BULETINUL ACADEMIEI DE ŞTIINŢE
A REPUBLICII MOLDOVA. MATEMATICA
Number 1(65), 2011, Pages 33–41
ISSN 1024–7696

On the structure of maximal non-finitely generated ideals of ring and Cohen's theorem

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Abstract. In this paper we consider analogues of Cohen's theorem. We introduce new notions of almost prime left (right) submodule and dr-prime left (right) ideal, this allows us to extend Cohen's theorem for modular and non-commutative analogues. We prove that if every almost prime submodule of a finitely generated module is a finitely generated submodule, then any submodule of this module is finitely generated.

Mathematics subject classification: 16D25, 16D80.
Keywords and phrases: Almost prime ideal, completely prime ideal, dr-prime ideal, duo-element, finite element, finitely generated module, maximal non-finitely generated ideal.

1 Introduction

The aim of this paper is to generalize Cohen's theorem for wider class of rings. In 1950 studying the structure of a commutative ring I. Cohen showed that if an arbitrary prime ideal in a commutative ring with $1 \neq 0$ is finitely generated (principal), then any ideal in R is finitely generated (principal) [1]. This theorem has a rich history. In particular, R. Chandran proved it for duo-ring [2]. G. Mihler showed that if an arbitrary left (right) prime ideal is finitely generated in an associative ring, then an arbitrary left (right) ideal in a ring is finitely generated [3]. Another non-commutative analogue of Cohen's theorem was proved by B. Zabavsky [6] using a weakly prime left (right) ideal. Also in $[4-5]$ some attempts were made to extend this theorem for module, but with some restrictions on this module.

In this paper we prove analogues of Cohen's theorem for module over arbitrary associative ring with $1 \neq 0$, for this we introduce a new notion of almost prime left (right) submodule. So if every almost prime submodule of a finitely generated module is a finitely generated, then any submodule of the module is finitely generated. Notice that in a commutative ring and duo-ring the notions of almost prime submodule and prime submodule coincide.

In the next section we consider the commutative ring with $1 \neq 0$ which is not a noetherian ring, the notion of a maximal non-finitely generated ideal and a finite element are investigated here. Also some important corollaries are considered in this section.

The last section deals with non-commutative analogue of Cohen's theorem. A new notion of dr-prime left (right) ideal is introduced. Thus if any dr-prime left (right) ideal of a ring R is principal, then any left (right) ideal in R is principal.

2 Preliminaries

Let M be a finitely generated left module considered over an associative ring R with $1 \neq 0$. Suppose that there is at least one submodule which is not finitely generated, we call it a non-finitely generated submodule. We denote by S the set of all non-finitely generated submodules and by $\{N_i\}_{i\in\Lambda}$ any chain of submodules of a module M which belong to the set S, moreover $N = \bigcup\limits_{i\in\Lambda} N_i$.

We show that $N \in S$. Suppose that $N \notin S$, then there exist elements $n_1, n_2, \ldots n_k \in N$ such that

$$N = Rn_1 + Rn_2 + \ldots + Rn_k.$$

Since $N = \bigcup\limits_{i\in\Lambda} N_i$ for every n_j, $j = 1, 2, \ldots, k$, there exists s such that $n_j \in N_{i_s}$. Since $\{N_i\}_{i\in\Lambda}$ is a chain of submodules, there exists t such that $n_1, n_2, \ldots, n_k \in N_t$. Then $Rn_1 + Rn_2 + \ldots + Rn_k \subset N_t$. Since

$$N_t \subset N = \bigcup_{i\in\Lambda} N_i = Rn_1 + Rn_2 + \ldots + Rn_k,$$

this is a contradiction to $N_t \in S$. This contradiction shows that $N \in S$, therefore the set S is inductive with respect to the order of submodules inclusion as a set.

According to Zorn's lemma there exists at least one maximal element in the set S. Therefore we have a submodule which is contained in the maximal element of S, we call it the maximal non-finitely generated submodule of the module M.

Definition 1. An element a of a ring R is called a duo-element if $aR = Ra$.

