BULETINUL ACADEMIEI DE ŞTIINŢE
A REPUBLICII MOLDOVA. MATEMATICA
Number 3(61), 2009, Pages 42–51
ISSN 1024–7696

On preradicals associated to principal functors of module categories. II

A. I. Kashu

Abstract. Continuing part I (see [1]) the classes of modules and preradicals determined by the functor $U \otimes_S \text{-} : S\text{-}Mod \to \mathcal{A}b$ are studied, the relations between them are established and the conditions of coincidence of some preradicals are shown.

Mathematics subject classification: 16D90, 16S90, 16D40.
Keywords and phrases: Tensor product, preradical, torsion, torsion class, flat module.

Introduction

In the first part of this work [1] the classes of modules and preradicals associated to the functor $H = Hom_R(U, \text{-}) : R\text{-}Mod \to \mathcal{A}b \quad ({}_RU \in R\text{-}Mod)$ are studied. Now we will use the same methods for the investigation of similar questions for the functor of tensor product:

$$T = U \otimes_S \text{-} : S\text{-}Mod \to \mathcal{A}b,$$

where U_S is a fixed *right S-module*. The preradicals determined in S-Mod by U_S and T are elucidated, their properties and relations between them are shown. Moreover, some conditions for the coincidence of "near" preradicals are indicated. We remark that there exists a partial duality between these results and those of part I for the functor $H = Hom_R(U, \text{-})$. The main general facts on preradicals and torsions in modules can be found in the books [3–6].

1 Preradicals defined by the functor T

Let S be a ring with unity and S-Mod is the category of unitary left S-modules. We fix a *right S-module* U_S and consider the functor of tensor product, defined by U_S:

$$T = T^U = U \otimes_S \text{-} : S\text{-}Mod \to \mathcal{A}b,$$

where $\mathcal{A}b$ is the category of abelian groups.

In S-Mod we consider the following class of modules:

$$\mathcal{F}(U_S) = \{M \in S\text{-}Mod \,|\, U \otimes_S m = 0 \text{ in } U \otimes_S M \;\; \text{implies} \;\; m = 0\},$$

where $U \otimes_S m = \{u \otimes_S m \in U \otimes_S M \,|\, u \in U\}$ for $m \in M$. A direct verification proves

Proposition 1.1. *$\mathcal{F}(U_S)$ is a pretorsionfree class (i.e. is closed under submodules and direct products), therefore it defines a <u>radical</u> t_U in S-Mod such that $\mathcal{P}(t_U) \overset{def}{=\!=} \mathcal{F}(U_S)$. For every module ${}_SM$ we have:*

$$t_U(M) = \{m \in M \,|\, U \otimes_S m = 0 \text{ in } U \otimes_S M\}. \qquad \square$$

Having the module U_S and respective functor $T = T^U$, we denote:

$$Ker T = \{M \in S\text{-}Mod \,|\, T(M) = 0\}.$$

Proposition 1.2. *$Ker\ T$ is a torsion class (i.e. is closed under homomorphic images, direct sums and extensions), therefore it defines an <u>idempotent radical</u> $\overline{t}_U$ in S-Mod such that $\mathcal{R}(\overline{t}_U) \overset{def}{=\!=} Ker T$. For every module $M \in S$-Mod we have:*

$$\overline{t}_U(M) = \textstyle\sum\{N_\alpha \subseteq M \,|\, N_\alpha \in Ker T\}.$$

The corresponding torsionfree class is $\mathcal{P}(\overline{t}_U) = (Ker T)^{\downarrow}$.

Proof. From properties of the functor T (which is right exact and preserves direct sums) follows that $\mathrm{Ker}\, T$ is a torsion class. For example, any short exact sequence in S-Mod

$$0 \to M' \xrightarrow{\varphi} M \xrightarrow{\pi} M'' \to 0$$

with $M', M'' \in Ker T$ implies in $\mathcal{A}b$ the exact sequence

$$T(M') \xrightarrow{T(\varphi)} T(M) \xrightarrow{T(\pi)} T(M'') \to 0$$

with $T(M') = T(M'') = 0$, therefore $T(M) = 0$. Thus the class $\mathrm{Ker}\, T$ is closed under extensions. The rest of statements are also obvious. $\square$

Next we clarify the relation between the preradicals t_U and $\overline{t}_U$. For that we study the connections between the associated classes of modules.

