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# Certain differential superordinations using a multiplier transformation and Ruscheweyh derivative

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**Abstract.** In the present paper we define a new operator, by means of convolution product between Ruscheweyh derivative and the multiplier transformation $I(m,\lambda,l)$. For functions $f$ belonging to the class $\mathcal{A}$ we define the differential operator $IR^m_{\lambda,l}$ : $\mathcal{A} \to \mathcal{A}$, $IR^m_{\lambda,l}f(z) := (I(m,\lambda,l) * R^m) f(z)$, where $\mathcal{A}_n = \{f \in \mathcal{H}(U) : \ f(z) = z + a_{n+1}z^{n+1} + \ldots,\ z \in U\}$ is the class of normalized analytic functions, with $\mathcal{A}_1 = \mathcal{A}$. We study some differential superordinations regarding the operator $IR^m_{\lambda,l}$.

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## 1 Introduction

Denote by $U$ the unit disc of the complex plane $U = \{z \in \mathbb{C} : |z| < 1\}$ and by $\mathcal{H}(U)$ the space of all holomorphic functions in $U$.

Let

$$\mathcal{A}(p,n) = \{f \in \mathcal{H}(U) : \ f(z) = z^p + \sum_{j=p+n}^{\infty} a_j z^j, \quad z \in U\},$$

with $\mathcal{A}(1,n) = \mathcal{A}_n$, $\mathcal{A}(1,1) = \mathcal{A}_1 = \mathcal{A}$ and

$$\mathcal{H}[a,n] = \{f \in \mathcal{H}(U),\ f(z) = a + a_n z^n + a_{n+1}z^{n+1} + \ldots,\ z \in U\}$$

for $a \in \mathbb{C}$ and $p, n \in \mathbb{N}$.

If $f$ and $g$ are analytic functions in $U$, we say that $f$ is superordinate to $g$, written $g \prec f$, if there is an analytic in $U$ function $w$ , with $w(0) = 0$ and $|w(z)| < 1$ for all $z \in U$, such that $g(z) = f(w(z))$ for all $z \in U$. If $f$ is univalent, then $g \prec f$ if and only if $f(0) = g(0)$ and $g(U) \subseteq f(U)$.

Let $\psi : \mathbb{C}^2 \times U \to \mathbb{C}$ and $h$ be an analytic function in $U$. If $p$ and $\psi(p(z), zp'(z); z)$ are univalent in $U$ and satisfy the (first-order) differential superordination

$$h(z) \prec \psi(p(z), zp'(z); z), \qquad \text{for} \quad z \in U, \tag{1}$$

then $p$ is called a solution of the differential superordination. The analytic function $q$ is called a subordinant of the solutions of the differential superordination, or more simply a subordinant, if $q \prec p$ for all $p$ satisfying (1).

A univalent subordinant $\widetilde{q}$ that satisfies $q \prec \widetilde{q}$ for all subordinants $q$ of (1) is said to be the best subordinant of (1). The best subordinant is unique up to a rotation of $U$.

**Definition 1 [7].** For $f \in \mathcal{A}(p,n)$, $p,n \in \mathbb{N}$, $m \in \mathbb{N} \cup \{0\}$, $\lambda, l \geq 0$, the operator $I_p(m,\lambda,l)f(z)$ is defined by the following infinite series

$$I_p(m,\lambda,l)f(z) := z^p + \sum_{j=p+n}^{\infty} \left( \frac{p+\lambda(j-1)+l}{p+l} \right)^m a_j z^j.$$

*Remark 1.* It follows from the above definition that

$$I_p(0,\lambda,l)f(z) = f(z),$$

$$(p+l)I_p(m+1,\lambda,l)f(z) = [p(1-\lambda)+l]\,I_p(m,\lambda,l)f(z) + \lambda z \left(I_p(m,\lambda,l)f(z)\right)',$$

for $z \in U$.

*Remark 2.* If $p=1$ and $n=1$, then we have $\mathcal{A}(1,1) = \mathcal{A}_1 = \mathcal{A}$, $I_1(m,\lambda,l)f(z) = I(m,\lambda,l)$ and

$$(l+1)I(m+1,\lambda,l)f(z) = [l+1-\lambda]\,I(m,\lambda,l)f(z) + \lambda z \left(I(m,\lambda,l)f(z)\right)',$$

for $z \in U$.

*Remark 3.* If $f \in \mathcal{A}$ and $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$, then $I(m,\lambda,l)f(z) = z + \sum_{j=2}^{\infty} \left(\frac{1+\lambda(j-1)+l}{l+1}\right)^m a_j z^j$, for $z \in U$.

*Remark 4.* For $l=0$ and $\lambda \geq 0$, the operator $D_\lambda^m = I(m,\lambda,0)$ was introduced and studied by Al-Oboudi [6]. The study of this operator is reduced to the Sălăgean differential operator $S^m = I(m,1,0)$ [10] for $\lambda = 1$.

**Definition 2 [9].** For $f \in \mathcal{A}$ and $m \in \mathbb{N}$ the operator $R^m$ is defined by $R^m : \mathcal{A} \to \mathcal{A}$,

$$\begin{aligned} R^0 f(z) &= f(z) \\ R^1 f(z) &= z f'(z) \\ &\ldots \\ (m+1)R^{m+1} f(z) &= z(R^m f(z))' + mR^m f(z), \quad z \in U. \end{aligned}$$

*Remark 5.* If $f \in \mathcal{A}$, $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$, then $R^m f(z) = z + \sum_{j=2}^{\infty} C_{m+j-1}^m a_j z^j$, $z \in U$.

**Definition 3 [8].** We denote by $Q$ the set of all functions that are analytic and injective on $\overline{U} \backslash E(f)$, where $E(f) = \{\zeta \in \partial U : \lim_{z \to \zeta} f(z) = \infty\}$, and $f'(\zeta) \neq 0$ for $\zeta \in \partial U \backslash E(f)$. The subclass of $Q$ for which $f(0) = a$ is denoted by $Q(a)$.

We will use the following lemmas.