Definition 2. A left ideal P of a ring R is called an almost prime left ideal if from the condition $ab \in P$, where a is a duo-element of the ring R it follows that either $a \in P$ or $b \in P$.

Definition 3. A submodule N of a module M is called an almost prime left submodule if

$$(N : M) = \{r | r \in R, rM \subset N\}$$

is an almost prime left ideal of a ring R.

Proposition 1. *Any maximal left ideal of a ring R is an almost prime left ideal.*

Proof. Let M be an arbitrary maximal ideal of a ring R and let M be not almost prime. Then there exist elements $a \in R\backslash M$, $b \in R\backslash M$, where a is a duo-element such that $ab \in M$. If M is maximal then we have $M + bR = R$ and hence there exist elements $m \in M$, $r \in R$ such that $m + br = 1$. Hence $am + abr = a \in M$. But this is a contradiction with the choice of the element a. $\square$

Remark that any maximal submodule of a module is an almost prime submodule [5]. We consider just a finitely generated submodule, so it is obvious that maximal submodules exist in it. It is easy to see that in module under consideration there always exist almost prime submodules.

3 Analogue of Cohen's theorem for modules

Suppose that for a module M there exists at least one non-finitely generated submodule. Then according to what was proved above there exists a maximal non-finitely generated submodule. If there exists a maximal non-finitely generated submodule, then there exists an almost prime submodule.

Theorem 1. *Every maximal non-finitely generated submodule of a finitely generated module over a ring is an almost prime submodule.*

Proof. Let M be a finitely generated module, N be a maximal non-finitely generated submodule, $N \subset M$. According to restrictions on a ring R, N could not be an almost prime ideal, that is there exist elements $a, b \in R$, where a is a duo-element such that $abN \subset M$, but $a \notin N$ and $b \notin N$. Then we assume that $N + aM = \sum\limits_{i=1}^{\alpha} Rx_i$ is a finitely generated submodule of the module M. Notice that $N + bM \subseteq (N : a)$, where $(N : a) = \{m | am \in N\}$. Thus $(N : a)$ is a finitely generated submodule of a module M, and let $(N : a) = \sum\limits_{j=1}^{\beta} Ry_j$. Since $N \subset N + aM = \sum\limits_{i=1}^{\alpha} Rx_i$, for any $n \in N$ there exist elements $r_i \in R$, $i = 1, 2, \ldots \alpha$, such that $n = r_1x_1 + \ldots + r_\alpha x_\alpha$. As $N + aM = \sum\limits_{i=1}^{\alpha} Rx_i$, there exist $n_i^0 \in N$ and $s_i \in M$, where $i = 1, 2, \ldots \alpha$, such that $x_i = n_i^0 + as_i$. We show that

$$n = r_1n_1^0 + \ldots + r_\alpha n_\alpha^0 + r_1as_1 + \ldots + r_\alpha as_\alpha.$$

As a is a duo-element, for every $r_i \in R$, $i = 1, 2, \ldots \alpha$, there exists $r_i' \in R$, $i = 1, 2, \ldots \alpha$, such that $r_ia = ar_i'$. Hence

$$n = r_1n_1^0 + \ldots + r_\alpha n_\alpha^0 + a(r_1's_1 + \ldots + r_\alpha's_1 + \ldots + r_\alpha's_\alpha).$$

Thus

$$n - r_1n_1^0 - \ldots - r_\alpha n_\alpha^0 = a(r_1's_1 + \ldots + r_\alpha's_\alpha) \in N.$$

If $(N : a) = \sum\limits_{j=1}^{\beta} Ry_j$, we obtain $r_1's_1 + \ldots + r_\alpha's_\alpha = t_1y_1 + \ldots + t_\beta y_\beta$, for some $t_1, \ldots t_\beta \in R$ such that $at_1 = t_1'a, \ldots at_\beta = t_\beta'a$, and then

$$n = r_1n_1^0 + \ldots + r_\alpha n_\alpha^0 + t_1ay_1 + \ldots + t_\beta ay_\beta.$$

