BULETINUL ACADEMIEI DE ŞTIINŢE
A REPUBLICII MOLDOVA. MATEMATICA
Numbers 2(72)–3(73), 2013, Pages 99–105
ISSN 1024–7696

A selection theorem for set-valued maps into normally supercompact spaces

V. Valov*

Abstract. The following selection theorem is established:
Let X be a compactum possessing a binary normal subbase $\mathcal{S}$ for its closed subsets. Then every set-valued $\mathcal{S}$-continuous map $\Phi\colon Z \to X$ with closed $\mathcal{S}$-convex values, where Z is an arbitrary space, has a continuous single-valued selection. More generally, if $A \subset Z$ is closed and any map from A to X is continuously extendable to a map from Z to X, then every selection for $\Phi|A$ can be extended to a selection for Φ.
This theorem implies that if X is a κ-metrizable (resp., κ-metrizable and connected) compactum with a normal binary closed subbase $\mathcal{S}$, then every open $\mathcal{S}$-convex surjection $f\colon X \to Y$ is a zero-soft (resp., soft) map. Our results provide some generalizations and specifications of Ivanov's results (see [5–7]) concerning superextensions of κ-metrizable compacta

Mathematics subject classification: 54C65, 54F65.
Keywords and phrases: Continuous selections, Dugundji spaces, κ-metrizable spaces, spaces with closed binary normal subbase, superextensions.

1 Introduction

In this paper we assume that all topological spaces are Tychonoff and all single-valued maps are continuous.

Recall that supercompact spaces and superextensions were introduced by de Groot [4]. A space is *supercompact* if it possesses a binary subbase for its closed subsets. Here, a collection $\mathcal{S}$ of closed subsets of X is *binary* provided any linked subfamily of $\mathcal{S}$ has a non-empty intersection (we say that a system of subsets of X is *linked* provided any two elements of this system intersect). The supercompact spaces with binary *normal* subbase will be of special interest for us. A subbase $\mathcal{S}$ which is closed both under finite intersections and finite unions is called normal if for every $S_0, S_1 \in \mathcal{S}$ with $S_0 \cap S_1 = \varnothing$ there exists $T_0, T_1 \in \mathcal{S}$ such that $S_0 \cap T_1 = \varnothing = T_0 \cap S_1$ and $T_0 \cup T_1 = X$. A space X possessing a binary normal subbase $\mathcal{S}$ is called *normally supercompact* [9] and will be denoted by $(X, \mathcal{S})$.

The *superextension* λX of X consists of all maximal linked systems of closed sets in X. The family

$$U^+ = \{\eta \in \lambda X : F \subset U \text{ for some } F \in \eta\},$$

$U \subset X$ is open, is a subbase for the topology of λX. It is well known that λX is normally supercompact. Let η_x, $x \in X$, be the maximal linked system of all closed

*The author was partially supported by NSERC Grant 261914-13.

sets in X containing x. The map $x \to \eta_x$ embeds X into λX. The book of van Mill [9] contains more information about normally supercompact space and superextensions, see also Fedorchuk-Filippov's book [3].

If $\mathcal{S}$ is a closed subbase for X and $B \subset X$, let $I_{\mathcal{S}}(B) = \bigcap\{S \in \mathcal{S} : B \subset S\}$. A subset $B \subset X$ is called *$\mathcal{S}$-convex* if for all $x, y \in B$ we have $I_{\mathcal{S}}(\{x,y\}) \subset B$. An *$\mathcal{S}$-convex map* $f \colon X \to Y$ is a map whose fibers are $\mathcal{S}$-convex sets. A set-valued map $\Phi \colon Z \to X$ is said to be *$\mathcal{S}$-continuous* provided for any $S \in \mathcal{S}$ both sets $\{z \in Z : \Phi(z) \cap (X \backslash S) \neq \varnothing\}$ and $\{z \in Z : \Phi(z) \subset X \backslash S\}$ are open in Z.

