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Some addition theorems for rectifiable spaces

Alexander V. Arhangel'skii, Mitrofan M. Choban

Abstract. We establish that if a compact Hausdorff space B with the cardinality less than 2^{ω_1} is represented as the union of two non-locally compact rectifiable subspaces X and Y, then X, Y and B are separable and metrizable.

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1 Introduction

It is well-known that if the cardinality of a compact topological group X does not exceed 2^ω and the continuum hypothesis is satisfied, then X is separable and metrizable (see [8]). Extending this result, we show that if the cardinality of a compact Hausdorff space X is less than 2^{ω_1}, then X cannot be represented as the union of two non-locally compact rectifiable spaces. Recall that every topological group is a rectifiable space (see the definition below). Some other results in this direction are also obtained.

We use the terminology and notations from [12]. A *remainder* of a Tychonoof space X is the subspace $bX \setminus X$ of a Hausdorff compactification bX of X.

A space X is of *countable type* (respectively, of *pointwise countable type*) if every compact subspace P (respectively, any point p) of X is contained in a compact subspace $F \subset X$ with a countable base of open neighbourhoods in X. All metrizable spaces and all locally compact Hausdorff spaces, as well as all Čech-complete spaces, are of countable type [1, 2, 12].

A famous classical result on duality between properties of spaces and properties of their remainders is the following theorem of M. Henriksen and J. Isbell [14]:

Theorem 1. *A Tychonoff space X is of countable type if and only if the remainder in any (in some) Hausdorff compactification of X is Lindelöf.*

It follows from this theorem that every remainder of a metrizable space is Lindelöf.

2 Addition theorems for rectifiable spaces

Recall that a space X is *rectifiable* if there exists $e \in X$ and a homeomorphism $g : X \times X \to X \times X$ such that $g((x,e)) = (x,x)$, for every $x \in X$, and the restriction of g to the subspace $X_x = \{(x,y) : y \in X\}$ is a homeomorphism of X_x onto itself, for every $x \in X$. Every topological group is rectifiable, and every rectifiable space is homogeneous (see [9, 10]).

Theorem 2. *Suppose that B is a compact Hausdorff space such that $|B| < 2^{\omega_1}$. Suppose further that $B = X \cup Y$, where X and Y are non-locally compact rectifiable spaces. Then the spaces B, X, and Y are separable and metrizable.*

Proof. Clearly, Y and X are non-empty, since they are not locally compact. Hence, B is non-empty. By Čech-Pospišil Theorem ([3, 15], [12], Problem 3.12.11), there exists a point $a \in B$ such that B is first-countable at a. Without loss of generality, we may assume that $a \in X$. Put $Z = B \setminus X$ and $H = B \setminus Y$. The spaces X and Y are nowhere locally compact, since they are homogeneous and non-locally compact. It also follows that Z and H are nowhere locally compact. Hence, X, Y, Z, and H are dense in B.

Since X is homogeneous and X is first-countable at a, it follows that the space X is first-countable. Therefore, X is metrizable, since X is rectifiable [13]. Hence, X is a space of countable type [1], which implies that the remainder Z of X in B is Lindelöf [14].

By the Dichotomy Theorem for remainders of rectifiable spaces (see [6]), the remainder H of Y in the Hausdorff compactification B of Y is either pseudocompact, or Lindelöf. Notice that H is metrizable in any case, since, obviously $H \subset X$.

Case 1: H is pseudocompact.

Then H is compact, since H is metrizable. Therefore, H is closed in B. Hence, Y is open in B, which implies that Y is locally compact, a contradiction. Thus, Case 1 is impossible.

Case 2: H is Lindelöf.

Then H is separable, since H is metrizable. Hence, B is separable, which implies that the Souslin number of Y is countable, since Y is dense in B. It also follows that X is separable, since H is dense in X.