Proposition 1.3. $\mathcal{F}(U_S) \subseteq (Ker T)^{\downarrow}$.

Proof. Let $N \in \mathcal{F}(U_S)$. If $M \in Ker T$ and $f \in Hom_S(M, N)$, then for the morphism $T(f)$: $U \otimes_S M \to U \otimes_S N$ and for every $m \in M$ we have $U \otimes_S m = 0$ in $U \otimes_S M = 0$. Therefore $U \otimes_S f(m) = 0$ in $U \otimes_S N$, and from the assumption $N \in \mathcal{F}(U_S)$ now it follows $f(m) = 0$. Thus $f = 0$ and $Hom_S(M, N) = 0$ for every $M \in KerT$, i.e. $M \in (Ker T)^{\downarrow}$. $\square$

Proposition 1.4. $\big(\mathcal{F}(U_S)\big)^{\uparrow} = KerT$.

Proof. ($\subseteq$) Let $M \in \big(\mathcal{F}(U_S)\big)^{\uparrow}$, i.e. $Hom_S(M, N) = 0$ for every $N \in \mathcal{F}(U_S)$. Since t_U is a radical, for every $M \in S$-Mod we have:

$$M/t_U(M) \in \mathcal{P}(t_U) = \mathcal{F}(U_S).$$

From the assumption it follows $Hom_R\big(M, M/t_U(M)\big) = 0$, therefore $M/t_U(M) = 0$, i.e. $M = t_U(M)$. This means that $U \otimes_S m = 0$ in $U \otimes_S M$ for every $m \in M$, thus $U \otimes_S M = 0$.

($\supseteq$) By Proposition 1.3 $\mathcal{F}(U_S) \subseteq (KerT)^{\downarrow}$, therefore

$$\big(\mathcal{F}(U_S)\big)^{\uparrow} \supseteq (KerT)^{\downarrow\uparrow} = KerT,$$

the last relation beeng true since $KerT$ is a torsion class (Proposition 1.2). □

Proposition 1.5. *For every module U_S we have the relation $t_U \geq \overline{t}_U$ and $\overline{t}_U$ is the greatest idempotent radical contained in the radical t_U.*

Proof. By Proposition 1.3 $\mathcal{F}(U_S) \subseteq (KerT)^{\downarrow}$, i.e. $\mathcal{P}(t_U) \subseteq \mathcal{P}(\overline{t}_U)$, therefore $t_U \geq \overline{t}_U$. Moreover, from Proposition 1.4 it follows $\big(\mathcal{F}(U_S)\big)^{\uparrow\downarrow} = (Ker\,T)^{\downarrow}$ and, since $\mathcal{F}(U_S) = \mathcal{P}(t_U)$ and $(Ker\,T)^{\downarrow} = \mathcal{P}(\overline{t}_U)$, we obtain $\big(\mathcal{P}(t_U)\big)^{\uparrow\downarrow} = \mathcal{P}(\overline{t}_U)$. Thus $\mathcal{P}(\overline{t}_U)$ is the least torsionfree class, containing $\mathcal{P}(t_U)$, which is equivalent with the assertion of proposition. □

Further we will show the necessary and sufficient conditions for coincidence of these two "neighbour" preradicals t_U and $\overline{t}_U$. We will need the following notion.

Definition 1. A module U_S will be called *weakly flat* if the functor $T = U \otimes_S$- preserves the short exact sequences of the form

$$0 \to t_U(M) \xrightarrow[\subseteq]{i} M \xrightarrow[\text{nat}]{\pi} M\,/\,t_U(M) \to 0$$

for every module $M \in S$-Mod (i.e. $T(i)$ is a monomorphism for every ${}_SM$).

Proposition 1.6. *For module U_S the following conditions are equivalent*:

1) $t_U = \overline{t}_U$;
2) *radical t_U is idempotent*;
3) $\mathcal{F}(U_S) = (KerT)^{\downarrow}$;
4) *U_S is weakly flat.*

Proof. 1) ⇔ 2) ⇔ 3) follow from Proposition 1.5.

2) ⇒ 4). If t_U is idempotent, then $t_U(M) = t_U\big(t_U(M)\big)$ for every module ${}_SM$, therefore

$$t_U(M) \in \mathcal{R}(t_U) = \mathcal{R}(\overline{r}_U) = KerT,$$

thus $T\big(t_U(M)\big) = 0$. So $T(i) = 0$ and $T(i)$ is mono, where i is the inclusion $t_U(M) \subseteq M$.