Since n is an arbitrary element, we proved that

$$N \subseteq Rn_1^0 + \ldots + Rn_\alpha^0 + Ray_1 + \ldots + Ray_\beta.$$

If $y_i \in (N : a)$, $i = 1, 2, \ldots, \beta$, then $ay_1 \in N, \ldots, ay_\beta \in N$, and if $n_1^0, \ldots, n_\alpha^0 \in N$, then $Rn_1^0 + \ldots + Rn_\alpha^0 + Ray_1 + \ldots + Ray_\beta \subset N$. Thus

$$N = Rn_1^0 + \ldots + Rn_\alpha^0 + Ray_1 + \ldots + Ray_\beta$$

is a finitely generated submodule N, but this is a contradiction with $N \in S$. Thus N is an almost prime submodule. $\square$

Remind that if R is a commutative ring or duo-ring, then the notion of almost prime submodule coincides with the notion of a prime submodule [4-5].

Also from Theorem 1, as a consequence, we obtain the modular analogue of Cohen's theorem. This theorem is the main result of the section.

Theorem 2 (**Modular analogue of Cohen's theorem**). *If every almost prime submodule of a finitely generated module is a finitely generated submodule, then any submodule of this module is finitely generated.*

Proof. Let M be a finitely generated module, and all almost prime submodules of the module M are finitely generated. If M does not contain non-finitely generated submodules, then everything is clear. In another case, for the module M there exists at least one non-finitely generated submodule. Then, according to what was proved above for M there exists at least one maximal non-finitely generated submodule. According to Theorem 1, N is an almost prime submodule. But all almost prime submodules of the module M are finitely generated, that is N is finitely generated as an almost prime submodule and N is not finitely generated as a maximal non-finitely generated submodule at the same time. But this is not possible, therefore M does not contain non-finitely generated submodules. □

4 Maximal non-finitely generated ideals of commutative ring

Let R be a commutative ring with $1 \neq 0$. Assume that R is not a noetherian ring, that is there exist non-finitely generated ideals in R. Consider a ring R as a module over itself, that is ${}_RR$.

Definition 4. An ideal I in R which is maximal in a set of non-finitely generated ideals is called maximal non-finitely generated ideal in R.

We can say that there exists at least one maximal non-finitely generated ideal in R. Moreover, using the theorem for the module ${}_RR$ we obtain that all maximal non-finitely generated ideals are prime ideals. Thus, the following theorem takes place.

Theorem 3 (see [3]). *Let R be a commutative ring which is not noetherian, then any maximal non-finitely generated ideal of the ring R is a prime ideal.*

Hence, as an obvious corollary we obtain the known Cohen's theorem.

Theorem 4 (see [1]). *If all prime ideals of a commutative ring R are finitely generated, then R is a noetherian ring.*

Consider the case when R is a commutative ring but is not a noetherian ring. According to above there exists at least one maximal non-finitely generated ideal in R. Denote by $N(R)$ the intersection of all maximal non-finitely generated ideals of the ring R.

Definition 5. We say that a nonzero element a of a ring R is a finite element if any chain of ideals which contain the element a is finite. That is for any chain of ideals $I_1 \subset I_2 \subset ...$ such that $a \in I_1$, there exists a number n for which $I_n = I_{n+1} = ...$

All invertible elements and all factorial elements are examples of the finite element a [6]. Thus we obtain the following corollary.

Corollary 1. *Let R be a commutative ring and a be an arbitrary element of the ring R. Then the following statements are equivalent:*

1) a is a finite element ;

2) any ideal which contains the element a is finitely generated;

Proof. 1) $\Longrightarrow$ 2). Let I be any ideal of a ring R which contains the element a. If $aR = I$, everything is clear, but otherwise there exists an element $i_1 \in I$ such that $i_1 \notin aR$. Consider the ideal $aR + i_1R$, it is obvious that $aR \subset aR + i_1R$. If $aR + i_1R \neq I$, then there exists an element $i_2 \in I$ such that $i_2 \notin aR + i_1R$. Consider the ideal $aR + i_1R + i_2R$, it is obvious, that $aR \subset aR + i_1R \subset aR + i_1R + i_2R$. This inclusion can be continued, but taking into account the definition of the element a, this chain can not be infinite. This means that there exist elements $i_1, i_2, \ldots, i_n \in I$ such that

$$aR + i_1R + \ldots + i_nR = I,$$

that is I is a finitely generated ideal.

