar submanifolds of Cayley algebra. We shall prove the following main results.

Theorem 1. *If a six-dimensional Hermitian planar submanifold of Cayley algebra satisfies the U-Kenmotsu hypersurfaces axiom, then it is Kählerian.*

Theorem 2. *A symmetric non-Kahlerian Hermitian six-dimensional submanifold of the Ricci type does not admit totally umbilical Kenmotsu hypersurfaces.*

This article is the continuation of the authors' researches in the area of planar Hermitian submanifolds of Cayley algebra (see [2, 6, 7] and others).

2 Preliminaries

Let us consider an almost Hermitian manifold, i. e. a $2n$-dimensional manifold M^{2n} with a Riemannian metric $g = \langle \cdot, \cdot \rangle$ and an almost complex structure J. Moreover, the following condition must hold

$$\langle JX,\ JY \rangle = \langle X,\ Y \rangle\,, \quad X, Y \in \aleph(M^{2n}),$$

where $\aleph(M^{2n})$ is the module of smooth vector fields on M^{2n}. All considered manifolds, tensor fields and similar objects are assumed to be of the class C^∞.

The specification of an almost Hermitian structure on a manifold is equivalent to the setting of a G-structure, where G is the unitary group $U(n)$ [19]. Its elements are the frames adapted to the structure (A-frames). They look as follows:

$$(p,\, \varepsilon_1,\, \ldots, \varepsilon_n,\, \varepsilon_{\hat{1}},\, \ldots,\, \varepsilon_{\hat{n}}\,),$$

where ε_a are the eigenvectors corresponding to the eigenvalue $i = \sqrt{-1}$, and $\varepsilon_{\hat{a}}$ are the eigenvectors corresponding to the eigenvalue $-i$. Here the index a ranges from 1 to n, and we state $\hat{a} = a + n$.

Therefore, the matrices of the operator of the almost complex structure and of the Riemannian metric written in an A-frame look as follows, respectively:

$$\left(J^k_j\right) = \left(\begin{array}{c|c} iI_n & 0 \\ \hline 0 & -iI_n \end{array}\right), \ \ (g_{kj}) = \left(\begin{array}{c|c} 0 & I_n \\ \hline I_n & 0 \end{array}\right),$$

where I_n is the identity matrix; $k, j = 1, \ldots, 2n$.

We recall that the fundamental form (or Kählerian form) of an almost Hermitian manifold is determined by the relation

$$F(X,\ Y) = \langle X,\ JY \rangle\,, \quad X, Y \in \aleph(M^{2n}).$$

By direct computing it is easy to obtain that in A-frame the fundamental form matrix looks as follows:

$$(F_{kj}) = \left(\begin{array}{c|c} 0 & iI_n \\ \hline -iI_n & 0 \end{array}\right).$$

An almost Hermitian manifold is called Hermitian if its structure is integrable. The following identity characterizes the Hermitian structure [13, 19]:

$$\nabla_X(F)(Y,Z) - \nabla_{JX}(F)(JY,Z) = 0,$$

where $X, Y, Z \in \aleph(M^{2n})$. The first group of the Cartan structural equations of a Hermitian manifold written in an A-frame looks as follows [19]:

$$d\omega^a = \omega^a_b \wedge \omega^b + B^{ab}_c \omega^c \wedge \omega_b,$$

$$d\omega_a = -\omega^b_\alpha \wedge \omega_b + B^c_{ab} \omega_c \wedge \omega^b,$$

where $\left\{B^{ab}_c\right\}$ and $\{B^c_{ab}\}$ are components M^{2n} of the Kirichenko tensors of [1, 5]; $a, b, c = 1, ..., n$.

We recall also that an almost contact metric structure on an odd-dimensional manifold N is defined by the system of tensor fields $\{\Phi,\, \xi,\, \eta,\, g\}$ on this manifold, where ξ is a vector field, η is a covector field, Φ is a tensor of the type (1, 1) and $g = \langle\cdot,\, \cdot\rangle$ is the Riemannian metric [8, 9]. Moreover, the following conditions are fulfilled:

$$\eta(\xi) = 1,\ \Phi(\xi) = 0,\ \eta \circ \Phi = 0,\ \Phi^2 = -id + \xi \otimes \eta,$$

$$\langle \Phi X, \Phi Y\rangle = \langle \Phi X, \Phi Y\rangle - \eta\,(X)\,\eta\,(Y)\,,\ X, Y \in \aleph(N),$$

where $\aleph(M^{2n})$ is the module of smooth vector fields on N. As an example of an almost contact metric structure we can consider the cosymplectic structure that is characterized by the following condition:

$$\nabla\eta = 0, \quad \nabla\Phi = 0,$$

where ∇ is the Levi-Civita connection of the metric. It has been proved that the manifold which admits the cosymplectic structure is locally equivalent to the product $M \times R$, where M is a Kählerian manifold [20].