Theorem 1. *Let $(X, \mathcal{S})$ be a normally supercompact space and Z an arbitrary space. Then every $\mathcal{S}$-continuous set-valued map $\Phi \colon Z \to X$ has a single-valued selection provided all $\Phi(z)$, $z \in Z$, are $\mathcal{S}$-convex closed subsets of X. More generally, if $A \subset Z$ is closed and every map from A to X can be extended to a map from Z to X, then every selection for $\Phi|A$ is extendable to a selection for Φ.*

Corollary 1. *Let $\Phi \colon Z \to X$ be an $\mathcal{S}$-continuous set-valued map such that each $\Phi(z) \subset X$ is closed, where X is a space with a binary normal closed subbase $\mathcal{S}$ and Z arbitrary. Then the map $\Psi \colon Z \to X$, $\Psi(z) = I_{\mathcal{S}}\big(\Phi(z)\big)$, has a continuous selection.*

A map $f \colon X \to Y$ is invertible if for any space Z and a map $g \colon Z \to Y$ there exists a map $h \colon Z \to X$ with $f \circ h = g$. If X has a closed subbase $\mathcal{S}$, we say $f \colon X \to Y$ is *$\mathcal{S}$-open* provided $f(X \backslash S) \subset Y$ is open for every $S \in \mathcal{S}$. Theorem 1 yields next corollary.

Corollary 2. *Let X be a space possessing a binary normal closed subbase $\mathcal{S}$. Then every $\mathcal{S}$-convex $\mathcal{S}$-open surjection $f \colon X \to Y$ is invertible.*

Another corollary of Theorem 1 is a specification of Ivanov's results [7] (see also [5] and [6]). Here, a map $f \colon X \to Y$ is $\mathcal{A}$-soft, where $\mathcal{A}$ is a class of spaces, if for any $Z \in \mathcal{A}$, its closed subset A and any two maps $k \colon Z \to Y$, $h \colon A \to X$ with $f \circ h = k|A$ there exists a map $g \colon Z \to X$ extending h such that $f \circ g = k$. When $\mathcal{A}$ is the family of all (0-dimensional) paracompact spaces, then $\mathcal{A}$-soft maps are called (0-)soft [11].

Corollary 3. *Let $\mathcal{A}$ be a given class of spaces and X be an absolute extensor for all $Z \in \mathcal{A}$. If X has a binary normal closed subbase $\mathcal{S}$, then any $\mathcal{S}$-convex $\mathcal{S}$-open surjection $f \colon X \to Y$ is $\mathcal{A}$-soft.*

Theorem 1 is also applied to establish the following proposition:

Proposition 1. *Let X be a κ-metrizable (resp., κ-metrizable and connected) compactum with a normal binary closed subbase $\mathcal{S}$. Then every open $\mathcal{S}$-convex surjection $f \colon X \to Y$ is a zero-soft (resp., soft) map.*

Corollary 4 (see [5,6]). *Let X be a κ-metrizable (resp., κ-metrizable and connected) compactum. Then $\lambda f \colon \lambda X \to \lambda Y$ is a zero-soft (resp., soft) map for any open surjection $f \colon X \to Y$.*

2 Proof of Theorem 1 and Corollaries 1–3

Recall that a set-valued map $\Phi : Z \to X$ is lower semi-continuous (br., lsc) if the set $\{z \in Z : \Phi(z) \cap U \neq \varnothing\}$ is open in Z for any open $U \subset X$. Φ is upper semi-continuous (br., usc) provided that the set $\{z \in Z : \Phi(z) \subset U\}$ is open in Z whenever $U \subset X$ is open. Upper semi-continuous and compact-valued maps are called usco maps. If Φ is both lsc and usc, it is said to be continuous. Obviously, every continuous set-valued map $\Phi : Z \to X$ is $\mathcal{S}$-continuous, where $\mathcal{S}$ is a binary closed normal subbase for X. Let $C(X, Y)$ denote the set of all (continuous single-valued) maps from X to Y.