4) ⇒ 2). Let U_S be a weakly flat module. Let $m \in t_U(M)$, i.e. $U \otimes_S m = 0$ in $U \otimes_S M$. Since the subset $U \otimes_S m \subseteq U \otimes_S t_U(M)$ pass by $T(i)$ on $U \otimes_S i(m) = U \otimes_S m = 0$ in $U \otimes_S M$, and by assumption $T(i)$ is a monomorphism, we have $U \otimes_S m = 0$ in $U \otimes_S t_U(M)$. Therefore $m \in t_U\big(t_U(M)\big)$ and $t_U(M) \subseteq t_U\big(t_U(M)\big)$, i.e. t_U is idempotent. □

Now we will consider the stronger condition to radical t_U: the requirement to be a *torsion* (i.e. hereditary radical).

Definition 2. The module U_S will be called *t-hereditary* if from $U \otimes_S M = 0$ it follows $U \otimes_S N = 0$ for every submodule $N \subseteq M$.

From the previous results and definitions follows

Proposition 1.7. *For module U_S the following conditions are equivalent*:
1) *radical t_U is a torsion*;
2) $t_U = \overline{t}_U$ *and class $Ker\, T$ is hereditary*;
3) $t_U = \overline{t}_U$ *and class $(Ker\, T)^{\downarrow}$ is stable*;
4) *U_S is weakly flat and t-hereditary.* □

Corollary 1.8. *If module U_S is flat then the radical t_U is a torsion.*

Proof. If U_S is flat then by definition it is weakly flat. Let $U \otimes_S M = 0$ and $N \overset{\mathrm{i}}{\subseteq} M$. Then $T(i)$ is monomorphism, so $U \otimes_S N = 0$, i.e. U_S is t-hereditary. □

2 Relations between $(t_U, \overline{t}_U)$ and preradicals defined by ideal $J = (0 : U_S)$

As before we fix a module U_S which defines the radical t_U (Section 1). Acting by t_U to ${}_SS$ we obtain the ideal:

$$J \overset{\mathrm{def}}{=} t_U({}_SS) = \{s \in S \,|\, U \otimes_S s = 0 \text{ in } U \otimes_S S\}.$$

The isomorphism $U \otimes_S S \cong U$ show that the relation $U \otimes_S s = 0$ in $U \otimes_S S$ means that $Us = 0$, therefore the ideal

$$J = (0 : U_S) = \{s \in S \,|\, Us = 0\}$$

is the *annihilator* of module U_S. As every ideal of a ring, J determines in S-Mod the following classes of modules [1, 2, 7]:

$${}_J\mathcal{T} = \{M \in S\text{-}Mod \,|\, JM = M\};$$
$${}_J\mathcal{F} = \{M \in S\text{-}Mod \,|\, m \in M, Jm = 0 \Rightarrow m = 0\};$$
$$\mathcal{A}(J) = \{M \in S\text{-}Mod \,|\, JM = 0\}.$$

We remind briefly form some facts on these classes of modules.

Proposition 2.1. 1) *${}_J\mathcal{T}$ is a torsion class, therefore it determines an idempotent radical r^J such that $\mathcal{R}(r^J) \overset{def}{=} {}_J\mathcal{T}$ and so $\mathcal{P}(r^J) = {}_J\mathcal{T}^{\downarrow}$*;

2) *${}_J\mathcal{F}$ is a torsionfree and stable class, therefore it determines a torsion r_J such that $\mathcal{P}(r_J) \overset{def}{=} {}_J\mathcal{F}$ and so $\mathcal{R}(r_J) = {}_J\mathcal{F}^{\uparrow}$*;

3) *$\mathcal{A}(J)$ is a pretorsion and hereditary class, therefore it determines a pretorsion $r_{(J)}$ such that $\mathcal{R}(r_{(J)}) = \mathcal{A}(J)$*;

4) *$\mathcal{A}(J)$ is a pretorsionfree and cohereditary class, therefore it determines a* <u>*cohereditary radical*</u> *$r^{(J)}$ such that $\mathcal{P}\left(r^{(J)}\right) \stackrel{def}{=\!=} \mathcal{A}(J)$.* □