As it was mentioned, the almost contact metric structures are closely connected to the almost Hermitian structures. For instance, if $(N, \{\Phi,\, \xi,\, \eta,\, g\})$ is an almost contact metric manifold, then an almost Hermitian structure is induced on the product $N \times R$ [8]. If this almost Hermitian structure is integrable, then the input almost contact metric structure is called normal. As it is known, a normal contact metric structure is called Sasakian [8]. On the other hand, we can characterize the Sasakian structure by the following condition [19]:

$$\nabla_X(\Phi)Y = \langle X, Y\rangle\, \xi - \eta(Y)X,\ \ X, Y \in \aleph(N).$$

For example, Sasakian structures are induced on totally umbilical hypersurfaces in a Kahlerian manifold [8]. As it is well known, the Sasakian structures have many remarkable properties and play a fundamental role in contact geometry.

In 1972 Katsuei Kenmotsu introduced a new class of almost contact metric structures [14] defined by the condition:

$$\nabla_X(\Phi)Y = \langle \Phi X, Y\rangle\, \xi - \eta(Y)\Phi X,\ X, Y \in \aleph(N).$$

The Kenmotsu manifolds are normal and integrable, but they are not contact, consequently, they can not be Sasakian. In spite of the fact that the conditions for these kinds of manifolds are similar, the properties of Kenmotsu manifolds are to some extent antipodal to the Sasakian manifolds properties [18]. Note that the remarkable investigation [18] in this field contains a detailed description of Kenmotsu manifolds as well as a collection of examples of such manifolds. We mark out also the recent fundamental and profound work by G. Pitis that contains a survey of most important results on geometry of Kenmotsu manifolds [21].

3 Proof of theorems

At first, we remind that an almost Hermitian manifold M^{2n} satisfies the U-Kenmotsu hypersurfaces axiom if a totally umbilical Kenmotsu hypersurface passes through every point of this manifold.

Let $O \equiv R^8$ be the Cayley algebra. As it is well-known [12], two non-isomorphic three-fold vector cross products are defined on it by means of the relations:

$$P_1(X,\ Y,\ Z\,) = -X(\bar{Y}Z) + \langle X,\, Y\rangle\, Z + \langle Y,\, Z\rangle\, X - \langle Z, X\rangle\, Y,$$

$$P_2(X,\ Y,\ Z\,) = -(X\bar{Y})Z + \langle X,\, Y\rangle\, Z + \langle Y,\, Z\rangle\, X - \langle Z, X\rangle\, Y,$$

where $X,\, Y,\, Z \in \mathbf{O}$, $\langle\cdot,\ \cdot\rangle$ is the scalar product in $\mathbf{O}$ and $X \to \bar{X}$ is the conjugation operator. Moreover, any other three-fold vector cross product in the octave algebra is isomorphic to one of the above-mentioned two.

If $M^6 \subset \mathbf{O}$ is a six-dimensional oriented submanifold, then the induced almost Hermitian structure $\{J_y, g = \langle\cdot,\, \cdot\rangle\}$ is determined by the relation

$$J_t(X) = P_t(X,\, e_1,\, e_2), \quad t = 1,\ 2,$$

where $\{e_1,\ e_2\}$ is an arbitrary orthonormal basis of the normal space of M^6 at the point p, $X \in T_p(M^6)$ [12].

We recall that the point $p \in M^6$ is called general [16, 17], if

$$e_0 \notin T_p(M^6),$$

where e_0 is the unit of Cayley algebra. A submanifold $M^6 \subset \mathbf{O}$, consisting only of general points, is called a general-type submanifold [16]. In what follows, all submanifolds M^6 that will be considered are assumed to be of general type.