2) $\Longrightarrow$ 1). Conversely, show that if any ideal which contains the element a is finitely generated, then a is a finite element of the ring R.

Let $\{I_\alpha\}_{\alpha\in\Lambda}$ be any chain of ideals, and all ideals of such type of this chain contain the element a. Show that this chain is finite. Let $I = \bigcup\limits_{\alpha\in\Lambda} I_\alpha$. Obviously, $a \in I$. As we assumed, I is a finitely generated ideal, that is there exist elements $i_1, \ldots, i_k \in I$ such that $I = i_1R + \ldots + i_kR$. Since $I = \bigcup\limits_{\alpha\in\Lambda} I_\alpha$, there exist numbers $\alpha_1, \ldots, \alpha_k$ such that $i_1 \in I_{\alpha_1}, \ldots, i_k \in I_{\alpha_k}$. If $\{I_\alpha\}_{\alpha\in\Lambda}$ is a chain of ideals, there exists a number t such that $i_1, \ldots, i_k \in I_{\alpha_t}$, that is $i_1R + \ldots + i_kR \subset I_{\alpha_t}$. As $\bigcup\limits_{\alpha\in\Lambda} I_\alpha = i_1R + \ldots + i_kR$, then $i_1R + \ldots + i_kR = I_{\alpha_t}$, that is the chain $\{I_\alpha\}_{\alpha\in\Lambda}$ is finite. $\square$

Corollary 2. *Suppose that an element a does not belong to any maximal non-finitely generated ideal of a ring R. Then a is a finite element in R.*

Proof. Use Corollary 1 and the fact that the element a is not contained in any non-finitely generated ideal. Thus, as it is proved above, the element a is contained in at least one maximal non-finitely generated ideal. $\square$

Corollary 3. *Let n be an arbitrary element from $N(R)$. Then for any finite element $a \in R$ and any element $x \in R$, the element $a + nx$ is finite.*

Proof. We will prove it by contradiction. Let the element $a + nx$ be not finite, then it belongs to some maximal non-finitely generated ideal N of the ring R. Since n is an element from $N(R)$, we see that $n \in N$ and x is an arbitrary element of the ring R, then $nx \in N$. From the definition of ideal we obtain $(a + nx) - nx \in N$, whence $a \in N$. However, the element a is finite and as proved above, the element a belongs to a maximal non-finitely generated ideal, which is impossible, according to Corollary 1. We obtain a contradiction. □

Corollary 4. *Let R be a commutative ring with only one maximal non-finitely generated ideal $N = N(R)$. Then the following statements hold:*

a) all non-finite elements from R form an ideal which coincides with N;

b) an arbitrary divisor of a finite element is a finite element of the ring R;

c) for an arbitrary non-finite element n and any finite element a, we obtain that $a + n$ is a finite element.

Proof. a) From Corollary 1 it is known that any finite element does not belong to N and every element which does not belong to N is finite. Then all non-finite elements form the ideal which coincides with N.

b) Let a be a finite element of the ring R such that $a = bc$, $b \notin U(R)$ and $c \notin U(R)$, where $U(R)$ is the group of units of the ring R. If b is not finite, then $b \in N$. Hence we see that $bc = a \in N$, but this is impossible, because the element a is finite. Corollary 1 completes the proof.

c) If $n \in N$ and a is a finite element, then obviously $a + n$ is not contained in N (because there are only finite elements in N). Thus, $a + n$ is a finite element. □

5 Analogue of Cohen's theorem for principal ideals of noncommutative ring

In this section, we denote by R an associative ring with $1 \neq 0$.

Definition 6. A left (right) ideal in R which is maximal in the set of non-finitely generated left(fight) ideals is called maximal non-finitely generated left (right) ideal in the ring R.

Definition 7. A left (right) ideal in R which is maximal in the set of non-principal left (right) ideals is called maximal non-principal left (right) ideal in the ring R.