**Lemma 1 [8].** *Let $h$ be a convex function with $h(0) = a$, and let $\gamma \in \mathbb{C}\backslash\{0\}$ be a complex number with $Re\gamma \geq 0$. If $p \in \mathcal{H}[a,n] \cap Q$, $p(z) + \frac{1}{\gamma} zp'(z)$ is univalent in $U$ and*

$$h(z) \prec p(z) + \frac{1}{\gamma} zp'(z), \quad \text{for} \quad z \in U,$$

then

$$q(z) \prec p(z), \quad \text{for} \quad z \in U,$$

*where $q(z) = \frac{\gamma}{nz^{\gamma/n}} \int_0^z h(t) t^{\gamma/n-1} dt$, for $z \in U$. The function $q$ is convex and is the best subordinant.*

**Lemma 2 [8].** *Let $q$ be a convex function in $U$ and let $h(z) = q(z) + \frac{1}{\gamma} zq'(z)$, for $z \in U$, where $Re\ \gamma \geq 0$. If $p \in \mathcal{H}[a,n] \cap Q$, $p(z) + \frac{1}{\gamma} zp'(z)$ is univalent in $U$ and*

$$q(z) + \frac{1}{\gamma} zq'(z) \prec p(z) + \frac{1}{\gamma} zp'(z), \quad \text{for} \quad z \in U,$$

*then*

$$q(z) \prec p(z), \quad \text{for} \quad z \in U,$$

*where $q(z) = \frac{\gamma}{nz^{\gamma/n}} \int_0^z h(t) t^{\gamma/n-1} dt$, for $z \in U$. The function $q$ is the best subordinant.*

## 2 Main Results

**Definition 4 [3].** Let $m, \lambda, l \in \mathbb{N}$. Denote by $IR_{\lambda,l}^m$ the operator given by the Hadamard product (the convolution product) of the operator $I(m, \lambda, l)$ and the Ruscheweyh operator $R^m$, $IR_{\lambda,l}^m : \mathcal{A} \to \mathcal{A}$,

$$IR_{\lambda,l}^m f(z) = (I(m, \lambda, l) * R^m) f(z).$$

*Remark 6.* If $f \in \mathcal{A}$, $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$, then $IR_{\lambda,l}^m f(z) = z + \sum_{j=2}^{\infty} \left( \frac{1+\lambda(j-1)+l}{l+1} \right)^m \cdot C_{m+j-1}^m a_j^2 z^j$, for $z \in U$.

*Remark 7.* For $l = 0$, $\lambda \geq 0$, we obtain the Hadamard product $DR_\lambda^n$ [2] of the generalized Sălăgean operator $D_\lambda^n$ and Ruscheweyh operator $R^n$.

For $l = 0$ and $\lambda = 1$, we obtain the Hadamard product $SR^n$ [1] of the Sălăgean operator $S^n$ and Ruscheweyh operator $R^n$.

**Theorem 1.** *Let $h$ be a convex function, $h(0) = 1$. Let $m, \lambda, l \in \mathbb{N}$, $f \in \mathcal{A}$ and suppose that $\frac{l+1}{[\lambda(l-m+2)-(l+1)]z} \cdot \left[ (m+1) IR_{\lambda,l}^{m+1} f(z) - (m-2) IR_{\lambda,l}^m f(z) \right] + \left( 1 - \frac{l+1}{\lambda(l-m+2)-(l+1)} \right) - \frac{2(l+1)(m-1)-2\lambda m}{\lambda(l-m+2)-(l+1)} \int_0^z \frac{IR_{\lambda,l}^m f(t) - t}{t^2} dt$ is univalent and $\left( IR_{\lambda,l}^m f(z) \right)' \in \mathcal{H}[1,1] \cap Q$. If*

$$h(z) \prec \frac{l+1}{[\lambda(l-m+2)-(l+1)]z} \left[ (m+1) IR_{\lambda,l}^{m+1} f(z) - (m-2) IR_{\lambda,l}^m f(z) \right] +$$

$$\left( 1 - \frac{l+1}{\lambda(l-m+2)-(l+1)} \right) - \frac{2(l+1)(m-1)-2\lambda m}{\lambda(l-m+2)-(l+1)} \int_0^z \frac{IR_{\lambda,l}^m f(t) - t}{t^2} dt, \quad (2)$$

*for* $z \in U$, *then*

$$q(z) \prec \left(IR^m_{\lambda,l} f(z)\right)', \qquad \textit{for} \quad z \in U,$$

*where* $q(z) = \frac{\lambda(l-m+2)-(l+1)}{\lambda(l+1) z^{\frac{\lambda(l-m+2)-(l+1)}{\lambda(l+1)}}} \int_0^z h(t)\, t^{-\frac{\lambda(m-1)+(l+1)}{\lambda(l+1)}} dt$. *The function* $q$ *is convex and it is the best subordinant.*

*Proof.* With notation $p(z) = \left(IR^m_{\lambda,l} f(z)\right)' = 1+\sum_{j=2}^{\infty} \left(\frac{1+\lambda(j-1)+l}{l+1}\right)^m C^m_{m+j-1} j a_j^2 z^{j-1}$ and $p(0) = 1$, we obtain for $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$,