Let N be an arbitrary oriented hypersurface of a six-dimensional Hermitian submanifold $M^6 \subset \mathbf{O}$ of Cayley algebra, let σ be the second fundamental form of immersion of N into M^6. The Cartan structural equations of the almost contact metric structure on such a hypersurface look as follows [22]:

$$d\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta + B^{\alpha\beta}_\gamma \omega^\gamma \wedge \omega_\beta +$$

$$+\left(\sqrt{2}B^{\alpha 3}_\beta + i\sigma^\alpha_\beta\right)\omega^\beta \wedge \omega + \left(-\frac{1}{\sqrt{2}}B^{\alpha\beta}_3 + i\sigma^{\alpha\beta}\right)\omega_\beta \wedge \omega,$$

$$d\omega_\alpha = -\omega_\alpha^\beta \wedge \omega_\beta + B^\gamma_{\alpha\beta}\omega_\gamma \wedge \omega^\beta +$$
$$+\left(\sqrt{2}B^\beta_{\alpha 3} - i\sigma^\beta_\alpha\right)\omega_\beta \wedge \omega + \left(-\frac{1}{\sqrt{2}}B^3_{\alpha\beta} - i\sigma_{\alpha\beta}\right)\omega^\beta \wedge \omega,$$
$$d\omega = \left(\sqrt{2}B^{3\alpha}_\beta - \sqrt{2}B^\alpha_{3\beta} - 2i\sigma^\alpha_\beta\right)\omega^\beta \wedge \omega_\alpha + \left(B^{3\beta}_3 - i\sigma^\beta_3\right)\omega \wedge \omega_\beta.$$

Here the indices α, β, γ range from 1 to 2. Taking into account that the Cartan structural equations of a Kenmotsu structure look as follows [18]:

$$d\omega^\alpha = \omega^\alpha_\beta \wedge \omega^\beta + \omega \wedge \omega^\alpha;$$
$$d\omega_\alpha = -\omega^\beta_\alpha \wedge \omega_\beta + \omega \wedge \omega_\alpha;$$
$$d\omega = 0,$$

we get the conditions whose simultaneous fulfillment is a criterion for the structure induced on N to be Kenmotsu:

$$\begin{gathered} 1)\ B^{\alpha\beta}_\gamma = 0; \\ 2)\ \sqrt{2}B^{\alpha 3}_\beta + i\sigma^\alpha_\beta = -\delta^\alpha_\beta; \\ 3)\ -\frac{1}{\sqrt{2}}B^{\alpha\beta}_3 + i\sigma^{\alpha\beta} = 0; \\ 4)\ \sqrt{2}B^{3\alpha}_\beta - \sqrt{2}B^\alpha_{3\beta} - 2i\sigma^\alpha_\beta = 0; \\ 5)\ B^{3\beta}_3 - i\sigma^\beta_3 = 0; \end{gathered} \tag{1}$$

and the formulae, obtained by complex conjugation (no need to write them explicitly).

From $(1)_3$ we obtain:

$$\sigma^{\alpha\beta} = -\frac{i}{\sqrt{2}}B^{\alpha\beta}_3.$$

By alternating this relation we get:

$$0 = \sigma^{[\alpha\beta]} = -\frac{i}{\sqrt{2}}B^{[\alpha\beta]}_3 = -\frac{i}{2\sqrt{2}}\left(B^{\alpha\beta}_3 - B^{\beta\alpha}_3\right) = -\frac{i}{\sqrt{2}}B^{\alpha\beta}_3.$$

That is why $B^{\alpha\beta}_3 = 0$, therefore

$$\sigma^{\alpha\beta} = 0.$$

Similarly, from $(1)_5$ we obtain:

$$\sigma^\beta_3 = 0.$$

So, we can rewrite the conditions (1) as follows:

$$1)\ B^{\alpha\beta}_\gamma = 0; \quad 2)\ \sigma^{\alpha\beta} = 0; \quad 3)\ \sigma^\beta_3 = 0; \quad 4)\ \sigma^\alpha_\beta = i\sqrt{2}B^{\alpha 3}_\beta + i\delta^\alpha_\beta \tag{2}$$

and the formulae, obtained by complex conjugation.

Next, let us use the expressions for Kirichenko tensors of six-dimensional Hermitian submanifolds of Cayley algebra [3, 4, 16]:

$$B_c^{ab} = \frac{1}{\sqrt{2}}\varepsilon^{abh}\, D_{hc},\;\; B_{ab}^{c} = \frac{1}{\sqrt{2}}\varepsilon_{abh}\, D^{hc}, \tag{3}$$

where

$$\varepsilon^{abc} = \varepsilon_{123}^{abc}, \varepsilon_{abc} = \varepsilon_{abc}^{123}$$

are the components of the third-order Kronecher tensor [16] and

$$D_{hc} = \pm T_{hc}^{8} + iT_{hc}^{7},$$

$$D^{hc} = D_{\hat{h}\hat{c}} = \pm T_{\hat{h}\hat{c}}^{8} - iT_{\hat{h}\hat{c}}^{7}.$$

Here $\{T_{hc}^{\varphi}\}$ are the components of the configuration tensor (in A. Gray's notation) of the Hermitian submanifold $M^6 \subset \mathbf{O}$; the index φ ranges from 7 to 8 and the indices a, b, c, h range from 1 to 3 [3, 4, 16].