Lindelöfness of H also implies that Y is a space of countable type, by the theorem of Henriksen and Isbell [14]. Therefore, we can fix a non-empty compact subspace F of Y with a countable base of open neighbourhoods in Y. We have: $|F| \leq |B| < 2^{\omega_1}$. Applying one more time the Čech-Pospišil Theorem [3,12,15] we conclude that there exists a point $b \in F$ such that F is first-countable at b. Since F has a countable base of open neighbourhoods in Y, we can now conclude that the space Y is first-countable at the point b. Therefore, the space Y is first-countable, since it is homogeneous. Hence, Y is metrizable, since it is rectifiable. Finally, it follows that Y is separable, since the Souslin number of Y is countable. □

A family η of non-empty open subsets of a space X is said to be a *π-base* of X at a point $a \in X$ if every open neighbourhood of a in X contains some $V \in \eta$ (see [7]).

Here is another restriction on a compactum B under which we can obtain even a stronger conclusion:

Theorem 3. *Suppose that B is a compact Hausdorff space of countable tightness and that $B = \cup\{Y_n : n \in \omega = \{0, 1, 2, ...\}\}$, where each Y_n is dense in B and is rectifiable. Then B and each Y_n are separable and metrizable.*

Proof. Take any $y \in Y_n$. Then there exists a countable π-base ξ of B at y, since the tightness of the compactum B is countable (see [18] and [3]). Then $\eta = \{V \cap Y_n : V \in \xi\}$ is a countable π-base of the subspace Y_n at y, since Y_n is dense in B. Since Y_n is rectifiable and Y_n has a countable π-base at y, it follows from a result of A. Gul'ko [13] that the space Y_n is metrizable. Therefore, each Y_n has a σ-disjoint open base. Since Y_n is dense in B, it follows that B is first-countable and that σ-disjoint open bases in the subspaces Y_n can be extended, in a standard way, to a point-countable base in B. It remains to use a well-known deep theorem of A.S. Mischenko that every compact Hausdorff space with a point-countable base is separable and metrizable (see [12], Problem 3.12.22(f)). □

The next result considerably generalizes Theorem 2.

Theorem 4. *Suppose that B is a compact Hausdorff space which doesn't admit a continuous mapping onto the Tychonoff cube I^{ω_1}. Suppose further that $B = X \cup Y$, where X and Y are non-locally compact rectifiable spaces. Then B, X, and Y are separable and metrizable.*

Proof. Clearly, Y and X are non-empty. Hence, B is non-empty. Since B cannot be continuously mapped onto the Tychonoff cube I^{ω_1}, it follows from a Theorem of B. E. Shapirovskij (see [18], [3], Theorems 2.2.20 and 3.1.9) that there exists a point $a \in B$ such that B has a countable π-base at a. Without loss of generality, we may assume that $a \in X$. Put $Z = B \setminus X$ and $H = B \setminus Y$. The spaces X and Y are nowhere locally compact, since they are homogeneous and non-locally compact. Clearly, the subspaces Z and H are nowhere locally compact as well. Thus, X, Y, Z, and H are dense in B.

Since X has a countable π-base at a and X is rectifiable, it follows that the space X is metrizable [13]. Hence, X is a space of countable type [1], which implies that the remainder Z of X in B is Lindelöf [14].

By the Dichotomy Theorem for remainders of rectifiable spaces (see [6]), the remainder H of Y in the compactification B of Y is either pseudocompact, or Lindelöf. Notice that H is metrizable, since $H \subset X$.

If H is pseudocompact, then H is compact, since H is metrizable. Therefore, H is closed in B and Y is open in B, which implies that Y is locally compact, a contradiction.

Hence, H is Lindelöf. Then H has a countable base, since H is metrizable. Hence, B has a countable π-base, since H is dense in B, which implies that Y also has a countable π-base, since Y is dense in B. Since the space Y is rectifiable, using again a theorem of A. Gul'ko in [13], we conclude that Y is metrizable. Clearly, Y is separable. It also follows that X is separable, since H is dense in X. Therefore, B is separable and metrizable, as the union of two separable metrizable subspaces (see [12], Corollary 3.1.20). $\square$

The proof of Theorem 2 obviously contains a proof of the next statement:

Theorem 5. *Suppose that B is a compact Hausdorff space and that $B = X \cup Y$, where X and Y are non-locally compact spaces. Suppose further that X is metrizable and Y is rectifiable. Then B, X and Y are separable and metrizable.*

3 On *k*-gentle paratopological groups

A group G with a topology $\mathcal{T}$ is called a *paratopological* group if the multiplication $(x, y) \to x \cdot y$ is a continuous mapping of $G \times G$ onto G.

