BULETINUL ACADEMIEI DE ŞTIINŢE
A REPUBLICII MOLDOVA. MATEMATICA
Numbers 2(72)−3(73), 2013, Pages 37–46
ISSN 1024–7696

Selected Old Open Problems in General Topology

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Abstract. We present a selection of old problems from different domains of General Topology. Formally, the number of problems is 20, but some of them are just versions of the same question, so the actual number of the problems is 15 or less. All of them are from 30 to 50 years old, and are known to have attracted attention of many topologists. A brief survey of these problems, including some basic references to articles and comments on their present status, are given.

Mathematics subject classification: 54A25, 54B05.
Keywords and phrases: Lindelöf space, G_δ-point, weakly first-countable, paracompact p-space, symmetrizable, homogeneous, topological group, extremally disconnected, pointwise convergence.

1 Introduction

There is more than one way to achieve progess in mathematics. One of them is to introduce a good new concept, usually generalizing some classical concept. The next step is to consider elementary natural questions arising from it. This may lead to a success and recognition, depending on whether the new concept provides valuable new insights in or not. Another, more standard and more widely spread, way is to take a classical construction, procedure, and to modify some parameters involved in it so that the construction becomes applicable to a wider class of objects.

But there is yet another, more secure and pleasant, way to gain immediately recognition and to make an impact on mathematics: to solve a well-known, or even famous, open problem in one of its fields. Whether a certain problem can be recognized as famous depends on many objective and subjective factors. One of them is how old is the problem, another factor is the history of it, in particular, who has posed the problem, and who has worked on it.

Below I briefly survey a very finite set of inspiring open problems in General Topology. The list is very selective. All of the problems in it are rather old, aged from 30 to 50 years, and I will provide some basic references to the literature. The brief survey I offer to the reader shows in many directions, it includes very different problems. So trying to make this survey detailed and comprehensive would result in a huge and non-focused text passing by many major topics in General Topology. This is not the task I have in mind at this time, so that only very selective references to results and articles are given in the comments.

A "space" below is a Tychonoff topological space. In particular, by a compact space we mean a compact Hausdorff space. In terminology and notation we follow [24].

2 The cardinality of Lindelöf spaces

In the summer of 1923 P. S. Alexandroff and P. S. Urysohn, building up the theory of compact spaces, came to the question: is it true that the cardinality of every first-countable compact space is not greater than the cardinality of the set of real numbers? This question became known as *Alexandroff-Urysohn Problem*. Apparently, it has appeared in print for the first time in [3]. The work on the Problem gave a good push to developing and refining set-theoretic methods in General Topology.

It is well-known that the cardinality of every metrizable uncountable compact space is exactly 2^ω. Alexandroff and Urysohn have been able to considerably generalize this fact: they proved, by a nice ramification method, that if every closed subset of a compact space X is a G_δ-set, then $|X| \le 2^\omega$ [3]. Alexandroff-Urysohn's Problem had been solved only in 1969, in [7]: the cardinality of every first-countable compact space is indeed not greater than 2^ω. In fact, it was shown in [7] that this inequality holds for Lindelöf first-countable spaces as well. The following question was raised at that time:

Problem 1 (A. V. Arhangel'skii, 1969). *Suppose that X is a Lindelöf space such that every point of X is a G_δ-point. Then is it true that $|X| \le 2^\omega$?*

Of course, the author of this problem wanted to see its solution in ZFC. In this sense, the problem still remains unsolved, even though it has been shown that, consistently, the anwer to Problem 1 is "no". We are still looking for an example in ZFC of a large Lindelöf space X in which every point is a G_δ. See [8, 34] and [32] for more about this question. It will definitely require new techniques and new ideas to answer Problem 1. However, notice that the answer is "yes" if we add the assumption that the tightness of X is countable.

3 Weakly first-countable spaces

Suppose that X is a space and $\eta_x = (V_n(x) : n \in \omega)$ is a decreasing sequence of subsets of X, for every $x \in X$, such that $x \in V_n(x)$ and the following condition is satisfied:

(wfc) A subset U of X is open if and only if for every $x \in U$ there exists $n \in \omega$ such that $V_n(x) \subset U$.

