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On 2-primal Ore extensions over Noetherian $\sigma(*)$-rings

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Abstract. In this article, we discuss the prime radical of skew polynomial rings over Noetherian rings. We recall $\sigma(*)$ property on a ring R (i. e. $a\sigma(a) \in P(R)$ implies $a \in P(R)$ for $a \in R$, where $P(R)$ is the prime radical of R, and σ an automorphism of R). Let now δ be a σ-derivation of R such that $\delta(\sigma(a)) = \sigma(\delta(a))$ for all $a \in R$. Then we show that for a Noetherian $\sigma(*)$-ring, which is also an algebra over $\mathbb{Q}$, the Ore extension $R[x;\sigma,\delta]$ is 2-primal Noetherian (i. e. the nil radical and the prime radical of $R[x;\sigma,\delta]$ coincide).

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1 Introduction

A ring R always means an associative ring with identity $1 \neq 0$. The fields of complex numbers, real numbers, rational numbers, the ring of integers and the set of natural numbers are denoted by $\mathbb{C}$, $\mathbb{R}$, $\mathbb{Q}$, $\mathbb{Z}$ and $\mathbb{N}$ respectively unless otherwise stated. The set of prime ideals of R is denoted by $Spec(R)$. The set of minimal prime ideals of R is denoted by $Min.Spec(R)$. The prime radical and the nil radical of R are denoted by $P(R)$ and $N(R)$ respectively. Let R be a ring and σ an automorphism of R. Let I be an ideal of R such that $\sigma^m(I) = I$ for some $m \in \mathbb{N}$. We denote $\cap_{i=1}^{m}\sigma^i(I)$ by I^0. For any two ideals I, J of R, $I \subset J$ means that I is strictly contained in J.

This article concerns the study of skew polynomial rings (Ore extensions) in terms of 2-primal rings. Recall that the skew polynomial ring $R[x;\sigma,\delta]$ is the set of polynomials

$$\{\sum_{i=0}^{n} x^i a_i,\ a_i \in R,\ n \in \mathbb{N}\}$$

with usual addition of polynomials and multiplication subject to the relation $ax = x\sigma(a) + \delta(a)$ for all $a \in R$. We take any $f(x) \in R[x;\sigma,\delta]$ to be of the form $f(x) = \sum_{i=0}^{n} x^i a_i$, $a_i \in R$ as in McConnell and Robson [15]. We denote $R[x;\sigma,\delta]$ by $O(R)$. In case δ is the zero map, we denote $R[x;\sigma]$ by S and in case σ is the identity map, we denote $R[x;\delta]$ by D. The study of Ore-extension $O(R) = R[x;\sigma,\delta]$ and its special cases S and D have been of interest to many authors. For example [6–8, 10, 13, 14, 16].

2-primal rings have been studied in recent years and are being treated by authors for different structures. In [14], Greg Marks discusses the 2-primal property of $R[x;\sigma,\delta]$, where R is a local ring, σ an automorphism of R and δ a σ-derivation of R. In Greg Marks [14], it has been shown that for a local ring R with a nilpotent maximal ideal, the Ore extension $R[x;\sigma,\delta]$ will or will not be 2-primal depending on the δ-stability of the maximal ideal of R. In the case where $R[x;\sigma,\delta]$ is 2-primal, it will satisfy an even stronger condition; in the case where $R[x;\sigma,\delta]$ is not 2-primal, it will fail to satisfy an even weaker condition.

Minimal prime ideals of 2-primal rings have been discussed by Kim and Kwak in [11]. 2-primal near rings have been discussed by Argac and Groenewald in [1]. Recall that a ring R is 2-primal if and only if $N(R) = P(R)$, i.e. if the prime radical is a completely semiprime ideal. An ideal I of a ring R is called completely semiprime if $a^2 \in I$ implies $a \in I$ for $a \in R$. We also note that a reduced ring is 2-primal and a commutative ring is also 2-primal. For further details on 2-primal rings, we refer the reader to [1–3, 11, 14].