$p(z) + zp'(z) = m1 + \sum_{j=2}^{\infty} \left(\frac{1+\lambda(j-1)+l}{l+1}\right)^m C^m_{m+j-1} j a_j^2 z^{j-1} +$
$\sum_{j=2}^{\infty} \left(\frac{1+\lambda(j-1)+l}{l+1}\right)^m C^m_{m+j-1} j(j-1) a_j^2 z^{j-1} =$
$1 + \sum_{j=2}^{\infty} \left(\frac{1+\lambda(j-1)+l}{l+1}\right)^m C^m_{m+j-1} j^2 a_j^2 z^{j-1} =$
$\frac{1}{z}\Bigg(z + \sum_{j=2}^{\infty} \left(\frac{1+\lambda(j-1)+l}{l+1}\right)^{m+1} C^{m+1}_{m+j} \frac{m+1}{\lambda} a_j^2 z^j -$
$\sum_{j=2}^{\infty} \left(\frac{1+\lambda(j-1)+l}{l+1}\right)^m C^m_{m+j-1} \frac{\lambda(m-1)-(l+1)}{\lambda(l+1)} j a_j^2 z^j -$
$\sum_{j=2}^{\infty} \left(\frac{1+\lambda(j-1)+l}{l+1}\right)^m C^m_{m+j-1} \frac{m-2}{\lambda} a_j^2 z^j -$
$\sum_{j=2}^{\infty} \left(\frac{1+\lambda(j-1)+l}{l+1}\right)^m C^m_{m+j-1} \frac{1}{j-1} \frac{2(l+1)(m-1)-2\lambda m}{\lambda(l+1)} a_j^2 z^j\Bigg) =$
$\frac{1}{z}\Bigg[\frac{m+1}{\lambda}\left(z + \sum_{j=2}^{\infty} \left(\frac{1+\lambda(j-1)+l}{l+1}\right)^{m+1} C^{m+1}_{m+j} a_j^2 z^j\right) -$
$\frac{m-2}{\lambda}\left(z + \sum_{j=2}^{\infty} \left(\frac{1+\lambda(j-1)+l}{l+1}\right)^m C^m_{m+j-1} a_j^2 z^j\right)\Bigg] +$
$\left(1 - \frac{m+1}{\lambda} - \frac{m-2}{\lambda}\right) + \left(1 + \sum_{j=2}^{\infty} \left(\frac{1+\lambda(j-1)+l}{l+1}\right)^m C^m_{m+j-1} a_j^2 j z^{j-1}\right) \frac{\lambda(m-1)-(l+1)}{\lambda(l+1)} -$
$\frac{\lambda(m-1)-(l+1)}{\lambda(l+1)} - \sum_{j=2}^{\infty} \left(\frac{1+\lambda(j-1)+l}{l+1}\right)^m C^m_{m+j-1} \frac{1}{j-1} \frac{2(l+1)(m-1)-2\lambda m}{\lambda(l+1)} a_j^2 z^{j-1} =$
$\frac{1}{z}\left(\frac{m+1}{\lambda} IR^{m+1}_{\lambda,l} f(z) - \frac{m-2}{\lambda} IR^m_{\lambda,l} f(z)\right) + \frac{\lambda(m-1)-(l+1)}{\lambda(l+1)} \left(IR^m_{\lambda,l} f(z)\right)' +$
$\frac{\lambda l-\lambda m+2\lambda-2l-2}{\lambda(l+1)} - \frac{2(l+1)(m-1)-2\lambda m}{\lambda(l+1)} \sum_{j=2}^{\infty} \left(\frac{1+\lambda(j-1)+l}{l+1}\right)^m C^m_{m+j-1} \frac{1}{j-1} a_j^2 z^{j-1} =$
$\frac{1}{z}\left(\frac{m+1}{\lambda} IR^{m+1}_{\lambda,l} f(z) - \frac{m-2}{\lambda} IR^m_{\lambda,l} f(z)\right) + \frac{\lambda(m-1)-(l+1)}{\lambda(l+1)} \left(IR^m_{\lambda,l} f(z)\right)' +$
$\left(1 - \frac{m-1}{l+1} - \frac{2}{\lambda}\right) - \frac{2(l+1)(m-1)-2\lambda m}{\lambda(l+1)} \int_0^z \frac{IR^m_{\lambda,l} f(t)-t}{t^2} dt.$

Therefore $p(z) + \frac{\lambda(l+1)}{\lambda(l-m+2)-(l+1)} zp'(z) =$

$\frac{l+1}{[\lambda(l-m+2)-(l+1)]z}\left[(m+1)\, IR^{m+1}_{\lambda,l} f(z) - (m-2)\, IR^m_{\lambda,l} f(z)\right] +$
$\left(1 - \frac{l+1}{\lambda(l-m+2)-(l+1)}\right) - \frac{2(l+1)(m-1)-2\lambda m}{\lambda(l-m+2)-(l+1)} \int_0^z \frac{IR^m_{\lambda,l} f(t)-t}{t^2} dt.$

Then (2) becomes

$$h(z) \prec p(z) + \frac{\lambda(l+1)}{\lambda(l-m+2)-(l+1)} zp'(z), \quad \text{for} \quad z \in U.$$

By using Lemma 1 for $\gamma = 1 - \frac{m-1}{l+1} - \frac{1}{\lambda}$ and $n = 1$, we have

$$q(z) \prec p(z), \quad \text{for} \quad z \in U, \quad \text{i.e.} \quad q(z) \prec \left(IR^m_{\lambda,l} f(z)\right)', \quad \text{for} \quad z \in U,$$

where $q(z) = \frac{\lambda(l-m+2)-(l+1)}{\lambda(l+1) z^{\frac{\lambda(l-m+2)-(l+1)}{\lambda(l+1)}}} \int_0^z h(t)\, t^{-\frac{\lambda(m-1)+(l+1)}{\lambda(l+1)}} dt$. The function $q$ is convex and it is the best subordinant. □

**Corollary 1** [5]. *Let $h$ be a convex function and $h(0) = 1$. Let $\lambda \geq 0$, $m \in \mathbb{N}$, $f \in \mathcal{A}$ and suppose that $\frac{m+1}{(m\lambda+1)z} DR^{m+1}_\lambda f(z) - \frac{m(1-\lambda)}{(m\lambda+1)z} DR^m_\lambda f(z)$ is univalent and $(DR^m_\lambda f(z))' \in \mathcal{H}[1,1] \cap Q$. If*

$$h(z) \prec \frac{m+1}{(m\lambda+1)z} DR^{m+1}_\lambda f(z) - \frac{m(1-\lambda)}{(m\lambda+1)z} DR^m_\lambda f(z), \quad \text{for} \quad z \in U, \tag{3}$$