Let us call a mapping f of a space X into a space Y *k-gentle* if for every compact subset F of X the image $f(F)$ is also compact.

A group G with a topology will be called *k-gentle* if the inverse mapping $x \to x^{-1}$ is k-gentle.

Proposition 1. *Suppose that B is a compact Hausdorff space in which any non-empty G_δ-subspace has a point of countable character in this subspace. Suppose further that $B = X \cup Y$, where each $Z \in \{X, Y\}$ is a space with the following properties:*

– the space Z is not locally compact;

– if the space Z contains some point of countable character in Z, then the space Z is metrizable;

– if bZ is a Hausdorff compactification of Z, then the remainder $bZ \setminus Z$ is either pseudocompact or Lindelöf.

Then B, X, and Y are separable and metrizable.

Proof. Clearly, Y and X are non-empty, since they are not locally compact. Hence, B is non-empty. Moreover, the sets X and Y are dense in B. Thus, B is a compactification of the subspaces X and Y. There exists a point $a \in B$ such that B is first-countable at a. Without loss of generality, we may assume that $a \in X$. Then the space X is metrizable.

If $b \in X \cap Y$, for some b, then the space Y is metrizable, as a space with the countable character at b. In this case the proof is complete.

Assume that $X \cap Y = \emptyset$. Clearly, X is a space of countable type [1], since X is metrizable. It follows that the remainder Y of X in B is Lindelöf [14].

Clearly, the space X is a remainder of the space Y in B. Hence, X is either pseudocompact or Lindelöf.

Case 1: X is pseudocompact.

Then X is compact, since X is metrizable. Therefore, X is closed in B, a contradiction. Thus, Case 1 is impossible.

Case 2: X is Lindelöf.

Then X is separable, since X is metrizable. Hence, B is separable, which implies that the Souslin number of Y is countable, since Y is dense in B. Lindelöfness of X also implies that Y is a space of countable type, by the theorem of Henriksen and Isbell [14]. Therefore, we can fix a non-empty compact subspace F of Y with a countable base of open neighbourhoods in Y. By the assumptions, there exists a point $c \in F$ such that B is first-countable at c. It follows that the space Y is first-countable at c. Thus Y is a metrizable space with a countable Souslin number. Hence, X and Y are separable and metrizable. It follows that B is separable and metrizable, since B is compact and Hausdorff ([12], Corollary 3.1.20). $\square$

Corollary 6. *Suppose that B is a compact Hausdorff space such that $|B| < 2^{\omega_1}$. Suppose further that $B = X \cup Y$, where X and Y are non-locally compact k-gentle paratopological groups. Then B, X, Y are separable, metrizable spaces, and X, Y are topological groups.*

In view of Proposition 1, this statement follows from the next proposition:

Proposition 2. *Let G be a Hausdorff k-gentle paratopological group such that G is first-countable at some point. Then:*

1) the space G is metrizable;

2) G is a topological group;

3) any remainder of G in a Hausdorff compactification bG of G is either pseudocompact or Lindelöf.

Proof. Since G is a homogeneous space, the space G is first-countable. Every first-countable Hausdorff space is a k-space ([12], Theorem 3.3.20). Hence G is a k-space. It is obvious that if a k-gentle paratopological group is a k-space, then this paratopological group is a topological group. Hence, 2) holds. Every first-countable topological group is metrizable (see [8]). Therefore, 1) holds. By the Dichotomy Theorem for remainders of topological groups (see [4, 5]), since G is a topological group, any remainder of G in a Hausdorff compactification of G is either pseudocompact or Lindelöf. Therefore, 3) holds. $\square$

Example 7. Let X_1 be the space of all rational numbers of the interval $I = [0, 1]$. Clearly, X_1 is homeomorphic to a topological group. The space $Y_1 = I \setminus X_1$ is also homeomorphic to a topological group. Take also the topological group D^{ω_1}. Put $B = I \times D^{\omega_1}$, $X = X_1 \times D^{\omega_1}$ and $Y = Y_1 \times D^{\omega_1}$. Then X and Y are dense non-metrizable nowhere locally compact topological groups, B is a homogeneous compact Hausdorff space with the cardinality 2^{ω_1}, and $B = X \cup Y$. The space B admits a continuous mapping onto I^{ω_1} and the tightness $t(B) = \aleph_1$. Thus the respective cardinal assumptions in Theorems 2, 3, 4 and Corollary 6 are essential.