In this case, we will say that the family $\{\eta_x : x \in X\}$ is a *weakly basic wfc-structure* on the space X. A space X is called *weakly first-countable* if there exists a weakly basic wfc-structure on X. The concept of a weakly first-countable space was introduced in [5]. Note that the interiors of the sets $V_n(x)$ in the above definition can be empty. There are many examples of weakly first-countable spaces that are not first-countable. However, every weakly first-countable space is sequential, and therefore, the tightness of every weakly first-countable space is countable. In connection with the next question, see [5] and [8].

Problem 2 (Arhangel'skii, 1966)**.** *Give an example in ZFC of a weakly first-countable compact space X such that $|X| > 2^\omega$.*

N. N. Jakovlev has constructed under CH a weakly first-countable, but not first-countable, compact space [33]. On the other hand, it has been shown [8], also under CH, that every homogeneous weakly first-countable compact space is first-countable and hence, the cardinality of it doesn't exceed 2^ω.

However, the next question, closely related to Problem 2, remains open:

Problem 3 (Arhangel'skii, $<$ 1978)**.** *Give an example in ZFC of a weakly first-countable, but not first-countable, compact space X.*

For consistency results on the existence of a weakly first-countable compact space with the cardinality larger than 2^ω, see [41] and [1].

A very interesting open question about weakly first-countable spaces arises in connection with the well-known Hajnal–Juhasz theorem on the cardinality of first-countable spaces with the countable Souslin number [32, 34].

Problem 4 (Arhangel'skii, $<$ 1980)**.** *Suppose that X is a weakly first-countable space such that the Souslin number of X is countable. Then is it true that $|X| \leq 2^\omega$?*

In topological groups weak first-countability turns out to be as strong as the first-countability itself. Indeed, M. M. Choban and S. J. Nedev in [44] have shown that every weakly first-countable topological group is metrizable.

4 Symmetrizable spaces

For the introduction to the theory of symmetrizable spaces, see [5] and [43]. A closely related but less general concept of a semimetrizable space was introduced by Alexandroff and Niemytzkii in [2], in 1927. A *symmetric* on a set X is a non-negative real-valued function $d(,)$ of two variables on X such that $d(x, y) = 0$ if and only if $x = y$, and $d(x, y) = d(y, x)$ for any $x, y \in X$. Suppose that X is a space and d is a symmetric on X such that a subset A of X is closed in X if and only if $d(x, A) > 0$ for every $x \in X \setminus A$. Then we say that d generates the topology of the space X. A space X is *symmetrizable* if the topology of X is generated by some symmetric on X. Symmetrizable spaces naturally arise under quotient mappings with compact fibers [5]. They constitute a much larger class of spaces than the class of metrizable spaces. It is easily seen that every symmetrizable space is weakly first-countable. Hence, every symmetrizable space is sequential. However, not every symmetrizable space is first-countable. But the next question is still open:

Problem 5 ($<$ 1970)**.** *Is every point in a symmetrizable space a G_δ?*

If I remember correctly, this question was formulated for the first time in the correspondence between E. Michael and A. Arhangel'skii in sixties. Of course, this problem is motivated by the following fact: every metrizable space is first-countable.

However, in some situations symmetrizable spaces, indeed, behave in the same way as metrizable spaces. In particular, every symmetrizable compact space is metrizable [5]. For semimetrizable spaces (they can be characterized as first-countable symmetrizable spaces) this statement was proved by V. V. Niemytzkii in [45]. The main step in the proof of the last statement is to show that every point in a symmetrizable compact space is a G_δ. After that is done, it remains to refer to Niemytzkii's result mentioned above.

It should be noted that symmetrizable first-countable spaces are quite well behaved in general. In particular, a symmetrizable space X is first-countable if and only if every subspace of X is symmetrizable [5]. The next case of Problem 5 is also open.

Problem 6. *Is every pseudocompact symmetrizable space first-countable?*

The fact that every symmetrizable countably compact space X is metrizable [43] makes the last Problem especially interesting.

A pseudocompact symmetrizable first-countable space needn't be metrizable (this is witnessed by Mrowka's space), but every such space is a Moore space (see [51]). Thus, the next question is a reformulation of Problem 6:

Problem 7. *Is every pseudocompact symmetrizable space a Moore space?*

5 Topological groups and topological invariants

A typical object of topological algebra can be described as a result of a happy marriage of an algebraic structure with a topology. The ties arising from this marriage strongly influence the properties of both structures. A classical example of this situation is Birkgoff–Kakutani Theorem: a topological group G is metrizable if and only if it is first-countable [18].