Proof of Theorem 1. Suppose X has a binary normal closed subbase $\mathcal{S}$ and $\Phi\colon Z \to X$ is a set-valued $\mathcal{S}$-continuous map with closed $\mathcal{S}$-convex values. Let $A \subset Z$ be a closed set such that every $f \in C(A, X)$ can be extended to a map $\bar{f} \in C(Z, X)$. Fix a selection $g \in C(A, X)$ for $\Phi|A$ and its extension $\bar{g} \in C(Z, X)$. By [9, Theorem 1.5.18], there exists a (continuous) map $\xi : X \times \exp X \to X$, defined by

$$\xi(x, F) = \bigcap\{I_{\mathcal{S}}(\{x, a\}) : a \in F\} \cap I_{\mathcal{S}}(F),$$

where $\exp X$ is the space of all closed subsets of X with the Vietoris topology. This map has the following properties for any $F \in \exp X$: (i) $\xi(x, F) = x$ if $x \in I_{\mathcal{S}}(F)$; (ii) $\xi(x, F) \in I_{\mathcal{S}}(F)$, $x \in X$. Because each $\Phi(z)$, $z \in Z$, is a closed $\mathcal{S}$-convex set, $I_{\mathcal{S}}(\Phi(z)) = \Phi(z)$, see [9, Theorem 1.5.7]. So, for all $z \in Z$ we have $h(z) = \xi\big(\bar{g}(z), \Phi(z)\big) \in \Phi(z)$. Therefore, we obtain a map $h\colon Z \to X$ which is a selection for Φ and $h(z) = g(z)$ for all $z \in A$. It remains to show that h is continuous. We can show that the subbase could be supposed to be invariant with respect to finite intersections. Because ξ is continuous, this would imply continuity of h. But instead of that, we follow the arguments from the proof of [9, Theorem 1.5.18].

Let $z_0 \in Z$ and $x_0 = h(z_0) \in W$ with W being open in X. We may assume that $W = X\backslash S$ for some $S \in \mathcal{S}$. Because x_0 is the intersection of a subfamily of the binary family $\mathcal{S}$, there exists $S^* \in \mathcal{S}$ containing x_0 and disjoint from S. Since $\mathcal{S}$ is normal, there exist $S_0, S_1 \in \mathcal{S}$ such that $S \subset S_1\backslash S_0$, $x_0 \in S^* \subset S_0\backslash S_1$ and $S_0 \cup S_1 = X$. Hence, $x_0 \in (X\backslash S_1) \cap \Phi(z_0)$. Because Φ is $\mathcal{S}$-continuous, there exists a neighborhood $O_1(z_0)$ of z_0 such that $\Phi(z) \cap (X\backslash S_1) \neq \varnothing$ for every $z \in O_1(z_0)$. Observe that $\bar{g}(z_0) \in X\backslash S_1$ provided $\Phi(z_0) \cap S_1 \neq \varnothing$, otherwise $x_0 \in I_{\mathcal{S}}(\{\bar{g}(z_0), a\}) \subset S_1$, where $a \in \Phi(z_0) \cap S_1$. Consequently, we have two possibilities: either $\Phi(z_0) \subset X\backslash S_1$ or $\Phi(z_0)$ intersects both S_1 and $X\backslash S_1$. In the first case there exists a neighborhood $O_2(z_0)$ with $\Phi(z) \subset X\backslash S_1$ for all $z \in O_2(z_0)$, and in the second one take $O_2(z_0)$ such that $\bar{g}\big(O_2(z_0)\big) \subset X\backslash S_1$ (recall that in this case $\bar{g}(z_0) \in X\backslash S_1$). In both cases let $O(z_0) = O_1(z_0) \cap O_2(z_0)$. Then, in the first case we have $h(z) \in \Phi(z) \subset X\backslash S_1 \subset X\backslash S$ for every $z \in O(z_0)$. In the second case let $a(z) \in \Phi(z) \cap (X\backslash S_1)$, $z \in O(z_0)$. Consequently, $h(z) \in I_{\mathcal{S}}(\{\bar{g}(z), a(z)\}) \subset X\backslash S_1 \subset S_0 \subset X\backslash S$ for any $z \in O(z_0)$. Hence, h is continuous.