Before proving the main result, we find a relation between the minimal prime ideals of R and those of the Ore extension $R[x;\sigma,\delta]$, where R is a Noetherian $\mathbb{Q}$-algebra, σ an automorphism of R and δ a σ-derivation of R such that $\delta(\sigma(a)) = \sigma(\delta(a))$ for all $a \in R$. This is proved in Theorem 3.

$\sigma(*)$-rings: Let R be a ring and σ an endomorphism of R. Then σ is said to be a rigid endomorphism if $a\sigma(a) = 0$ implies that $a = 0$, for $a \in R$, and R is said to be a σ-rigid ring (Krempa [12]).

For example let $R = \mathbb{C}$, and $\sigma : \mathbb{C} \to \mathbb{C}$ be the map defined by $\sigma(a + ib) = a - ib$, a, $b \in \mathbb{R}$. Then it can be seen that σ is a rigid endomorphism of R.

In Theorem 3.3 of [12], Krempa has proved the following:
Let R be a ring, let σ be an endomorphism and δ a σ-derivation of R. If σ is a monomorphism, then the skew polynomial ring $R[x;\sigma,\delta]$ is reduced if and only if R is reduced and σ is rigid. Under this conditions any minimal prime ideal (annihilator) of $R[x;\sigma;\delta]$ is of the form $P[x;\sigma;\delta]$ where P is a minimal prime ideal (annihilator) in R.

In [13], Kwak defines a $\sigma(*)$-ring R to be a ring in which $a\sigma(a) \in P(R)$ implies $a \in P(R)$ for $a \in R$.

Example 1. Let R $= \begin{pmatrix} F & F \\ 0 & F \end{pmatrix}$, where F is a field. Then $P(R) = \begin{pmatrix} 0 & F \\ 0 & 0 \end{pmatrix}$ Let $\sigma : R \to R$ be defined by $\sigma\Big(\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}\Big) = \begin{pmatrix} a & 0 \\ 0 & c \end{pmatrix}$. Then it can be seen that σ is an endomorphism of R and R is a $\sigma(*)$-ring.

We note that the above ring is not σ-rigid. For let $0 \neq a \in F$. Then
$\begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix} \sigma \begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, but $\begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix} \neq \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

Kwak in [13] also establishes a relation between a 2-primal ring and a $\sigma(*)$-ring. The property is also extended to the skew-polynomial ring $R[x;\sigma]$. It has been proved in Theorem 5 of [13] that if R is a 2-primal ring and σ is an automorphism of R, then R is a $\sigma(*)$-ring if and only if $\sigma(P) = P$ for all $P \in Min.Spec(R)$. In Theorem 12 of [13] it has been proved that if R is a $\sigma(*)$-ring with $\sigma(P(R)) = P(R)$, then $R[x;\sigma]$ is 2-primal if and only if $P(R)[x;\sigma] = P(R[x;\sigma])$.

It is known that if R is a 2-primal Noetherian $\mathbb{Q}$-algebra, and δ is a derivation of R, then $R[x;\delta]$ is 2-primal Noetherian. (Theorem 2.4 of Bhat [3]).

Let now R be a ring, σ an automorphism of R and δ a σ-derivation of R. Recall from Bhat [2] that R is said to be a δ-ring if $a\delta(a) \in P(R)$ implies $a \in P(R)$ for $a \in R$. It is known that if R is a δ-Noetherian $\mathbb{Q}$-algebra, σ an automorphism of R and δ a σ-derivation of R such that $\delta(\sigma(a)) = \sigma(\delta(a))$ for all $a \in R$, $\sigma(P) = P$ for all $P \in Min.Spec(R)$ and $\delta(P(R)) \subseteq P(R)$, then $R[x;\sigma,\delta]$ is 2-primal Noetherian (Theorem 2.4 of Bhat [2]).