Corollary 5. *Any left (right) non-finitely generated ideal of ring R is contained in at least one maximal non-finitely generated left (right) ideal.*

Proof. Let I be an arbitrary non-finitely generated left ideal of a ring R. Denote by S the set of all non-finitely generated left ideals of the ring R which contain the ideal I. We show that the set S is inductive with respect to the order of ideals inclusion. If $\{I_\alpha\}_{\alpha\in\Lambda}$ is any chain of left ideals from the set S, denote $J = \bigcup\limits_{\alpha\in\Lambda} I_\alpha$. It is obvious that J is an ideal of the ring R. Moreover, $J \in S$. Indeed, according to the definition of a left ideal, $I \in S$. If $J \notin S$, then there exist elements $j_1, \dots, j_k \in J$

such that $J = Rj_1 + \ldots + Rj_k$, and there exist elements $\alpha_1, \ldots, \alpha_k \in \Lambda$ such that $j_1 \in I_{\alpha_1}, \ldots, j_k \in I_{\alpha_k}$. Since $\{I_\alpha\}_{\alpha\in\Lambda}$ is a chain, there exists t such that $j_1, \ldots, j_k \in I_{\alpha_t}$. As $I_{\alpha_t} \subset \bigcup_{\alpha\in\Lambda} I_\alpha$, then $I_{\alpha_t} = Rj_1 + \ldots + Rj_k$, but this is impossible, because $I_{\alpha_t} \in S$. Zorn's lemma completes the proof of the corollary. □

In the same way we can consider the case of a right non-finitely generated ideal.

In the case of a principal left (right) ideals the following corollary takes place.

Corollary 6. *Every left (right) non-principal ideal of a ring R is contained in at least one maximal non-principal left (right) ideal.*

Proof. Using the previous proof of Corollary 5 for any left (right) non-finitely generated ideal, we can prove in the same way for any left (right) non-principal ideal. □

Definition 8. Remind that an ideal P of a ring R is called prime left (right) ideal if the condition $aRb \subseteq P$ implies that either $a \in P$ or $b \in P$.

According to a result of [3] we have the following theorem.

Theorem 5. *Any maximal non-finitely generated left (right) ideal of a ring is a prime left (right) ideal.*

In [6] a noncommutative analogue of Cohen's theorem was proved, using weakly prime ideals.

Definition 9. We say that left (right) ideal P of a rind R is a weakly prime left (right) ideal, if from the condition $(a+P)R(b+P) \subseteq P$ if follows that either $a \in P$ or $b \in P$.

Using a result of the paper [7], the folowing theorem holds.

Theorem 6. *Any maximal non-principal left (right) ideal of a ring R is a weakly prime left (right) ideal.*

Definition 10. Remind that an element of a ring R is called an atom if it is non-inverse and non-zero and cannot be presented as the product of two noninvertible elements [8].

Theorem 7. *Let N be an arbitrary maximal non-principal left ideal for which there exists a duo-element c such that $N \subset Rc$. Then for any $n \in N$, from $n = cx$ it always follows $x \in N$.*

Proof. Consider the set $J = \{x | cx \in N\}$. Since c is a duo-element, J is a left ideal. Obviously $N \subset J$. If there exists an element y such that $cy \in N$, but $y \notin N$, this means that $N \subset J$, but $N \neq J$. Using the definition of left ideal N we see that $J = Rd$. We show that $N = Rcd$. Indeed, since $N \subset Rc = cR$, for any $n \in R$ there exists $t \in R$ such that $n = ct$. Since $t \in J$, we have $t = sd$. Hence $n = csd$. As c is a duo-element, then there exists $s' \in R$ such that $cs = s'c$. Thus $n = s'cd$, that is $N \subset Rcd$. Since $1 \in R$ and $d \in J$, we obtain $cd \in N$. Hence $dcR \subset N$. Thus $N = dcR$. We obtain a contradiction to the choice of the left ideal. □

Definition 11. A left (right) ideal P of an associative ring R with $1 \neq 0$ is a dr-prime left (right) ideal if $P \subset Rc (P \subset cR)$, where c is a duo-element, and for any $p \in P$, the condition $p = yc = cx (p = cx = yc)$ implies $x \in P$ $(y \in P)$.