We could also modify the definitions of B, X, and Y above so that each of the spaces B, X, Y would admit a structure of a topological group.

4 On Mal'cev spaces. Some questions

A Mal'cev operation on a space X is a continuous mapping $\mu : X^3 \to X$ such that $\mu(x,x,z) = z$ and $\mu(x,y,y) = x$, for all $x,y,z \in X$. A space is called a *Mal'cev space* if it admits a Mal'cev operation (see [9–11, 16, 19]).

A homogeneous algebra on a space G is a pair of binary continuous operations $p,q : G \times G \to G$ such that $p(x,x) = p(y,y)$, and $p(x, q(x,y)) = y$, $q(x, p(x,y)) = y$ for all $x, y \in G$. If the above conditions are satisfied, then the ternary operation $\mu(x,x,z) = q(x, p(y,z))$ is a Mal'cev operation (see [9, 10]).

A biternary algebra on a space G is a pair of ternary continuous operations $\alpha, \beta : G \times G \times G \to G$ such that $\alpha(x,x,y) = y$, $\alpha(\beta(x,y,z),y,z)) = x$, and $\beta(\alpha(x,y,z),y,z)) = x$, for all $x,y,z \in G$ (see [16]).

In [9, 10] (see also [19]) it was proved that for an arbitrary space G the following conditions are equivalent:

1) G is a rectifiable space;
2) G is homeomorphic to a homogeneous algebra;
3) There exists a structure of a biternary algebra on G.

A structure of *a topological quasigroup* on a space G is a triplet of binary continuous operations $p,l,r : G \times G \times G \to G$ such that $p(x, l(x,y)) = p(r(y,x),x) = l(x, p(x,y)) = l(r(x,y),x) = r(p(y,x),x) = r(x, l(y,x)) = y$, for all $x,y \in G$. If there exists an element $e \in G$ such that $p(e,x) = p(x,e) = x$ for any $x \in G$, then we say that G is *a topological loop* and e is the identity of G. Any topological quasigroup admits the structure of a topological loop (see [16]). If $e \in G$ and $p(e,x) = x$ for any $x \in G$, then $x + y = p(y,x)$ and $x \cdot y = r(y,x)$ is a structure of a homogeneous algebra.

If $(G, \cdot)$ is a topological group with the neutral element e, then the mapping $\varphi(x,y) = (x, x^{-1} \cdot y)$ is a rectification on the space G with the neutral element e, and the mappings $p(x,y) = x^{-1} \cdot y$ and $q(x,y) = x \cdot y$ form a structure of homogeneous algebra on G. Therefore, every topological quasigroup is a rectifiable space.

A space X is called *κ-perfect* if the closure of any open subset of X is a G_δ-set in X.

Proposition 3. *Let X and Y be any pseudocompact κ-perfect subspaces of a Tychonoff space Z such that $Z = X \cup Y$, and X, Y are dense in Z. Suppose further that X and Y are Mal'cev spaces. Then the space Z is also κ-perfect.*

Proof. Let U be an open subset of the space Z. We put $V = X \cap U$ and $W = Y \cap U$. There exist two sequences $\{V_n : n \in \omega\}$ and $\{W_n : n \in \omega\}$ of open subsets of the space Z such that:

– $cl_X V = \cap\{V_n \cap X : n \in \omega\}$ and $cl_Y W = \cap\{W_n \cap Y : n \in \omega\}$;
– $V_{n+1} \subseteq V_n$ and $W_{n+1} \subseteq W_n$ for any $n \in \omega$.