In a sense we generalize the above results of Bhat [2, 3] when σ is an automorphism of R and ultimately investigate the 2-primal property of $R[x;\sigma,\delta]$ when R is a $\sigma(*)$-Noetherian $\mathbb{Q}$-algebra and prove the following, even without the hypothesis of R being a δ-ring:

Let R be a Noetherian $\sigma(*)$-ring, which is also an algebra over $\mathbb{Q}$. Further $P \in Min.Spec(O(R))$ imply that $P \cap R \in Min.Spec(R)$. Then $R[x;\sigma,\delta]$ is 2-primal Noetherian, where $\delta(\sigma(a)) = \sigma(\delta(a))$ for all $a \in R$.

This result is proved in Theorem 5. We note that for a Noetherian $\sigma(*)$-ring, $\sigma(P) = P$ for all $P \in Min.Spec(R)$ (Theorem 2), and this is crucial in proving Theorem 4 and, therefore, the main result (Theorem 5).

We generalize Theorem 7 of [5] which states the following:

Theorem 7 of [5]. Let R be a Noetherian ring, which is also an algebra over $\mathbb{Q}$. Let σ be an automorphism of R such that R is a $\sigma(*)$-ring and δ be a σ-derivation of R such that R is a δ-ring and $\delta(\sigma(a)) = \sigma(\delta(a))$ for all $a \in R$. Further let $P \in Min.Spec(O(R))$ imply that $P \cap R \in Min.Spec(R)$. Then $R[x;\sigma,\delta]$ is 2-primal Noetherian.

2 Ore extensions

Recall that an ideal I of a ring R is called σ-invariant if $\sigma(I) = I$. Also I is called completely prime if $ab \in I$ implies $a \in I$ or $b \in I$ for $a, b \in R$. ([13])

In commutative case completely prime and prime have the same meaning. We also note that every completely prime ideal of a ring R is a prime ideal, but the converse need not be true.

The following example shows that a prime ideal need not be a completely prime ideal.

Example 2. Let $R = \begin{pmatrix} \mathbb{Z} & \mathbb{Z} \\ \mathbb{Z} & \mathbb{Z} \end{pmatrix} = M_2(\mathbb{Z})$. If p is a prime number, then the ideal $P = M_2(p\mathbb{Z})$ is a prime ideal of R, but is not completely prime, since for $a = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $b = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, we have $ab \in P$, even though $a \notin P$ and $b \notin P$.

We also recall that an ideal J of a ring is called a σ-prime ideal of R if J is σ-invariant and for any σ-invariant ideals K and L with $KL \subseteq J$, we have $K \subseteq J$ or $L \subseteq J$.

We also note that if R is a Noetherian ring, then $Min.Spec(R)$ is finite (Theorem 2.4 of Goodearl and Warfield [10]) and for any automorphism σ of R and for any $U \in Min.Spec(R)$, we have $\sigma^i(U) \in Min.Spec(R)$ for all $i \in \mathbb{N}$, therefore, it follows that there exists some $m \in N$ such that $\sigma^m(U) = U$ for all $U \in Min.Spec(R)$. As mentioned earlier we denote $\cap_{i=0}^{m} \sigma^i(U)$ by U^0.

We now prove the following Theorem. This Theorem has not been used to prove the main Theorem, but gives an idea to find a relation between $Min.Spec(R)$ and $Min.Spec(O(R))$ (namely Theorem 3) which is crucial in proving the main result (Theorem 5):

Theorem 1. *Let R be a Noetherian ring and σ an automorphism of R. Let $S = R[x;\sigma]$ be as usual. Then:*

1. *If $P \in Min.Spec(S)$, then $P = (P \cap R)S$ and there exists $U \in Min.Spec(R)$ such that $P \cap R = U^0$.*
2. *If $U \in Min.Spec(R)$, then $U^0 S \in Min.Spec(S)$.*

Proof. See Theorem 2.4 of Bhat [6]. □

Proposition 1. *Let R be a ring and σ an automorphism of R. Then R is a $\sigma(*)$-ring implies R is 2-primal.*