Taking into account (2) and (3), we get:

$$\begin{gathered}
\sigma_{\hat{1}1} = \sigma_1^1 = i\sqrt{2}B_1^{13} + i\delta_1^1 = i\sqrt{2}(\frac{1}{\sqrt{2}}\varepsilon^{13\gamma}D_{\gamma 1}) + i = -iD_{12} + i;\\
\sigma_{\hat{2}2} = \sigma_2^2 = i\sqrt{2}B_2^{23} + i\delta_2^2 = i\sqrt{2}(\frac{1}{\sqrt{2}}\varepsilon^{23\gamma}D_{\gamma 2}) + i = iD_{12} + i;\\
\sigma_{\hat{1}2} = \sigma_2^1 = i\sqrt{2}B_2^{13} + i\delta_2^1 = i\sqrt{2}(\frac{1}{\sqrt{2}}\varepsilon^{13\gamma}D_{\gamma 2}) = -iD_{22};\\
\sigma_{\hat{2}1} = \sigma_1^2 = i\sqrt{2}B_1^{23} + i\delta_1^2 = i\sqrt{2}(\frac{1}{\sqrt{2}}\varepsilon^{23\gamma}D_{\gamma 1}) = iD_{11}.
\end{gathered} \tag{4}$$

If N is a totally umbilical submanifold of M^6, then for its second fundamental form we have:

$$\sigma_{ps} = \lambda g_{ps}, \quad \lambda - const, \quad p, s = 1, 2, 3, 4, 5. \tag{5}$$

Taking into account that the matrix of the contravariant metric tensor of the hypersurface N looks as follows [2]:

$$(g^{ps}) = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix},$$

we conclude from (2), (4), and (5) that

$$B_{31}^{2} = 0\,,\;\; B_{32}^{1} = 0.$$

Consequently,

$$\frac{1}{\sqrt{2}}\varepsilon_{31h}\, D^{h2} = 0 \;\Leftrightarrow \varepsilon_{312}\, D^{22} = 0 \;\Leftrightarrow$$

$$\Leftrightarrow\ D^{22} = 0 \ \Leftrightarrow D_{22} = 0;$$

$$\frac{1}{\sqrt{2}}\varepsilon_{32h}\, D^{h1} = 0 \ \Leftrightarrow \varepsilon_{321}\, D^{11} = 0 \ \Leftrightarrow$$

$$\Leftrightarrow\ D^{11} = 0 \ \Leftrightarrow D_{11} = 0.$$

Knowing the identity from [4]

$$(D_{12})^2 = D_{11}D_{22}, \tag{6}$$

we conclude that $D_{\alpha\beta} = 0$. Moreover, from (2) it follows that

$$B^{\alpha\beta}_{\gamma} = 0 \ \Leftrightarrow \frac{1}{\sqrt{2}}\varepsilon^{\alpha\beta 3}\, D_{3\gamma} = 0 \ \Leftrightarrow\ D_{3\gamma} = 0;$$

$$B^{\alpha\beta}_{3} = 0 \ \Leftrightarrow \frac{1}{\sqrt{2}}\varepsilon^{\alpha\beta 3}\, D_{33} = 0 \ \Leftrightarrow\ D_{33} = 0.$$

So, the matrix (D_{ab}) vanishes:

$$D_{ab} \equiv 0. \tag{7}$$

As we can see the condition (7) is fulfilled at every point of totally umbilical Kenmotsu hypersurface of six-dimensional Hermitian submanifold of the octave algebra. But this condition is a criterion for the six-dimensional submanifold $M^6 \subset \mathbf{O}$ to be Kählerian [3, 16]. That is why if $M^6 \subset \mathbf{O}$ satisfies with the U-Kenmotsu hypersurfaces axiom, then it is a Kählerian manifold. So, the Theorem 1 is completely proved.

As it was mentioned above, the paper [17] by V. F. Kirichenko contains a complete classification of six-dimensional Kählerian submanifolds of Cayley algebra. Now, we can state that this paper contains a complete classification of six-dimensional planar Hermitian submanifolds of Cayley algebra satisfying the U-Kenmotsu hypersurfaces axiom. We remark also that the property to satisfy the U-Kenmotsu hypersurfaces axiom essentially simplify the structure of the six-dimensional planar Hermitian submanifold of the octave algebra.