When the set A is a point a we define $g(a)$ to be an arbitrary point in $\Phi(a)$ and $\bar{g}(x) = g(a)$ for all $x \in X$. Then the above arguments provide a selection for Φ. $\square$

Proof of Corollary 1. Since each $\Psi(z)$ is $\mathcal{S}$-convex, by Theorem 1 it suffices to show that Ψ is $\mathcal{S}$-continuous. To this end, suppose that $F_0 \in \mathcal{S}$ and $\Psi(z_0)\cap(X\backslash F_0) \neq \varnothing$ for some $z_0 \in Z$. Then $\Phi(z_0) \cap (X\backslash F_0) \neq \varnothing$, for otherwise $\Phi(z_0) \subset F_0$ and $\Psi(z_0)$, being intersection of all $F \in \mathcal{S}$ containing $\Phi(z_0)$, would be contained in F_0. Since Φ is $\mathcal{S}$-continuous, there exists a neighborhood $O(z_0) \subset Z$ of z_0 such that $\Phi(z)\cap(X\backslash F_0) \neq \varnothing$ for all $z \in O(z_0)$. Consequently, $\Psi(z) \cap (X\backslash F_0) \neq \varnothing$, $z \in O(z_0)$.

Suppose now that $\Psi(z_0) \subset X\backslash F_0$. Then $\Psi(z_0) \cap F_0 = \varnothing$, so there exists $S_0 \in \mathcal{S}$ with $\Phi(z_0) \subset S_0$ and $S_0 \cap F_0 = \varnothing$ (recall that $\mathcal{S}$ is binary). Since $\mathcal{S}$ is normal, we can find $S_1, F_1 \in \mathcal{S}$ such that $S_0 \subset S_1\backslash F_1$, $F_0 \subset F_1\backslash S_1$ and $F_1 \cup S_1 = X$. Using again that Φ is $\mathcal{S}$-continuous to choose a neighborhood $U(z_0) \subset Z$ of z_0 with $\Phi(z) \subset X\backslash F_1 \subset S_1$ for all $z \in U(z_0)$. Hence, $\Psi(z) \subset S_1 \subset X\backslash F_0$, $z \in U(z_0)$, which completes the proof. □

Proof of Corollary 2. Let X possess a binary normal closed subbase $\mathcal{S}$, $f\colon X \to Y$ be an $\mathcal{S}$-open $\mathcal{S}$-convex surjection, and $g : Z \to Y$ be a map. Since f is both $\mathcal{S}$-open and closed (recall that X is compact as a space with a binary closed subbase), the map $\phi\colon Y \to X$, $\phi(y) = f^{-1}(y)$, is $\mathcal{S}$-continuous and $\mathcal{S}$-convex valued. So is the map $\Phi = \phi \circ g : Z \to X$. Then, by Theorem 1, Φ admits a continuous selection $h : Z \to X$. Obviously, $g = f \circ h$. Hence, f is invertible. □

Proof of Corollary 3. Suppose X is a compactum with a normal binary closed subbase $\mathcal{S}$ such that X is an absolute extensor for all $Z \in \mathcal{A}$. Let us show that every $\mathcal{S}$-open $\mathcal{S}$-convex surjection $f\colon X \to Y$ is $\mathcal{A}$-soft. Take a space $Z \in \mathcal{A}$, its closed subset A and two maps $k\colon Z \to Y$, $h\colon A \to X$ such that $k|A = f \circ h$. Then h can be continuously extended to a map $\bar{h}\colon Z \to X$. Moreover, the set-valued map $\Phi : Z \to X$, $\Phi(z) = f^{-1}(k(z))$, is $\mathcal{S}$-continuous and has $\mathcal{S}$-convex values. Hence, by Theorem 1, there is a selection $g\colon Z \to X$ for Φ extending h. Then $f \circ g = k$. So, f is $\mathcal{A}$-soft. □