Proof. Let $a \in R$ be such that $a^2 \in P(R)$. Then $a\sigma(a)\sigma(a\sigma(a)) = a\sigma(a)\sigma(a)\sigma^2(a) \in \sigma(P(R)) = P(R)$. Therefore $a\sigma(a) \in P(R)$ and hence $a \in P(R)$. □

A necessary and sufficient condition for a Noetherian ring to be a $\sigma(*)$-ring is given by Bhat in Theorem 2.4 of [4]:

Theorem 2. *Let R be a Noetherian ring. Then R is a $\sigma(*)$-ring if and only if for each minimal prime U of R, $\sigma(U) = U$ and U is completely prime ideal of R.*

Proof. Theorem 2.4 of [4]. □

We now give a relation between the minimal prime ideals of R and those of $R[x;\sigma,\delta]$, where R is a Noetherian $\mathbb{Q}$-algebra, σ an automorphism of R and δ a σ-derivation of R. This is proved in Theorem 3. Towards this we have the following:

Proposition 2. *Let R be a Noetherian $\mathbb{Q}$-algebra, σ an automorphism and δ a σ-derivation of R such that $\delta(\sigma(a)) = \sigma(\delta(a))$ for all $a \in R$. Then $e^{t\delta}$ is an automorphism of $T = R[[t,\sigma]]$, the skew power series ring.*

Proof. The proof is on the same lines as in Seidenberg [16] and in the non-commutative case on the same lines as provided by Blair and Small in [8]. □

Henceforth we denote $R[[t,\sigma]]$ by T. Let I be an ideal of R such that $\sigma(I) = I$. Then it is easy to see that $TI \subseteq IT$ and $IT \subseteq TI$. Hence $TI = IT$ is an ideal of T.

Lemma 1. *Let R be a Noetherian $\mathbb{Q}$-algebra, σ an automorphism and δ a σ-derivation of R such that $\delta(\sigma(a)) = \sigma(\delta(a))$ for all $a \in R$. Let I be an ideal of R such that $\sigma(I) = I$. Then I is δ-invariant if and only if IT is $e^{t\delta}$-invariant.*

Proof. Let IT be $e^{t\delta}$-invariant. Let $a \in I$. Then $a \in IT$. So $e^{t\delta}(a) \in IT$; i.e. $a + t\delta(a) + (t^2\delta^2/2!)(a) + ... \in IT$. Therefore $\delta(a) \in I$.

Conversely suppose that $\delta(I) \subseteq I$ and let $f = \sum t^i a_i \in IT$. Then $e^{t\delta}(f) = f + t\delta(f) + (t^2\delta^2/2!)(f) + ... \in IT$, as $\delta(a_i) \in I$. Therefore $e^{t\delta}(IT) \subseteq IT$. Replacing $e^{t\delta}$ by $e^{-t\delta}$, we get that $e^{t\delta}(IT) = IT$. □

Assumption A: Henceforth we assume that R is a ring and T as usual such that for any $U \in Min.Spec(R)$ with $\sigma(U) = U$, $UT \in Min.Spec(T)$.

Proposition 3. *Let R be a Noetherian $\mathbb{Q}$-algebra. Let σ be an automorphism of R and δ be a σ-derivation of R such that $\delta(\sigma(a)) = \sigma(\delta(a))$ for all $a \in R$. Then $P \in Min.Spec(R)$ with $\sigma(P) = P$ implies $\delta(P) \subseteq P$.*

Proof. Let T be as usual. Now by Proposition (2) $e^{t\delta}$ is an automorphism of T. Let $P \in Min.Spec(R))$. Then by assumption $PT \in Min.Spec(T)$. Therefore there exists an integer $n \geq 1$ such that $(e^{t\delta})^n(PT) = PT$, i.e. $e^{nt\delta}(PT) = PT$. But R is a $\mathbb{Q}$-algebra, therefore, $e^{t\delta}(PT) = PT$ and now Lemma 1 implies $\delta(P) \subseteq P$. □