Proposition 2.2. 1) *$r^{J} \leq r^{(J)}$ and r^{J} is the greatest idempotent radical contained in $r^{(J)}$.*

2) *$r_{J} \geq r_{(J)}$ and r_{J} is the least idempotent radical (torsion) containing $r_{(J)}$.* □

Proposition 2.3. *The following conditions are equivalent*:

1) $r^{J} = r^{(J)}$;
2) *$r^{(J)}$ is idempotent*;
3) $\mathcal{A}(J) = {}_{J}\mathcal{T}^{\downarrow}$;
4) $r_{J} = r_{(J)}$;
5) *$r_{(J)}$ is a radical*
6) $\mathcal{A}(J) = {}_{J}\mathcal{F}^{\uparrow}$;
7) $J = J^2$. □

Next we will study the relations between the preradicals defined by ideal $J \triangleleft S$ and preradicals $t_U, \overline{t}_U$ from Section 1. For that purpose it is sufficient to clarify the connections between the respective classes of modules.

Proposition 2.4. *$\mathcal{F}(U_S) \subseteq \mathcal{A}(J)$ (i.e. $\mathcal{P}(t_U) \subseteq \mathcal{P}\left(r^{(J)}\right)$, so $t_U \geq r^{(J)}$.*

Proof. Let $M \in \mathcal{F}(U_S)$. For every $j \in J$ and $m \in M$ we have:

$$U \otimes_S (j\, m) = (Uj) \otimes_S m = 0 \otimes_S m \;\text{ in }\; U \otimes_S M,$$

thus by assumption it follows $j\, m = 0$. Therefore $JM = 0$, i.e. $M \in \mathcal{A}(J)$. □

Proposition 2.5. *${}_{J}\mathcal{T} \subseteq KerT$ (i.e. $\mathcal{R}(r^{J}) \subseteq \mathcal{R}(\overline{t}_U)$*, so *$r^{J} \leq \overline{t}_U$).*

Proof. If $M \in {}_{J}\mathcal{T}$, then $JM = M$ and we have

$$U \otimes_S M = U \otimes_S (JM) = UJ \otimes_S M = 0 \otimes_S M = 0,$$

thus $M \in KerT$. □

From the last statement it follows that

$${}_{J}\mathcal{T}^{\downarrow} \supseteq (KerT)^{\downarrow} = \big(\mathcal{F}(U_S)\big)^{\uparrow\downarrow} \supseteq \mathcal{F}(U_S),$$

i.e. $\mathcal{P}(r^{J}) \supseteq \mathcal{P}(\overline{t}_U) \supseteq \mathcal{P}(t_U)$, which means that

$$r^{J} \leq \overline{t}_U \leq t_U.$$

In this way, we obtain the following scheme, which illustrates the relations between preradicals studied above:

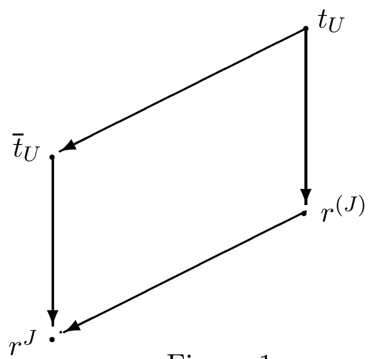

Figure 1.

The question of coincidence of all these preradicals is more complicated than in the case of functor H [1]. We remark, in particular, that the relations $t_U = r^{(J)}$ or $\overline{t}_U = r^J$ are not sufficient for the coincidence of all preradicals of Figure 1.

The relation $r^J = \overline{t}_U$ is equivalent to the inclusion $Ker\, T \subseteq {}_J\mathcal{T}$; the relation $r^{(J)} = t_U$ is equivalent to the inclusion $\mathcal{A}(J) \subseteq \mathcal{F}(U_S)$. Finally, the stronger relation $r_J = t_U$ is equivalent to the inclusion ${}_J\mathcal{T}^{\downarrow} \subseteq \mathcal{F}(U_S)$.

The general situation on classes of modules in this case is shown in Figure 2 (see next page).

3 Supplement to the case of functor *H*

In the part I of this work [1] we noted the fact that for the functor H is not obtained the symmetric statements for the preradicals $\big(r_I, r_{(I)}\big)$. Now we supplement the results of [1], using the above constructions for the functor T.