*then*

$$q(z) \prec (DR^m_\lambda f(z))', \quad \text{for} \quad z \in U,$$

*where $q(z) = \frac{m+\frac{1}{\lambda}}{z^{m+\frac{1}{\lambda}}} \int_0^z h(t)\, t^{m-1+\frac{1}{\lambda}} dt$. The function $q$ is convex and it is the best subordinant.*

**Corollary 2** [4]. *Let $h$ be a convex function and $h(0) = 1$. Let $n \in \mathbb{N}$, $f \in \mathcal{A}$ and suppose that $\frac{1}{z} SR^{n+1} f(z) + \frac{n}{n+1} z (SR^n f(z))''$ is univalent and $(SR^n f(z))' \in \mathcal{H}[1,1] \cap Q$. If*

$$h(z) \prec \frac{1}{z} SR^{n+1} f(z) + \frac{n}{n+1} z (SR^n f(z))'', \quad \text{for} \quad z \in U, \tag{4}$$

*then*

$$q(z) \prec (SR^n f(z))', \quad \text{for} \quad z \in U,$$

*where $q(z) = \frac{1}{z} \int_0^z h(t) dt$. The function $q$ is convex and it is the best subordinant.*

**Theorem 2**. *Let $q$ be convex in $U$ and let $h$ be defined by $h(z) = q(z) + \frac{\lambda(l+1)}{\lambda(l-m+2)-(l+1)} zq'(z)$, $m, \lambda, l \in \mathbb{N}$. If $f \in \mathcal{A}$, suppose that $\frac{l+1}{[\lambda(l-m+2)-(l+1)]z} \left[(m+1) IR^{m+1}_{\lambda,l} f(z) - (m-2) IR^m_{\lambda,l} f(z)\right] + \left(1 - \frac{l+1}{\lambda(l-m+2)-(l+1)}\right)$ $- \frac{2(l+1)(m-1)-2\lambda m}{\lambda(l-m+2)-(l+1)} \int_0^z \frac{IR^m_{\lambda,l} f(t) - t}{t^2} dt$ is univalent, $\left(IR^m_{\lambda,l} f(z)\right)' \in \mathcal{H}[1,1] \cap Q$ and satisfies the differential superordination*

$$h(z) = q(z) + \frac{\lambda(l+1)}{\lambda(l-m+2)-(l+1)} zq'(z) \prec$$

$$\frac{l+1}{[\lambda(l-m+2)-(l+1)]z} \left[(m+1) IR^{m+1}_{\lambda,l} f(z) - (m-2) IR^m_{\lambda,l} f(z)\right]$$

$$+\left(1-\frac{l+1}{\lambda\left(l-m+2\right)-\left(l+1\right)}\right)-\frac{2\left(l+1\right)\left(m-1\right)-2\lambda m}{\lambda\left(l-m+2\right)-\left(l+1\right)}\int_0^z\frac{IR^m_{\lambda,l}f\left(t\right)-t}{t^2}dt, \tag{5}$$

*for* $z\in U$. *Then*

$$q(z)\prec\left(IR^m_{\lambda,l}f\left(z\right)\right)', \quad for \quad z\in U,$$

*where* $q(z)=\frac{\lambda(l-m+2)-(l+1)}{\lambda(l+1)z^{\frac{\lambda(l-m+2)-(l+1)}{\lambda(l+1)}}}\int_0^z h\left(t\right)t^{-\frac{\lambda(m-1)+(l+1)}{\lambda(l+1)}}dt$. *The function* $q$ *is the best subordinant.*

*Proof.* Let $p\left(z\right)=\left(IR^m_{\lambda,l}f\left(z\right)\right)'=1+\sum_{j=2}^{\infty}\left(\frac{1+\lambda(j-1)+l}{l+1}\right)^m C^m_{m+j-1}ja_j^2z^{j-1}$.

Differentiating, we obtain $p\left(z\right)+zp'\left(z\right)=\frac{1}{z}\left(\frac{m+1}{\lambda}IR^{m+1}_{\lambda,l}f\left(z\right)-\frac{m-2}{\lambda}IR^m_{\lambda,l}f\left(z\right)\right)+\frac{\lambda(m-1)-(l+1)}{\lambda(l+1)}\left(IR^m_{\lambda,l}f\left(z\right)\right)'+\left(1-\frac{m-1}{l+1}-\frac{2}{\lambda}\right)-\frac{2(l+1)(m-1)-2\lambda m}{\lambda(l+1)}\int_0^z\frac{IR^m_{\lambda,l}f(t)-t}{t^2}dt$,

$p\left(z\right)+\frac{\lambda(l+1)}{\lambda(l-m+2)-(l+1)}zp'\left(z\right)=\frac{l+1}{[\lambda(l-m+2)-(l+1)]z}\left[\left(m+1\right)IR^{m+1}_{\lambda,l}f\left(z\right)-\left(m-2\right)IR^m_{\lambda,l}f\left(z\right)\right]+\left(1-\frac{l+1}{\lambda(l-m+2)-(l+1)}\right)-\frac{2(l+1)(m-1)-2\lambda m}{\lambda(l-m+2)-(l+1)}\int_0^z\frac{IR^m_{\lambda,l}f(t)-t}{t^2}dt$,

for $z\in U$, and (2) becomes

$$q(z)+\frac{\lambda\left(l+1\right)}{\lambda\left(l-m+2\right)-\left(l+1\right)}zq'(z)\prec p(z)+\frac{\lambda\left(l+1\right)}{\lambda\left(l-m+2\right)-\left(l+1\right)}zp'\left(z\right),$$

for $z\in U$.