Proposition 4. *Let R be a $\sigma(*)$-ring, which is also an algebra over $\mathbb{Q}$ and $U \in Min.Spec(R)$. Then $U(O(R)) = U[x;\sigma,\delta]$ is a completely prime ideal of $O(R) = R[x;\sigma,\delta]$, where δ is a σ-derivation of R such that $\delta(\sigma(a)) = \sigma(\delta(a))$ for all $a \in R$.*

Proof. Let $U \in Min.Spec(R)$. Then $\sigma(U) = U$ by Theorem 2, and $\delta(U) \subseteq U$ by Proposition 3). Now R is 2-primal by Proposition 1 and furthermore U is completely prime by Theorem 2. Now we note that σ can be extended to an automorphism $\overline{\sigma}$ of R/U and δ can be extended to a $\overline{\sigma}$-derivation $\overline{\delta}$ of R/U. Now it is well known that $O(R)/U(O(R)) \simeq (R/U)[x;\overline{\sigma},\overline{\delta}]$ and hence $U(O(R))$ is a completely prime ideal of $O(R)$. □

Theorem 3. *Let R be a Noetherian $\mathbb{Q}$-algebra. Consider $O(R)$ as usual such that R is a $\sigma(*)$-ring and $\delta(\sigma(a)) = \sigma(\delta(a))$ for all $a \in R$. Then $P_1 \in Min.Spec(R)$ with $\sigma(P_1) = P_1$ implies that $O(P_1) \in Min.Spec(O(R)$.*

Proof. Let $P_1 \in Min.Spec(R)$. Now by Theorem 2 $\sigma(P_1) = P_1$, and by Proposition 3 $\delta(P_1) \subseteq P_1$. Now Proposition (3.3) of [9] implies that $O(P_1) \in Spec(O(R))$. Suppose $O(P_1) \notin Min.Spec(O(R))$ and $P_2 \subset O(P_1)$ be a minimal prime ideal of $O(R)$. Then $P_2 = O(P_2 \cap R) \subset O(P_1) \subseteq Min.Spec(O(R))$. Therefore $(P_2 \cap R) \subset P_1$ which is a contradiction, as $(P_2 \cap R) \in Spec(R)$. Hence $O(P_1) \in Min.Spec(O(R))$. □

We now prove the following Theorem, which is crucial in proving Theorem 5.

Theorem 4. *Let R be a Noetherian $\sigma(*)$-ring, which is also an algebra over $\mathbb{Q}$, σ an automorphism of R and δ a σ-derivation of R such that $\delta(\sigma(a)) = \sigma(\delta(a))$ for all $a \in R$. Then $R[x;\sigma,\delta]$ is 2-primal if and only if $P(R)[x;\sigma,\delta] = P(R[x;\sigma,\delta])$.*

Proof. Let $R[x;\sigma,\delta]$ be 2-primal. Now by Proposition 4 $P(R[x;\sigma,\delta]) \subseteq P(R)[x;\sigma,\delta]$. Let

$$f(x) = \sum_{j=0}^{n} x^j a_j \in P(R)[x;\sigma,\delta].$$

Now R is a 2-primal subring of $R[x;\sigma,\delta]$ by Proposition 1, which implies that a_j is nilpotent and thus

$$a_j \in N(R[x;\sigma,\delta]) = P(R[x;\sigma,\delta]).$$

So we have $x^j a_j \in P(R[x;\sigma,\delta])$ for each j, $0 \leq j \leq n$, which implies that $f(x) \in P(R[x;\sigma,\delta])$. Hence $P(R)[x;\sigma,\delta] = P(R[x;\sigma,\delta])$.