Obviously, $cl_Z U = cl_Z V = cl_Z W$. We put $F + cl_z U$.

We affirm that $\cap\{V_n : n \in \omega\} \subseteq F$. Assume that $H = (\cap\{V_n : n \in \omega\}) \setminus F \neq \emptyset$. By construction, H is a G_δ-subset of Z and $H \subseteq Y$. Fix a point $b \in H$. There exists a continuous function $f : Z \longrightarrow [0,1]$ such that $b \in f^{-1}(0) \subseteq H$. Then the function $g(x) = 1/f(x)$ is a continuous unbounded function on the space X, a contradiction. Thus $\cap\{V_n : n \in \omega\} \subseteq F$ $\cap\{W_n : n \in \omega\} \subseteq F$. If $U_n = V_n \cup W_n$, then $\cap\{W_n : n \in \omega\} = F$. $\square$

Proposition 4. *Let X and Y be pseudocompact subspaces of a compact Hausdorff space B such that X and Y are dense in B and $Z = X \cup Y$. Suppose further that X and Y are Mal'cev spaces. Then:*

1) The space B is a κ-perfect Mal'cev space.

2) There exist Mal'cev operations $\mu, \eta : B^3 \longrightarrow B$ on B such that $\mu(X^3) = X$ and $\eta(Y^3) = Y$.

3) If X is rectifiable, then B is also rectifiable, and there exists a structure of homogeneous algebra $\{+, \cdot\}$ on B such that X is a subagebra of B.

4) If X is a topological quasigroup, then there exists a structure of a topological loop on B such that X is a subloop of B.

5) If the space X is a topological group, then on there exists a structure of a topological group on B such that X is a subgroup of B.

6) $B = \beta X = \beta Y$.

Proof. Since X is a pseudocompact Mal'cev space, the Stone-Čech compactification βX of X is a compact Mal'cev space [17]. Any compact Mal'cev space is κ-perfect [10]. Thus, X and Y are κ-perfect spaces, since they are dense subspaces of κ-perfect spaces. By Proposition 3, the space B is κ-perfect. Now we need the following known fact:

Fact 1: If Z is a pseudocompact subspace of a κ-perfect compact Hausdorff space B such that Z is dense in B, then $B = \beta Z$.

Really, let F_1 and F_2 be two closed subsets of Z and $f : Z \longrightarrow \mathbb{R}$ be a continuous function such that $F_1 \subseteq f^{-1}(-2)$ and $F_{\subseteq} f^{-1}(2)$. There exist two open subsets U and V of B such that $U \cap Z \subseteq f^{-1}(-3,-1)$ and $V \cap Z \subseteq f^{-1}(1,3)$. Then $H = cl_B U \cap cl_B V$ is a G_δ-subset of B and, by construction, $H \subseteq B \setminus Z$. Since Z is pseudocompact, we have $H = \emptyset$. Since $F_1 \subseteq U$ and $F_2 \subseteq V$, we have $cl_B F_1 \cap cl_B F_2 = \emptyset$. Therefore $B = \beta Z$.

Thus, $B = \beta X = \beta Y$. Statements 1 and 6 are proved. The space X^n is pseudocompact for any $n \in \omega$. Hence, by virtue of Glicksberg's Theorem ([12], Problem 3.12.20(d)), any continuous binary operation on X admits continuous extension on B. Statements 2, 3 and 4 are established. $\square$

Proposition 5. *Let X be a subalgebra of a homogeneous algebra G. If the space G is regular and Lindelöf, and the space X is of pointwise countable type and is dense in G, then there exist a separable metrizable homogeneous algebra G' and a homomorphism $g : G \longrightarrow G'$ such that $X = g^{-1}(g(X))$ and the mapping g is open and perfect. In particular, it follows that X is a Lindelöf p-space.*

Proof. By the assumptions, there is a pair of binary continuous operations $p, q : G \times G \to G$ on the space G such that:

– $p(x,x) = p(y,y)$, and $p(x, q(x,y)) = y$, $q(x, p(x,y)) = y$ for all $x, y \in G$;

– $p(x,y) \in X$ and $q(x,y) \in X$ for all $x, y \in X$.