Locally symmetric $M^6 \subset \mathbf{O}$ are important and substantive examples of six-dimensional Hermitian planar submanifolds of Cayley algebra [4]. As we have just mentioned, the most interesting work on this subject is the article by V. F. Kirichenko [17]. In this paper, the notion of six-dimensional Hermitian Ricci type submanifolds was introduced. We note that the point $p \in M^6$ is called special if

$$T_p(M^6) \subset L(e_0)^{\perp},$$

where $L(e_0)^{\perp}$ is the orthogonal supplement of the unit of Cayley algebra. Otherwise, the point p is called simple. It is evident that the set of all simple points in M^6 forms an open submanifold $M^6_0 \subset M^6$, on which canonically is determined the one-dimensional distribution Z induced by the orthogonal projections of e_0 on the tangent spaces $T_p(M^6)$ for all points $p \in M_0^6$. Such a distribution Z as well as the one-dimensional space $Z_p \in T_p(M^6)$, $p \in M_0^6$, are called exceptional [17].

In accordance with the definition [4,17], a Hermitian $M^6 \subset \mathbf{O}$ is called a manifold of the Ricci type if its Ricci curvature at every point $p \in M_0^6$ in the direction of the exceptional space Z_p gets the minimum value.

Now, we use the complete classification of locally symmetric Hermitian $M^6 \subset \mathbf{O}$ of the Ricci type obtained by V. F. Kirichenko: every Hermitian locally symmetric submanifold $M^6 \subset \mathbf{O}$ of the Ricci type is locally holomorphically isometric either to C^3 or to the product of Kählerian manifolds C^2 and CH^1 , "twisted" along CH^1. (Here CH^1 denotes the complex hyperbolic space.)

In [17] it is also proved that the matrices (D_{ab}), (T^8_{ab}) and T^8_{ab} with a corresponding choice of the frame look as follows, respectively:

$$\begin{pmatrix} D_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} T^8_{ab} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \begin{pmatrix} T^7_{ab} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Moreover, for the case of $C^2 \times CH^1$ the conditions

$$D_{11} \neq 0;\ T^8_{ab} \neq 0;\ T^7_{ab} \neq 0$$

are simultaneously fulfilled.

Applying (4) and (6), we obtain the matrix of the second fundamental form of the immersion of Kenmotsu hypersurface in such a locally symmetric submanifold $M^6 \subset \mathbf{O}$ of the Ricci type:

$$(\sigma_{ps}) = \begin{pmatrix} 0 & 0 & 0 & -i & -iD^{11} \\ 0 & 0 & 0 & 0 & -i \\ 0 & 0 & \sigma_{33} & 0 & 0 \\ i & 0 & 0 & 0 & 0 \\ iD_{11} & i & 0 & 0 & 0 \end{pmatrix}.$$

It is easy to see that the condition (5) can not hold. That is why we conclude that the Kenmotsu hypersurface in a non-Kählerian locally symmetric submanifold $M^6 \subset \mathbf{O}$ of the Ricci type can not be totally umbilical. So, Theorem 2 is also completely proved.

Computing the determinant of the matrix of the second fundamental form of the immersion of Kenmotsu hypersurface in a non-Kahlerian locally symmetric submanifold $M^6 \subset \mathbf{O}$ of the Ricci type we have:

$$det(\sigma_{ps}) = \sigma_{33} \begin{vmatrix} 0 & 0 & -i & -iD^{11} \\ 0 & 0 & 0 & -i \\ i & 0 & 0 & 0 \\ iD_{11} & i & 0 & 0 \end{vmatrix} = i\,\sigma_{33} \begin{vmatrix} 0 & -i & -iD^{11} \\ 0 & 0 & -i \\ i & 0 & 0 \end{vmatrix} = \sigma_{33}.$$

We obtain that the matrix is degenerate if and only if $\sigma_{33} = 0$. Knowing that this equality is equivalent to the condition of minimality of a Kenmotsu hypersurface in a Hermitian manifold $\sigma(\xi, \xi) = 0$ [2], we get the following additional result.

Corollary. *The Kenmotsu hypersurface of a locally symmetric submanifold* $M^6 \subset \mathbf{O}$ *of the Ricci type is minimal if and only if its second fundamental form matrix is degenerate.*

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