3 Proof of Proposition 1 and Corollary 4

Proof of Proposition 1. According to Corollary 3, it suffices to show that X is a Dugundji space (resp., an absolute retract) provided X is a κ-metrizable (resp., κ-metrizable and connected) compactum with a normal binary closed subbase $\mathcal{S}$ (recall that the class of Dugundji spaces coincides with the class of compact absolute extensors for 0-dimensional spaces, see [8]). To this end, we follow the arguments from the proof of [12, Proposition 3.2]. Suppose first that X is a κ-metrizable compactum with a normal binary closed subbase $\mathcal{S}$. Consider X as a subset of a Tyhonoff cube $\mathbb{I}^\tau$. Then, by [10] (see also [12] for another proof), there exists a function $\mathrm{e}\colon \mathcal{T}_X \to \mathcal{T}_{\mathbb{I}^\tau}$ between the topologies of X and $\mathbb{I}^\tau$ such that:

(e1) $\mathrm{e}(\varnothing) = \varnothing$ and $\mathrm{e}(U) \cap X = U$ for any open $U \subset X$;

(e2) $\mathrm{e}(U) \cap \mathrm{e}(V) = \varnothing$ for any two disjoint open sets $U, V \subset X$.

Consider the set valued map $r\colon \mathbb{I}^\tau \to X$ defined by

$$r(y) = \bigcap\{I_{\mathcal{S}}(\overline{U}) : y \in e(U), U \in \mathcal{T}_X\} \text{ if } y \in \bigcup\{e(U) : U \in \mathcal{T}_X\} \qquad (1)$$

$$\text{and } r(y) = X \text{ otherwise,}$$

where $\overline{U}$ is the closure of U in X. According to condition (e2), the system $\gamma_y = \{U \in \mathcal{T}_X : y \in e(U)\}$ is linked for every $y \in \mathbb{I}^\tau$. Consequently, $\omega_y = \{S \in \mathcal{S} : \overline{U} \subset S \text{ for some } U \in \gamma_y\}$ is also linked. This implies $r(y) = \bigcap\{S : S \in \omega_y\} \neq \varnothing$ because $\mathcal{S}$ is binary.

Claim. *$r(x) = \{x\}$ for every $x \in X$.*

Suppose there is another point $z \in r(x)$. Then, by normality of $\mathcal{S}$, there exist two elements $S_0, S_1 \in \mathcal{S}$ such that $x \in S_0 \backslash S_1$, $z \in S_1 \backslash S_0$ and $S_0 \cup S_1 = X$. Choose an open neighborhood V of x with $\overline{V} \subset S_0 \backslash S_1$. Observe that $x \in \mathrm{e}(V)$, so $z \in I_{\mathcal{S}}(\overline{V}) \subset S_0$, a contradiction.

Finally, we can show that r is upper semi-continuous. Indeed, let $r(y) \subset W$ with $y \in \mathbb{I}^\tau$ and $W \in \mathcal{T}_X$. Then there exist finitely many $U_i \in \mathcal{T}_X$, $i = 1, 2, .., k$, such that $y \in \bigcap_{i=1}^{i=k} e(U_i)$ and $\bigcap_{i=1}^{i=k} I_{\mathcal{S}}(\overline{U}_i) \subset W$. Obviously, $r(y') \subset W$ for all $y' \in \bigcap_{i=1}^{i=k} e(U_i)$. So, r is an usco retraction from $\mathbb{I}^\tau$ onto X. According to [1], X is a Dugundji space.