Every first-countable space X is *Fréchet-Urysohn*, that is, a point $x \in X$ is in the closure of a subset A of X only if some sequence in A converges to x. Clearly, every countable first-countable space is metrizable, but it is easy to construct a non-metrizable countable Fréchet-Urysohn space. This shows that the class of Fréchet-Urysohn spaces is much wider than the class of first-countable spaces. Therefore, it is amazing that the next problem remains unsolved:

Problem 8 (V. I. Malykhin, < 1980)**.** *Construct in ZFC a non-metrizable countable Fréchet-Urysohn topological group.*

It is known, however, that under $MA + \neg CH$ one can find a dense subgroup of D^{ω_1} with these properties (see [8, 10]).

Another well-known open problem of topological algebra concerns the class of extremally disconnected topological groups. This class is apparently very narrow, unlike the class of extremally disconnected spaces. Recall that a space X is *extremally* *disconnected* if the closure of every open subset of X is open. These spaces seem to be quite special. In particular, none of them contains a non-trivial convergent sequence. Therefore, only discrete extremally disconnected spaces are first-countable. Nevertheless, extremally disconnected spaces are rather easy to encounter in General Topology, since every space can be represented as an image of an extremally disconnected space under an irreducible perfect mapping [48], see also [35,36]. A. Gleason has characterized extremally disconnected compacta as projective objects in the category of compact spaces [27].

On the other hand, it has been known for a long time that extremal disconnectedness is on quite bad terms with homogeneity: in 1968 Z. Frolik proved [26] that every extremally disconnected homogeneous compact space is finite (see also [8]). However, there exist non-discrete homogeneous spaces [25]. One may argue that the highest degree of homogeneity is achieved in topological groups. Indeed, a space X is said to be homogeneous if for any $x, y \in X$ there exists a homeomorphism h of X onto itself such that $h(x) = y$. To verify homogeneity of topological groups, it is enough to use left or right translations (shifts). The next problem, which remains open today, has been posed 46 years ago in [6].

Problem 9 (A. V. Arhangel'skii, 1967)**.** *Construct in ZFC a non-discrete extremally disconnected topological group.*

It was proved in [6] that every compact subspace of an arbitrary extremally disconnected topological group is finite. Hence, if an extremally disconnected topological group G is a k-space, then the space G is discrete. S. Sirota was the first to show that the existence of a non-discrete extremally disconnected topological group is consistent with ZFC [50]. More information on the vast and delicate research around Problem 9 can be found in [37,39,40,52]. It was shown that extremal disconnectedness strongly influences the structure of a topological group. In particular, every extremally disconnected topological group G has an open subgroup H such that $a^2 = e$ for every $a \in H$, where e is the neutral element of G [40].

Another interesting and long standing open problem on topological groups I wish to recall concerns free topological groups. For the definitions and basic facts on free topological groups, see Chapter 7 in [18].

Problem 10 (A. V. Arhangel'skii, 1981)**.** *Is the free topological group $F(X)$ of an arbitrary paracompact p-space X paracompact?*

This question is motivated by the fact established in [9], where the above problem has been posed: the free topological group of any metrizable space is paracompact. We remind that a paracompact p-space is a preimage of a metrizable space under a perfect mapping [4].

The next question concerns the behaviour of topological properties of topological groups under the product operation. We give two slightly different versions of this question.

Problem 11. *Construct in ZFC countably compact topological groups G and H such that their product $G \times H$ is not countably compact.*

Problem 12. *Construct in ZFC a countably compact topological group G such that its square $G \times G$ is not countably compact.*

The general idea behind the last two questions is that in topological groups many topological properties improve so that some of them, which are not productive in the general case, may become productive in the special case of topological groups. For example, pseudocompactness is a property of this kind (W. W. Comfort and K. A. Ross, see [22] and [18]). Observe that Problems 11 and 12 are not equivalent, since, in general, the free topological sum of two topological groups is not a topological group. Under $MA + \neg CH$, there exists a countably compact topological group G such that its square $G \times G$ is not countably compact [23]. See also [38] and [31].