Using Lemma 2 for $\gamma=1-\frac{m-1}{l+1}-\frac{1}{\lambda}$ and $n=1$, we have $q(z)\prec p(z)$, $z\in U$, i.e.

$$q(z)=\frac{\lambda\left(l-m+2\right)-\left(l+1\right)}{\lambda\left(l+1\right)z^{\frac{\lambda(l-m+2)-(l+1)}{\lambda(l+1)}}}\int_0^z h\left(t\right)t^{-\frac{\lambda(m-1)+(l+1)}{\lambda(l+1)}}dt\prec\left(IR^m_{\lambda,l}f\left(z\right)\right)', \ \ z\in U,$$

and $q$ is the best subordinant. □

**Corollary 3 [5].** *Let* $q$ *be convex in* $U$, $h$ *be defined by* $h\left(z\right)=q\left(z\right)+\frac{\lambda}{m\lambda+1}zq'\left(z\right)$, $\lambda\geq 0$, $m\in\mathbb{N}$ *and* $f\in\mathcal{A}$. *Suppose that* $\frac{m+1}{(m\lambda+1)z}DR^{m+1}_{\lambda}f\left(z\right)-\frac{m(1-\lambda)}{(m\lambda+1)z}DR^m_{\lambda}f\left(z\right)$ *is univalent,* $\left(DR^m_{\lambda}f\left(z\right)\right)'\in\mathcal{H}\left[1,1\right]\cap Q$ *and satisfies the differential superordination*

$$h(z)=q\left(z\right)+\frac{\lambda}{m\lambda+1}zq'\left(z\right)\prec\frac{m+1}{\left(m\lambda+1\right)z}DR^{m+1}_{\lambda}f\left(z\right)-\frac{m\left(1-\lambda\right)}{\left(m\lambda+1\right)z}DR^m_{\lambda}f\left(z\right), \tag{6}$$

*for* $z\in U$. *Then*

$$q(z)\prec\left(DR^m_{\lambda}f\left(z\right)\right)', \quad for \quad z\in U,$$

*where* $q(z)=\frac{m+\frac{1}{\lambda}}{z^{m+\frac{1}{\lambda}}}\int_0^z h\left(t\right)t^{m-1+\frac{1}{\lambda}}dt$. *The function* $q$ *is the best subordinant.*

**Corollary 4 [4].** *Let $q$ be convex in $U$ and let $h$ be defined by $h(z) = q(z) + zq'(z)$. If $n \in \mathbb{N}$ and $f \in \mathcal{A}$, suppose that $\frac{1}{z}SR^{n+1}f(z) + \frac{n}{n+1}z(SR^n f(z))''$ is univalent, $(SR^n f(z))' \in \mathcal{H}[1,1] \cap Q$ and satisfies the differential superordination*

$$h(z) = q(z) + zq'(z) \prec \frac{1}{z}SR^{n+1}f(z) + \frac{n}{n+1}z(SR^n f(z))'', \quad \text{for } z \in U. \qquad (7)$$

*Then*

$$q(z) \prec (SR^n f((z)))', \quad \text{for } z \in U,$$

*where $q(z) = \frac{1}{z}\int_0^z h(t)dt$. The function $q$ is the best subordinant.*

**Theorem 3.** *Let $h$ be a convex function and $h(0) = 1$. Let $m, \lambda, l \in \mathbb{N}$, $f \in \mathcal{A}$ and suppose that $\left(IR^m_{\lambda,l} f(z)\right)'$ is univalent and $\frac{IR^m_{\lambda,l} f(z)}{z} \in \mathcal{H}[1,1] \cap Q$. If*

$$h(z) \prec \left(IR^m_{\lambda,l} f(z)\right)', \quad \text{for } z \in U, \qquad (8)$$

*then*

$$q(z) \prec \frac{IR^m_{\lambda,l} f(z)}{z}, \quad \text{for } z \in U,$$

*where $q(z) = \frac{1}{z}\int_0^z h(t)dt$. The function $q$ is convex and it is the best subordinant.*

*Proof.* Consider $p(z) = \frac{IR^m_{\lambda,l} f(z)}{z} = \frac{z + \sum_{j=2}^{\infty}\left(\frac{1+\lambda(j-1)+l}{l+1}\right)^m C^m_{m+j-1} a_j^2 z^j}{z} = 1 + \sum_{j=2}^{\infty}\left(\frac{1+\lambda(j-1)+l}{l+1}\right)^m C^m_{m+j-1} a_j^2 z^{j-1}$. Evidently $p \in \mathcal{H}[1,1]$.

We have $p(z) + zp'(z) = \left(IR^m_{\lambda,l} f(z)\right)'$, for $z \in U$. Then (8) becomes

$$h(z) \prec p(z) + zp'(z), \quad \text{for } z \in U.$$

By using Lemma 1 for $\gamma = 1$ and $n = 1$, we have

$$q(z) \prec p(z), \quad \text{for } z \in U, \quad \text{i.e.} \quad q(z) \prec \frac{IR^m_{\lambda,l} f(z)}{z}, \quad \text{for } z \in U,$$

where $q(z) = \frac{1}{z}\int_0^z h(t)dt$. The function $q$ is convex and it is the best subordinant. □

**Corollary 5 [5].** *Let $h$ be a convex function, $h(0) = 1$. Let $\lambda \geq 0$, $m \in \mathbb{N}$, $f \in \mathcal{A}$ and suppose that $(DR^m_\lambda f(z))'$ is univalent and $\frac{DR^m_\lambda f(z)}{z} \in \mathcal{H}[1,1] \cap Q$. If*

$$h(z) \prec (DR^m_\lambda f(z))', \quad \text{for } z \in U. \qquad (9)$$

*Then*

$$q(z) \prec \frac{DR^m_\lambda f(z)}{z}, \quad \text{for } z \in U,$$

*where $q(z) = \frac{1}{z}\int_0^z h(t)dt$. The function $q$ is convex and it is the best subordinant.*

**Corollary 6 [4]**. *Let $h$ be a convex function, $h(0)=1$. Let $n\in\mathbb{N}$, $f\in\mathcal{A}$ and suppose that $(SR^n f(z))'$ is univalent and $\frac{SR^n f(z)}{z}\in\mathcal{H}[1,1]\cap Q$. If*

$$h(z)\prec (SR^n f(z))',\quad \text{for } z\in U, \tag{10}$$

*then*

$$q(z)\prec \frac{SR^n f(z)}{z},\quad \text{for } z\in U,$$

*where $q(z)=\frac{1}{z}\int_0^z h(t)dt$. The function $q$ is convex and it is the best subordinant.*