Suppose now, that X is connected. By [9], any set of the form $I_{\mathcal{S}}(F)$ is $\mathcal{S}$-convex, so is each $r(y)$. According to [9, Corollary 1.5.8], all closed $\mathcal{S}$-convex subsets of X are also connected. Hence, the map r, defined by (1), is connected-valued. Consequently, by [1], X is an absolute extensor in dimension 1, and there exists a map $r_1 : \mathbb{I}^\tau \to \exp X$ with $r_1(x) = \{x\}$ for all $x \in X$, see [2, Theorem 3.2]. On the other hand, since X is normally supercompact, there exists a retraction r_2 from $\exp X$ onto X, see [9, Corollary 1.5.20]. Then the composition $r_2 \circ r_1 \colon \mathbb{I}^\tau \to X$ is a (single-valued) retraction. So, $X \in AR$. □

Proof of Corollary 4. It is well known that λ is a continuous functor preserving open maps, see [3]. So, λX is κ-metrizable. Moreover, λX is connected if so is X. On the other hand, the family $\mathcal{S} = \{F^+ : F \text{ is closed in } X\}$, where $F^+ = \{\eta \in \lambda X : F \in \eta\}$, is a binary normal subbase for λX. Observe that λf is $\mathcal{S}$-convex because $(\lambda f)^{-1}(\nu) = \bigcap\{f^{-1}(H)^+ : H \in \nu\}$ for every $\nu \in \lambda Y$. Then, Proposition 1 completes the proof. □

The next proposition shows that the statements from Proposition 1 and Corollary 4 are actually equivalent. At the same time it provides more information about validity of Corollary 3.

Proposition 2. *For any class $\mathcal{A}$ the following statements are equivalent:*

- *(i) If X is a compactum possessing a normal binary closed subbase $\mathcal{S}$, then any open $\mathcal{S}$-convex surjection $f \colon X \to Y$ is $\mathcal{A}$-soft.*
- *(ii) The map $\lambda f \colon \lambda X \to \lambda Y$ is $\mathcal{A}$-soft for any compactum X and any open surjection $f \colon X \to Y$.*

Proof. $(i) \Rightarrow (ii)$ Let X be a compactum and $f \colon X \to Y$ be an open surjection. It is easily seen that λf is an open surjection too. We already noted that $\mathcal{S} = \{F^+ : F \subset X \text{ is closed}\}$ is a normal binary closed subbase for λX and λf is a $\mathcal{S}$-convex and open map. Hence, by (i), λf is $\mathcal{A}$-soft.

$(ii) \Rightarrow (i)$. Suppose X is a compactum possessing a normal binary closed subbase $\mathcal{S}$, and $f\colon X \to Y$ is an $\mathcal{S}$-convex open surjection. To show that f is $\mathcal{A}$-soft, take a space $Z \in \mathcal{A}$, its closed subset A and two maps $h : A \to X$, $g\colon Z \to Y$ with $f \circ h = g|A$. So, we have the following diagram, where i_X and i_Y are embeddings defined by $x \to \eta_x$ and $y \to \eta_y$, respectively.

$$\begin{array}{ccccc} A & \xrightarrow{h} & X & \xrightarrow{i_X} & \lambda X \\ {\scriptstyle id}\downarrow & & \downarrow{\scriptstyle f} & & \downarrow{\scriptstyle \lambda f} \\ Z & \xrightarrow{g} & Y & \xrightarrow{i_Y} & \lambda Y \end{array}$$