6 Homogeneous compacta

Some of the most natural and oldest open problems in General Topology concern homogeneous compacta. The next Problem had been posed by W. Rudin in 1956 in [49]:

Problem 13 (W. Rudin, 1956)**.** *Is it true that every infinite homogeneous compact space contains a non-trivial convergent sequence?*

A motivation for this unusual question comes from the well-known fact, established by A. N. Tychonoff, that the Stone–Čech remainder of the discrete space of natural numbers doesn't contain non-trivial convergent sequences. In 1956 it was an open question whether this remainder (which is compact) is homogeneous or not. Obiously, a positive solution of Problem 13 would immediately provide the negative answer to the last question as well. However, Problem 13 is still open, after more than 55 years have passed since it had been published.

Of course, in the very special case of compact topological groups the answer to Problem 13 is "yes", because every compact topological group is a dyadic compactum (see [18]).

The next question came to my mind in eighties (see [10]). Later I learned from Jan van Mill that Eric van Douwen also came to this question.

Problem 14. *Is it possible to represent an arbitrary compact space Y as an image of a homogeneous compact space X under a continuous mapping?*

Observe that every nonempty metrizable compactum is a continuous image of the Cantor set. The Cantor set is not only homogeneous, it is a compact topological group. However, it is not possible to represent an arbitrary compact space Y as an image of a compact topological group X under a continuous mapping, since only dyadic compacta can be represented in this way.

The third open problem on homogeneity presented in this section is less known than the other two, but is also very interesting, in my opinion. A compact space X is *ω-monolithic* if the closure of any countable subset of X is metrizable. For example, every compact LOTS, that is, every compact space whose topology is generated by a linear ordering, is ω-monolithic.

Problem 15 (Arhangel'skii, 1987)**.** *Is every homogeneous ω-monolithic compact space X first-countable?*

This question was posed in [10]. It was shown there that the answer to it is "yes" under the additional assumption that the tightness of X is countable. The next closely related to the above question problem is also open:

Problem 16. *Is the cardinality of every homogeneous ω-monolithic compact space X not greater than 2^ω?*

If the answer to the last question is "yes", then the answer to the preceding question is "yes" under CH. It is not difficult to notice that behind the last two question is hidden the next problem which is also open now (see [10]):

Problem 17. *Is it true that every nonempty monolithic compact space is first-countable at some point?*

7 *t*-equivalence and *t*-invariants

Below $C_p(X)$ stands for the space of real-valued continuous functions with the topology of pointwise convergence on a space X. These spaces, studied in C_p-theory [14], have many applications in mathematics. In $C_p(X)$ an algebraic structure is naturally blended with a topology. To compare topological spaces X and Y, we may use homeomorphisms between $C_p(X)$ and $C_p(Y)$ which do not necessarily preserve the algebraic operations in $C_p(X)$ and $C_p(Y)$. In particular, this approach had been adopted in [14]. Following it, we say that spaces X and Y are *t-equivalent* if there exists a homeomorphism h of $C_p(X)$ onto $C_p(Y)$. Note that h in the above definition needn't be a topological isomorphism, that is, h needn't preserve the operations.

One of the basic general questions in C_p-theory is the next one: how close are the properties of X and Y if X and Y are t-equivalent spaces [13, 14]? For example, it has been established by S. Gul'ko and T. Khmyleva [30] that the usual space R of real numbers is t-equivalent to the closed unit interval I. Amazingly, this is a very non-trivial result! The reader may find even more unexpected that the answer to the following two questions are still unknown:

Problem 18 (A. V. Arhangel'skii, ≤ 1989)**.** *Is the unit segment I t-equivalent to the square I^2?*

Problem 19 (A. V. Arhangel'skii, ≤ 1989)**.** *Is the unit segment I t-equivalent to the Cantor set?*

Solving the last two problems may lead to a solution of the next basic question:

Problem 20 (A. V. Arhangel'skii, ≤ 1985)**.** *Is the dimension dim preserved by the t-equivalence, at least in the class of separable metrizable spaces or in the class of compacta?*

Gul'ko–Khmyleva's result mentioned above shows that compactness is not t-invariant. Local compactness, in general, is also not preserved by t-equivalence. On the other hand, the dimension $dim(X)$ is preserved by linear homeomorphisms between $C_p(X)$ and $C_p(Y)$. This is a deep result of V. G. Pestov [46]. In connection with Pestov's result and Problem 20, see also [29]. For more on C_p-theory and t-equivalence, see [11, 14, 15, 19, 21, 42, 47] and [12].

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Received January 26, 2013