**Theorem 4**. *Let $q$ be convex in $U$, $h$ be defined by $h(z)=q(z)+zq'(z)$, $m,\lambda,l\in\mathbb{N}$ and $f\in\mathcal{A}$. Suppose that $\left(IR^m_{\lambda,l}f(z)\right)'$ is univalent, $\frac{IR^m_{\lambda,l}f(z)}{z}\in\mathcal{H}[1,1]\cap Q$ and satisfies the differential superordination*

$$h(z)=q(z)+zq'(z)\prec \left(IR^m_{\lambda,l}f(z)\right)',\quad \text{for } z\in U. \tag{11}$$

*Then*

$$q(z)\prec \frac{IR^m_{\lambda,l}f(z)}{z},\quad \text{for } z\in U,$$

*where $q(z)=\frac{1}{z}\int_0^z h(t)dt$. The function $q$ is the best subordinant.*

*Proof.* Let $p(z)=\dfrac{IR^m_{\lambda,l}f(z)}{z}=\dfrac{z+\sum_{j=2}^{\infty}\left(\frac{1+\lambda(j-1)+l}{l+1}\right)^m C^m_{m+j-1}a_j^2 z^j}{z}=$ $1+\sum_{j=2}^{\infty}\left(\frac{1+\lambda(j-1)+l}{l+1}\right)^m C^m_{m+j-1}a_j^2 z^{j-1}$. Evidently $p\in\mathcal{H}[1,1]$.

Differentiating, we obtain $p(z)+zp'(z)=\left(IR^m_{\lambda,l}f(z)\right)'$, for $z\in U$ and (11) becomes

$$q(z)+zq'(z)\prec p(z)+zp'(z),\quad \text{for } z\in U.$$

Using Lemma 2 for $\gamma=1$ and $n=1$, we have

$$q(z)\prec p(z),\quad \text{for } z\in U,\quad \text{i.e.}\quad q(z)=\frac{1}{z}\int_0^z h(t)dt\prec \frac{IR^m_{\lambda,l}f(z)}{z},\quad \text{for } z\in U,$$

and $q$ is the best subordinant. □

**Corollary 7 [5]**. *Let $q$ be convex in $U$, $h$ be defined by $h(z)=q(z)+zq'(z)$, $\lambda\geq 0$, $m\in\mathbb{N}$ and $f\in\mathcal{A}$. Suppose that $(DR^m_\lambda f(z))'$ is univalent, $\frac{DR^m_\lambda f(z)}{z}\in\mathcal{H}[1,1]\cap Q$ and satisfies the differential superordination*

$$h(z)=q(z)+zq'(z)\prec (DR^m_\lambda f(z))',\quad \text{for } z\in U. \tag{12}$$

*Then*

$$q(z)\prec \frac{DR^m_\lambda f(z)}{z},\quad \text{for } z\in U,$$

*where $q(z)=\frac{1}{z}\int_0^z h(t)dt$. The function $q$ is the best subordinant.*

**Corollary 8** [4]. *Let $q$ be convex in $U$ and let $h$ be defined by $h(z) = q(z) + zq'(z)$. If $n \in \mathbb{N}$, $f \in \mathcal{A}$, suppose that $(SR^n f(z))'$ is univalent, $\frac{SR^n f(z)}{z} \in \mathcal{H}[1,1] \cap Q$ and satisfies the differential superordination*

$$h(z) = q(z) + zq'(z) \prec (SR^n f(z))', \quad \text{for} \quad z \in U. \tag{13}$$

*Then*

$$q(z) \prec \frac{SR^n f(z)}{z}, \quad \text{for} \quad z \in U,$$

*where $q(z) = \frac{1}{z}\int_0^z h(t)dt$. The function $q$ is the best subordinant.*

**Theorem 5**. *Let $h(z) = \frac{1+(2\beta-1)z}{1+z}$ be a convex function in $U$, where $0 \le \beta < 1$. Let $m, \lambda, l \in \mathbb{N}$, $f \in \mathcal{A}$ and suppose that $\left(IR^m_{\lambda,l} f(z)\right)'$ is univalent and $\frac{IR^m_{\lambda,l} f(z)}{z} \in \mathcal{H}[1,1] \cap Q$. If*

$$h(z) \prec \left(IR^m_{\lambda,l} f(z)\right)', \quad \text{for} \quad z \in U, \tag{14}$$

*then*

$$q(z) \prec \frac{IR^m_{\lambda,l} f(z)}{z}, \quad \text{for} \quad z \in U,$$

*where $q$ is given by $q(z) = 2\beta - 1 + 2(1-\beta)\frac{\ln(1+z)}{z}$, for $z \in U$. The function $q$ is convex and it is the best subordinant.*

*Proof.* Following the same steps as in the proof of Theorem 3 and considering $p(z) = \frac{IR^m_{\lambda,l} f(z)}{z}$, the differential superordination (14) becomes

$$h(z) = \frac{1+(2\beta-1)z}{1+z} \prec p(z) + zp'(z), \quad \text{for} \quad z \in U.$$

By using Lemma 1 for $\gamma = 1$ and $n = 1$, we have $q(z) \prec p(z)$, i. e.,

$$q(z) = \frac{1}{z}\int_0^z h(t)dt = \frac{1}{z}\int_0^z \frac{1+(2\beta-1)t}{1+t}dt = 2\beta - 1 + 2(1-\beta)\frac{1}{z}\ln(z+1) \prec \frac{IR^m_{\lambda,l} f(z)}{z},$$

for $z \in U$.
The function $q$ is convex and it is the best subordinant. □

**Theorem 6**. *Let $h$ be a convex function, $h(0) = 1$. Let $m, \lambda, l \in \mathbb{N}$, $f \in \mathcal{A}$ and suppose that $\left(\frac{zIR^{m+1}_{\lambda,l} f(z)}{IR^m_{\lambda,l} f(z)}\right)'$ is univalent and $\frac{IR^{m+1}_{\lambda,l} f(z)}{IR^m_{\lambda,l} f(z)} \in \mathcal{H}[1,1] \cap Q$. If*

$$h(z) \prec \left(\frac{zIR^{m+1}_{\lambda,l} f(z)}{IR^m_{\lambda,l} f(z)}\right)', \quad \text{for} \quad z \in U, \tag{15}$$