Conversely suppose that $P(R)[x;\sigma,\delta] = P(R[x;\sigma,\delta])$. We will show that $R[x;\sigma,\delta]$ is 2-primal. Let

$$g(x) = \sum_{i=0}^{n} x^i b_i \in R[x;\sigma,\delta],\ b_n \neq 0$$

be such that

$$(g(x))^2 \in P(R[x;\sigma,\delta]) = P(R)[x;\sigma,\delta].$$

We will show that $g(x) \in P(R[x;\sigma,\delta])$. Now leading coefficient $\sigma^{2n-1}(b_n)b_n \in P(R) \subseteq P$, for all $P \in Min.Spec(R)$. Also $\sigma(P) = P$ and P is completely prime by Theorem 2. Therefore we have $b_n \in P$, for all $P \in Min.Spec(R)$, i.e. $b_n \in P(R)$. Now $\delta(P) \subseteq P$ for all $P \in Min.Spec(R)$ by Proposition 3, we get

$$(\sum_{i=0}^{n-1} x^i b_i)^2 \in P(R[x;\sigma,\delta]) = P(R)[x;\sigma,\delta]$$

and as above we get $b_{n-1} \in P(R)$. With the same process in a finite number of steps we get $b_i \in P(R)$ for all i, $0 \leq i \leq n$. Thus we have $g(x) \in P(R)[x;\sigma,\delta]$, i.e. $g(x) \in P(R[x;\sigma,\delta])$. Therefore, $P(R[x;\sigma,\delta])$ is completely semiprime. Hence $R[x;\sigma,\delta]$ is 2-primal. □

Theorem 5. *Let R be a Noetherian, which is also an algebra over $\mathbb{Q}$. Let σ be an automorphism of R such that R is a $\sigma(*)$-ring and δ a σ-derivation of R such that $\delta(\sigma(a)) = \sigma(\delta(a))$ for all $a \in R$. Further let $P \in Min.Spec(O(R))$ imply that $P \cap R \in Min.Spec(R)$. Then $O(R) = R[x;\sigma,\delta]$ is 2-primal Noetherian.*

Proof. $R[x;\sigma,\delta]$ is Noetherian by Hilbert Basis Theorem (Theorem 1.12 of Goodearl and Warfield [10]). We now use Theorem 3 to get that $P(R)[x;\sigma,\delta] = P(R[x;\sigma,\delta])$, and the result now follows from Theorem 4. □

We note that the hypothesis that R is a $\sigma(*)$-ring can not be deleted as can be seen below:

Example 3. Let $R = K \oplus K$, where K is a field. Then the Ore extension $O(R) = R[x;\sigma,0]$, where σ is an automorphism of R defined by $\sigma((a;b)) = (b;a)$, is a prime ring. Thus $P = 0$ is a minimal prime of $O(R)$. But $P \cap R = 0$ is not a prime ideal of R.

The following example shows that if R is a Noetherian ring, then $R[x;\sigma,\delta]$ need not be 2-primal.

Example 4. Let $R = \mathbb{Q} \oplus \mathbb{Q}$ with $\sigma(a,b) = (b,a)$. Then the only σ-invariant ideals of R are $\{0\}$ and R, and so R is σ-prime. Let $\delta : R \to R$ be defined by $\delta(r) = ra - a\sigma(r)$, where $a = (0,\alpha) \in R$. Then δ is a σ-derivation of R and $R[x;\sigma,\delta]$ is prime and $P(R[x;\sigma,\delta]) = 0$. But $(x(1,0))^2 = 0$ as $\delta(1,0) = -(0,\alpha)$. Therefore $R[x;\sigma,\delta]$ is not 2-primal. If δ is taken to be the zero map, then even $R[x;\sigma]$ is not 2-primal.

The following example shows that if R is a Noetherian ring , then even $R[x]$ need not be 2-primal.

Example 5. Let $R = M_2(\mathbb{Q})$, the set of 2×2 matrices over $\mathbb{Q}$. Then $R[x]$ is a prime ring with non-zero nilpotent elements, and so can not be 2-primal.

From these examples we conclude that if R is a Noetherian ring, then even $R[x]$ need not be 2-primal. But it is known that if R is a 2-primal Noetherian $\mathbb{Q}$-algebra and δ is a derivation of R, then $R[x;\delta]$ is 2-primal Noetherian (Theorem 2.4 of Bhat [3]), and therefore, we have the following question:

Question: If R is a 2-primal ring, is $R[x;\sigma,\delta]$ 2-primal (even if R is commutative or the special case when R is Noetherian)?

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