We remind that in part I [1] is studied the functor

$$H = H^U = Hom_R(U, \text{-}) : R\text{-}Mod \rightarrow \mathcal{A}b$$

for a fixed module ${}_RU \in R$-Mod. We have the idempotent preradical r^U in R-Mod with $\mathcal{R}(r^U) = Gen\,({}_RU)$ and the idempotent radical $\overline{r}^{\,U}$ with $\mathcal{P}(\overline{r}^{\,U}) = Ker\, H$. Moreover, the trace of ${}_RU$ in R, i.e. the ideal $I = r^U({}_RR)$, determines two pairs of preradicals of different types: $\big(r^I, r^{(I)}\big)$ and $\big(r_I, r_{(I)}\big)$. We obtained the situation

$$r^I \le r^U \le \overline{r}^{\,U}, \quad r^I \le r^{(I)} \le \overline{r}^{\,U},$$

studying the conditions of coincidence of these preradicals.

Now we will construct two preradicals t_V and $\overline{t}_V$, which are related similarly with the pair $(r_I, r_{(I)})$. With this purpose for our fixed module ${}_RU \in R$-Mod we denote:

$$V_R = Hom_R(U, R),$$

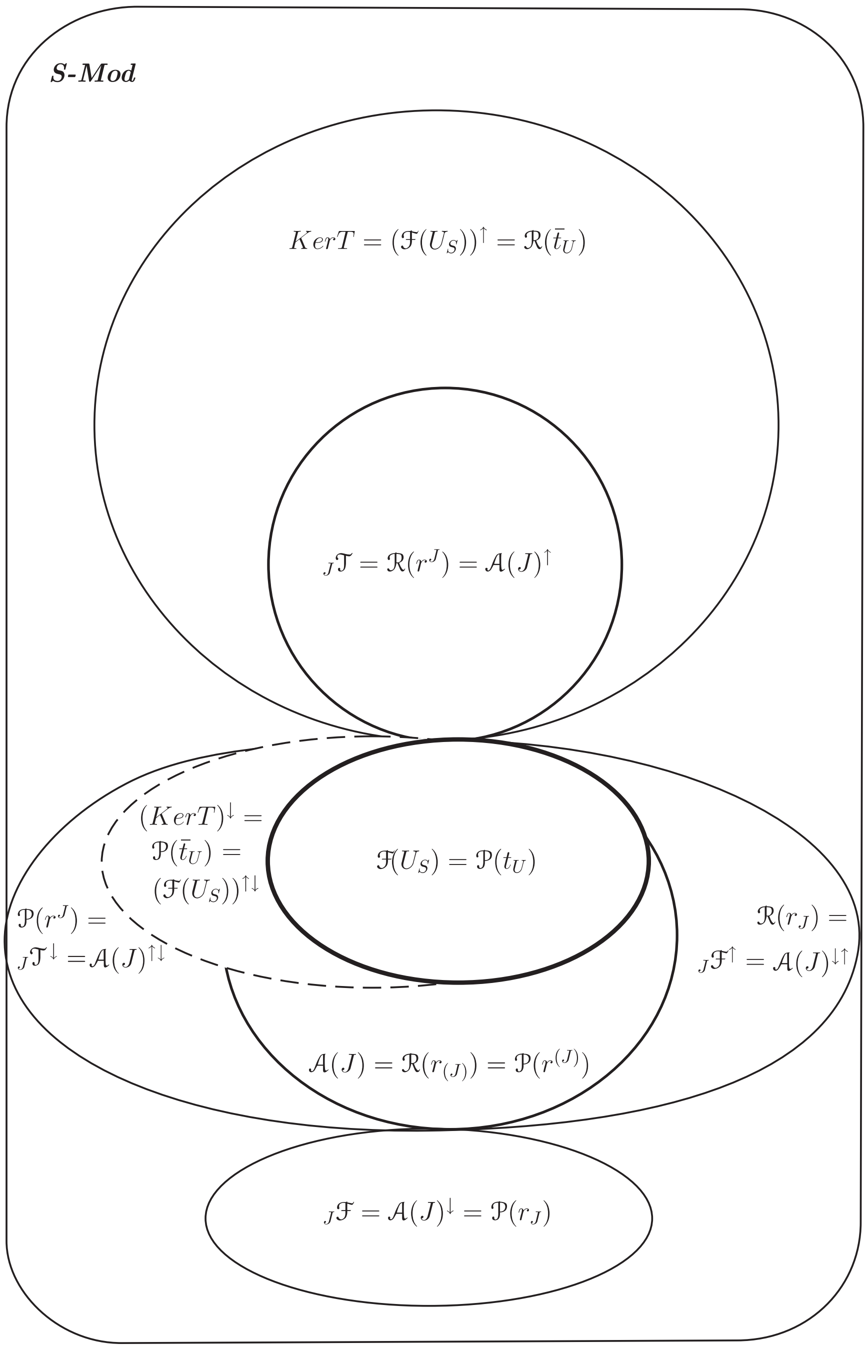
S-Mod
$KerT = (\mathcal{F}(U_S))^{\uparrow} = \mathcal{R}(\bar{t}_U)$
${}_J\mathcal{T} = \mathcal{R}(r^J) = \mathcal{A}(J)^{\uparrow}$
$(KerT)^{\downarrow} =$
$\mathcal{P}(\bar{t}_U) =$
$(\mathcal{F}(U_S))^{\uparrow\downarrow}$
$\mathcal{F}(U_S) = \mathcal{P}(t_U)$
$\mathcal{P}(r^J) =$
${}_J\mathcal{T}^{\downarrow} = \mathcal{A}(J)^{\uparrow\downarrow}$
$\mathcal{R}(r_J) =$
${}_J\mathcal{F}^{\uparrow} = \mathcal{A}(J)^{\downarrow\uparrow}$
$\mathcal{A}(J) = \mathcal{R}(r_{(J)}) = \mathcal{P}(r^{(J)})$
${}_J\mathcal{F} = \mathcal{A}(J)^{\downarrow} = \mathcal{P}(r_J)$