*then*

$$q(z) \prec \frac{IR^{m+1}_{\lambda,l} f(z)}{IR^m_{\lambda,l} f(z)}, \quad \text{for} \quad z \in U,$$

*where $q(z) = \frac{1}{z}\int_0^z h(t)dt$. The function $q$ is convex and it is the best subordinant.*

*Proof.* Consider $p(z) = \frac{IR_{\lambda,l}^{m+1}f(z)}{IR_{\lambda,l}^{m}f(z)} = \frac{z+\sum_{j=2}^{\infty}\left(\frac{1+\lambda(j-1)+l}{l+1}\right)^{m+1}C_{m+j}^{m+1}a_j^2z^j}{z+\sum_{j=2}^{\infty}\left(\frac{1+\lambda(j-1)+l}{l+1}\right)^{m}C_{m+j-1}^{m}a_j^2z^j} = \frac{1+\sum_{j=2}^{\infty}\left(\frac{1+\lambda(j-1)+l}{l+1}\right)^{m+1}C_{m+j}^{m+1}a_j^2z^{j-1}}{1+\sum_{j=2}^{\infty}\left(\frac{1+\lambda(j-1)+l}{l+1}\right)^{m}C_{m+j-1}^{m}a_j^2z^{j-1}}$. Evidently $p \in \mathcal{H}[1,1]$.

We have $p'(z) = \frac{\left(IR_{\lambda,l}^{m+1}f(z)\right)'}{IR_{\lambda,l}^{m}f(z)} - p(z)\cdot\frac{\left(IR_{\lambda,l}^{m}f(z)\right)'}{IR_{\lambda,l}^{m}f(z)}$. Hence $p(z) + zp'(z) = \left(\frac{zIR_{\lambda,l}^{m+1}f(z)}{IR_{\lambda,l}^{m}f(z)}\right)'$.

Then (15) becomes

$$h(z) \prec p(z) + zp'(z), \quad \text{for } z \in U.$$

By using Lemma 1 for $\gamma = 1$ and $n = 1$, we have

$$q(z) \prec p(z), \quad \text{for } z \in U, \quad \text{i.e.} \quad q(z) \prec \frac{IR_{\lambda,l}^{m+1}f(z)}{IR_{\lambda,l}^{m}f(z)}, \quad \text{for } z \in U,$$

where $q(z) = \frac{1}{z}\int_0^z h(t)dt$. The function $q$ is convex and it is the best subordinant. □

**Corollary 9 [5]**. *Let $h$ be a convex function, $h(0) = 1$. Let $\lambda \geq 0$, $m \in \mathbb{N}$, $f \in \mathcal{A}$ and suppose that $\left(\frac{zDR_{\lambda}^{m+1}f(z)}{DR_{\lambda}^{m}f(z)}\right)'$ is univalent and $\frac{DR_{\lambda}^{m+1}f(z)}{DR_{\lambda}^{m}f(z)} \in \mathcal{H}[1,1] \cap Q$. If*

$$h(z) \prec \left(\frac{zDR_{\lambda}^{m+1}f(z)}{DR_{\lambda}^{m}f(z)}\right)', \quad \text{for } z \in U, \tag{16}$$

*then*

$$q(z) \prec \frac{DR_{\lambda}^{m+1}f(z)}{DR_{\lambda}^{m}f(z)}, \quad \text{for } z \in U,$$

*where $q(z) = \frac{1}{z}\int_0^z h(t)dt$. The function $q$ is convex and it is the best subordinant.*

**Corollary 10 [4]**. *Let $h$ be a convex function, $h(0) = 1$. Let $n \in \mathbb{N}$, $f \in \mathcal{A}$ and suppose that $\left(\frac{zSR^{n+1}f(z)}{SR^{n}f(z)}\right)'$ is univalent and $\frac{SR^{n+1}f(z)}{SR^{n}f(z)} \in \mathcal{H}[1,1] \cap Q$. If*

$$h(z) \prec \left(\frac{zSR^{n+1}f(z)}{SR^{n}f(z)}\right)', \quad \text{for } z \in U, \tag{17}$$

*then*

$$q(z) \prec \frac{SR^{n+1}f(z)}{SR^{n}f(z)}, \quad \text{for } z \in U,$$

*where $q(z) = \frac{1}{z}\int_0^z h(t)dt$. The function $q$ is convex and it is the best subordinant.*

**Theorem 7**. *Let $q$ be convex in $U$, $h$ be defined by $h(z) = q(z) + zq'(z)$, $m, \lambda, l \in \mathbb{N}$ and $f \in \mathcal{A}$. Suppose that $\left(\frac{zIR_{\lambda,l}^{m+1}f(z)}{IR_{\lambda,l}^{m}f(z)}\right)'$ is univalent, $\frac{IR_{\lambda,l}^{m+1}f(z)}{IR_{\lambda,l}^{m}f(z)} \in \mathcal{H}[1,1] \cap Q$ and* *satisfies the differential superordination*

$$h(z)=q(z)+zq'(z)\prec\left(\frac{zIR_{\lambda,l}^{m+1}f(z)}{IR_{\lambda,l}^{m}f(z)}\right)',\quad \textit{for}\ \ z\in U, \tag{18}$$

*then*

$$q(z)\prec\frac{IR_{\lambda,l}^{m+1}f(z)}{IR_{\lambda,l}^{m}f(z)},\quad \textit{for}\ \ z\in U,$$

*where* $q(z)=\frac{1}{z}\int_0^z h(t)dt$. *The function* $q$ *is the best subordinant.*

*Proof.* Let $p(z)=\frac{IR_{\lambda,l}^{m+1}f(z)}{IR_{\lambda,l}^{m}f(z)}=\frac{z+\sum_{j=2}^{\infty}\left(\frac{1+\lambda(j-1)+l}{l+1}\right)^{m+1}C_{m+j}^{m+1}a_j^2z^j}{z+\sum_{j=2}^{\infty}\left(\frac{1+\lambda(j-1)+l}{l+1}\right)^{m}C_{m+j-1}^{m}a_j^2z^j}=$ $\frac{1+\sum_{j=2}^{\infty}\left(\frac{1+\lambda(j-1)+l}{l+1}\right)^{m+1}C_{m+j}^{m+1}a_j^2z^{j-1}}{1+\sum_{j=2}^{\infty}\left(\frac{1+\lambda(j-1)+l}{l+1}\right)^{m}C_{m+j-1}^{m}a_j^2z^{j-1}}$. Evidently $p\in\mathcal{H}[1,1]$.