Since, by (ii), λf is $\mathcal{A}$-soft, there exists a map $g_1\colon Z \to \lambda X$ such that $h = g_1|A$ and $\lambda f \circ g_1 = g$. The last equality implies that $g_1(Z) \subset (\lambda f)^{-1}(Y)$. According to [9, Corollary 2.3.7], there exists a retraction $r\colon \lambda X \to X$, defined by

$$r(\eta) = \bigcap\{F \in \mathcal{S} : F \in \eta\}. \tag{2}$$

Consider now the map $\bar{g} = r \circ g_1\colon Z \to X$. Obviously, $\bar{g}$ extends h. Let us show that $f \circ \bar{g} = g$. Indeed, for any $z \in Z$ we have

$$g_1(z) \in (\lambda f)^{-1}(g(z)) = (f^{-1}(g(z)))^+.$$

Since f is $\mathcal{S}$-convex, $I_{\mathcal{S}}\big(f^{-1}(g(z))\big) = f^{-1}(g(z))$, see [9, Theorem 1.5.7]. Hence, $f^{-1}(g(z))$ is the intersection of the family $\{F \in \mathcal{S} : f^{-1}(g(z)) \subset F\}$ whose elements belong to any $\eta \in (\lambda f)^{-1}(g(z))$. It follows from (2) that $r(\eta) \in f^{-1}(g(z))$, $\eta \in (\lambda f)^{-1}(g(z))$. In particular, $\bar{g}(z) \in f^{-1}(g(z))$. Therefore, $f \circ \bar{g} = g$. □

The following corollary follows from Corollary 3 and Proposition 2.

Corollary 5. *If X is a compactum with a binary normal closed subbase $\mathcal{S}$ such that λX is an absolute extensor for a given class $\mathcal{A}$, then any open $\mathcal{S}$-convex surjection $f\colon X \to Y$ is $\mathcal{A}$-soft.*

Acknowledgments. The author would like to express his gratitude to M. Choban for his continuous support and valuable remarks.

References

[1] Dranishnikov A. *Multivalued absolute retracts and absolute extensors in dimnesions* 0 *and* 1. Usp. Mat. Nauk, 1984, **39**, issue 5(239), 241–242 (in Russian).

[2] Fedorchuk V. *Some geometric properties of covariant functors*. Usp. Mat. Nauk, 1984, **39**, issue 5(239), 169–208 (in Russian).

[3] Fedorchuk V., Filippov V. *General Topology: Basic constructions*, Moscow State University, Moscow 1988 (in Russian).

[4] de Groot J. *Supercompactness and superextensions*. Contributions to extension theory of topological structures, Symp. Berlin 1967, Deutscher Verlag Wiss., Berlin, 1969, 89–90.

[5] Ivanov A. *Superextension of openly generated compacta*, Dokl. Akad. Nauk SSSR, 1981, **259**, No. 2, 275–278 (in Russian).

[6] Ivanov A. *Solution of van Mill's problem on the characterization of compacta, the superextension of which are absolute retracts*. Dokl. Akad. Nauk SSSR, 1982, **262**, No. 3, 526–528 (in Russian).

[7] Ivanov A. *Mixers, functors, and soft mappings*. Proceedings of Steklov Inst. of Math., 1993, **193**, No. 3, 139–142.

[8] Haydon R. *On a problem of Pelczynski: Milutin spaces, Dugundji spaces and* $AE(0-dim)$. Studia Math., 1974, **52**, 23–31.

[9] van Mill J. *Supercompactness and Wallman spaces*. Math. Centre Tracts, **85**, Amsterdam 1977.

[10] Shirokov L. *External characterization of Dugundji spaces and* κ*-metrizable compacta*. Dokl. Acad. Nauk SSSR, 1982, **263**, No. 5, 1073–1077 (in Russian).

[11] Shchepin E. *Topology of limit spaces of uncountable inverse spectra*. Russian Math. Surveys, 1976, **315**, 155–191.

[12] Valov V. *Extenders and* κ*-metrizable compacta*. Math. Notes, 2011, **89**, No. 3, 331–341.

V. Valov
Department of Computer Science and Mathematics
Nipissing University
100 College Drive, P.O. Box 5002
North Bay, ON, P1B 8L7
Canada
E-mail: *veskov@nipissingu.ca*

Received August 14, 2013