We put $e = p(x,x)$. If $a \in G$, then $p_a(x) = p(a,x)$ and $q_a(x) = q(a,x)$ for any $x \in G$. We have $q_a^{-1} = p_a$ and $q_a(e) = a$. Thus, p_a and q_a are homeomorphisms. Moreover, $p_a(X) = q_a(X) = X$ for each $a \in X$.

Let F be a non-empty compact subspace of X with a countable base of open neighbourhoods in X. We can assume that $e \in F$. Since X is dense in G, the set F also has a countable base of open neighbourhoods in the space G. Therefore, X and G are p-spaces (see [6], Proposition 2.1).

Since F is a compact G_δ-subset of the Lindelöf algebra G, there exist a separable metrizable homogeneous algebra G' and a homomorphism $g : G \longrightarrow G'$ such that $F = g^{-1}(g(F))$ and g is a perfect mapping [10]. The quotient homomorphism of a Mal'cev algebra is an open mapping [10]. Thus, the mapping g is open. We can assume that $e' = g(e)$ and $p(z,z) = e'$ for any $z \in G'$.

Let $b \in g(X) \subseteq G'$. Fix $a \in X \cap g^{-1}(b)$. If $H = g^{-1}(e'$, then $H \subseteq F \subseteq X$ and $q_a(H) = g^{-1}(q_b(e') \subseteq X$. Thus, $g^{-1}(b) = g^{-1}(q_b(e') \subseteq X$. Therefore, $g^{-1}(g(X)) = X$. The proof is complete. □

Since a pseudocompact rectifiable space X can be considered as a subalgebra of the compact homogeneous algebra $G = \beta X$, Proposition 5 yields

Corollary 8. *If G is a pseudocompact rectifiable space of pointwise countable type, then G is compact.*

Theorem 9. *Suppose that B is a compact Hausdorff space, $B = X \cup Y$ and $X \cap Y = \emptyset$, where X and Y are non-locally compact rectifiable spaces. Then X and Y either are both pseudocompact, or are both Lindelöf p-spaces.*

Proof. Clearly, X is the remainder of Y in the Hausdorff compactification B of Y; similarly, Y is the remainder of X in the Hausdorff compactification B of X.

By the Dichotomy Theorem for remainders of rectifiable spaces (see [6]), each of the spaces X and Y is either pseudocompact, or Lindelöf. If both of them are pseudocompact, then we are done.

Assume now that at least one of the subspaces X and Y, say X, is Lindelöf.

By the Dichotomy Theorem for remainders in [6], the remainder Y of X in B is either pseudocompact or Lindelöf.

Case 1: Y is pseudocompact.

Lindelöfness of X implies that Y is a space of countable type, by the theorem of Henriksen and Isbell [14]. Then Corollary 8 implies that Y is compact. Therefore, Y is closed in B. Hence, X is open in B, which implies that X is locally compact, a contradiction. Thus, Case 1 is impossible.

Case 2: Y is Lindelöf.

Lindelöfness of X and Y implies that X and Y are spaces of countable type, by the theorem of Henriksen and Isbell [14]. Therefore, by Proposition 2.1 from [6], X and Y are Lindelöf p-spaces. The proof is complete. $\square$

Observe that Theorem 9 doesn't generalize to homogeneous Mal'cev spaces. Indeed, a non-metrizable compactum can be represented as the union of two disjoint dense copies of Sorgenfrey line (take the "double arrow" space). It was shown in [17] that Sorgenfrey line is a Mal'cev space. It is well-known that Sorgenfrey line is not a p-space (see [5]). It is also clear that Sorgenfrey line is not pseudocompact.

Question 1. Is every rectifiable space of countable type paracompact? Normal?

Question 2. Does every rectifiable space of countable type admit a perfect mapping onto a metrizable (rectifiable) space?

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Alexander V. Arhangel'skii
Moscow, Russia
E-mail: *arhangel.alex@gmail.com*

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Mitrofan M. Choban
Department of Mathematics
Tiraspol State University, Chişinău Moldova
E-mail: *mmchoban@gmail.com*