Differentiating, we obtain $p(z)+zp'(z)=\left(\frac{zIR_{\lambda,l}^{m+1}f(z)}{IR_{\lambda,l}^{m}f(z)}\right)'$, for $z\in U$ and (18) becomes

$$q(z)+zq'(z)\prec p(z)+zp'(z),\quad \text{for}\ \ z\in U.$$

Using Lemma 2 for $\gamma=1$ and $n=1$, we have

$$q(z)\prec p(z),\quad \text{for}\ \ z\in U,\quad \text{i.e.}\quad q(z)=\frac{1}{z}\int_0^z h(t)dt\prec\frac{IR_{\lambda,l}^{m+1}f(z)}{IR_{\lambda,l}^{m}f(z)},\ \ \text{for}\ \ z\in U,$$

and $q$ is the best subordinant. □

**Corollary 11** [**5**]. *Let* $q$ *be convex in* $U$, $h$ *be defined by* $h(z)=q(z)+zq'(z)$, $\lambda\geq 0$, $m\in\mathbb{N}$ *and* $f\in\mathcal{A}$. *Suppose that* $\left(\frac{zDR_{\lambda}^{m+1}f(z)}{DR_{\lambda}^{m}f(z)}\right)'$ *is univalent,* $\frac{DR_{\lambda}^{m+1}f(z)}{DR_{\lambda}^{m}f(z)}\in\mathcal{H}[1,1]\cap Q$ *and satisfies the differential superordination*

$$h(z)=q(z)+zq'(z)\prec\left(\frac{zDR_{\lambda}^{m+1}f(z)}{DR_{\lambda}^{m}f(z)}\right)',\quad \textit{for}\ \ z\in U. \tag{19}$$

*Then*

$$q(z)\prec\frac{DR_{\lambda}^{m+1}f(z)}{DR_{\lambda}^{m}f(z)},\quad \textit{for}\ \ z\in U,$$

*where* $q(z)=\frac{1}{z}\int_0^z h(t)dt$. *The function* $q$ *is the best subordinant.*

**Corollary 12** [**4**]. *Let* $q$ *be convex in* $U$, $h$ *be defined by* $h(z)=q(z)+zq'(z)$, $n\in\mathbb{N}$, $f\in\mathcal{A}$. *Suppose that* $\left(\frac{zSR^{n+1}f(z)}{SR^{n}f(z)}\right)'$ *is univalent,* $\frac{SR^{n+1}f(z)}{SR^{n}f(z)}\in\mathcal{H}[1,1]\cap Q$ *and satisfies the differential superordination*

$$h(z)=q(z)+zq'(z)\prec\left(\frac{zSR^{n+1}f(z)}{SR^{n}f(z)}\right)',\quad \textit{for}\ \ z\in U. \tag{20}$$

*Then*

$$q(z) \prec \frac{SR^{n+1}f(z)}{SR^{n}f(z)}, \quad \textit{for} \ \ z \in U,$$

*where* $q(z) = \frac{1}{z}\int_0^z h(t)dt$*. The function* $q$ *is the best subordinant.*

**Theorem 8**. *Let* $h(z) = \frac{1+(2\beta-1)z}{1+z}$ *be a convex function in* $U$*, where* $0 \le \beta < 1$*. Let* $m, \lambda, l \in \mathbb{N}$*,* $f \in \mathcal{A}$ *and suppose that* $\left(\frac{zIR^{m+1}_{\lambda,l}f(z)}{IR^{m}_{\lambda,l}f(z)}\right)'$ *is univalent,* $\frac{IR^{m+1}_{\lambda,l}f(z)}{IR^{m}_{\lambda,l}f(z)} \in \mathcal{H}[1,1] \cap Q$*. If*

$$h(z) \prec \left(\frac{zIR^{m+1}_{\lambda,l}f(z)}{IR^{m}_{\lambda,l}f(z)}\right)', \quad \textit{for} \ \ z \in U, \tag{21}$$

*then*

$$q(z) \prec \frac{IR^{m+1}_{\lambda,l}f(z)}{IR^{m}_{\lambda,l}f(z)}, \quad \textit{for} \ \ z \in U,$$

*where* $q$ *is given by* $q(z) = 2\beta - 1 + 2(1-\beta)\frac{\ln(1+z)}{z}$*, for* $z \in U$*. The function* $q$ *is convex and it is the best subordinant.*

*Proof.* Following the same steps as in the proof of Theorem 2 and considering $p(z) = \frac{IR^{m+1}_{\lambda,l}f(z)}{IR^{m}_{\lambda,l}f(z)}$, the differential superordination (21) becomes

$$h(z) = \frac{1+(2\beta-1)z}{1+z} \prec p(z) + zp'(z), \quad \text{for} \ \ z \in U.$$

By using Lemma 1 for $\gamma = 1$ and $n = 1$, we have $q(z) \prec p(z)$, i.e.,

$$q(z) = \frac{1}{z}\int_0^z h(t)dt = \frac{1}{z}\int_0^z \frac{1+(2\beta-1)t}{1+t}dt =$$

$$2\beta - 1 + 2(1-\beta)\frac{1}{z}\ln(z+1) \prec \frac{IR^{m+1}_{\lambda,l}f(z)}{IR^{m}_{\lambda,l}f(z)}, \quad \text{for} \ \ z \in U.$$

The function $q$ is convex and it is the best subordinant. □

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