Proposition 2. *Any maximal left (right) ideal M of a ring R is a dr-prime left (right) ideal.*

Proof. Let M be a maximal left ideal. Since there is only one two-sided ideal in R which contains M, for arbitrary $m \in M$ it always follows $m = 1m$.

A similar proof could be made for a maximal right ideal. □

Theorem 8 (**Non-commutative analogue of Cohen's theorem**). *If any dr-prime left (right) ideal of a ring R is principal, then any left (right) ideal from R is principal.*

Proof. Let R be a ring in which any dr-prime left ideal is principal, but R is not a principal left ideal I. By Corollary 6, I is contained in a maximal non-principal left ideal N. According to Theorem 9, N is a dr-prime left ideal, since any dr-prime left ideal is principal. But this is a contradiction.

□

Definition 12. A two-sided ideal P is called a completely prime ideal if the condition $ab \in P$, where $a, b \in R$, implies either $a \in P$ or $b \in P$ [8].

Notice that in the case of a commutative ring the notion of completely prime ideal coincides with the notion of prime ideal.

Theorem 9. *If a maximal non-finitely generated left (right) ideal of a ring R is two-sided, then it is a completely prime ideal.*

Proof. Let N be a maximal non-finitely generated left ideal of a ring R which is two-sided. If R/N is not a ring without zero divisors, then there exist elements $a \notin N$ and $b \notin N$ such that $ab \in N$ in R. Thus, the left ideal $J = \{x | x \in R, xb \in N\}$ contains the ideal N and the element a. Hence, the inclusion $N \subset J$ is strict, and according to the restriction on N, the left ideal J is finitely generated. Let $J = Rc_1 + \ldots + Rc_n$. Since $b \notin N$, according to the definition of the maximal non-finitely generated left ideal N, we obtain

$$N + Rb = Rd_1 + \ldots + Rd_k$$

for some elements $d_1, \ldots, d_k \in R$. Hence $d_i = n_i + r_i b$, where $n_i \in N$, $r_i \in R$, $i = 1, 2, \ldots, k$. As $N \subset Rd_1 + \ldots + Rd_k$, then any element $m \in N$ can be represented in the following form

$$m = s_1 d_1 + \ldots + s_k d_k,$$

where $s_1, \ldots, s_k \in R$.

Using what is written above, we obtain

$$m = s_1 d_1 + \ldots + s_k d_k = s_1(n_1 + r_1 b) + \ldots + s_k(n_k + r_k b) =$$

$$= s_1 n_1 + \ldots + s_k n_k + s_1 r_1 b + \ldots + s_k r_k b.$$

Since $m \in N$ and $n_1, \ldots, n_k \in N$, we have

$$m - s_1 n_1 - \ldots - s_k n_k = (s_1 r_1 + \ldots + s_k r_k) b \in N,$$

then according to the definition of left ideal J we have $s_1 r_1 + \ldots + s_k r_k \in J$, that is there exist elements $t_1, \ldots, t_n \in R$ such that $s_1 r_1 + \ldots + s_k r_k = t_1 c_1 + \ldots + t_n c_n$, because $J = Rc_1 + \ldots + Rc_n$. Hence

$$m = s_1 n_1 + \ldots + s_k n_k + t_1 c_1 b + \ldots + t_n c_n b.$$

Using the fact that element m is arbitrary, we obtain

$$N \subset Rn_1 + \ldots + Rn_k + Rc_1 b + \ldots + Rc_n b.$$

However, $n_1, \ldots, n_k \in N$ and $c_1, \ldots, c_k \in J$, so this means that $c_1 b \in N_1, \ldots c_k b \in N$, that is $Rn_1 + \ldots + Rn_k + Rc_1 b + \ldots + Rc_n b \subset N$. Thus

$$N = Rn_1 + \ldots + Rn_k + Rc_1 b + \ldots + Rc_n b,$$

but this is a contradiction to the choice of N. □

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Received March 2, 2010