Figure 2.

the dual module of ${}_RU$, which is a *right R-module*. For this module we consider the functor

$$T = T^V = V \otimes_R \text{-} : R\text{-}Mod \to \mathcal{A}b,$$

which determines the associated preradicals t_V and $\overline{t}_V$ of R-Mod, where:

1) t_V is a *radical* of R-Mod with $\mathcal{P}(t_V) \overset{\text{def}}{=\!=} \mathcal{F}(V_R) =$
$= \{M \in R\text{-}Mod \,|\, V \otimes_R m = 0 \;\text{ in }\; V \otimes_R M \Rightarrow m = 0\};$

2) $\overline{t}_V$ is an *idempotent radical* of R-Mod such that $\mathcal{R}(\overline{t}_V) \overset{\text{def}}{=\!=} Ker\, T^V$, therefore $\mathcal{P}(\overline{t}_V) = (KerT^V)^{\downarrow}$.

From Section 1 it follows that $\mathcal{F}(V_R) \subseteq KerT^V)^{\downarrow}$ (Proposition 1.3), thus $t_V \geq \overline{t}_V$. Moreover, $\big(\mathcal{F}(V_R)\big)^{\uparrow} = KerT^V$, therefore $\overline{t}_V$ is the greatest idempotent radical contained in t_V (Proposition 1.5).

Now we will combine this situation with the corresponding situation defined in R-Mod by module ${}_RU$ and ideal I [1]. The purpose is to clarify the relations between preradicals studied in part I [1] and preradicals $(t_V, \overline{t}_V)$. As usual, we study the connections between the corresponding classes of modules.

Proposition 3.1. $KerT^V \subseteq \mathcal{A}(I)$.

Proof. Every element $u \in U$ determines the morphism $\varphi_u : V \otimes_R M \to M$ by the rule $\varphi_u(f \otimes m) \overset{\text{def}}{=\!=} [(u)f] \cdot m$, where $f \in Hom_R(U, R)$ and $m \in M$. We have $Im\, \varphi_u = [(u)V] \cdot M$ and

$$\sum_{u \in U} Im\, \varphi_u = \sum_{u \in U} [(u)V] \cdot M = \Big(\sum_{f : U \to R} Im\, f \Big) \cdot M = I\, M.$$

If $M \in KerT^V$, then $V \otimes_R M = 0$ and $\varphi_u = 0$ for every $u \in U$, therefore $\sum_{u \in U} Im\, \varphi_u = I\, M = 0$. □

Proposition 3.2. ${}_I\mathcal{F} \subseteq \mathcal{F}(V_R)$.

Proof. Let $M \in {}_I\mathcal{F}$, i.e. from $I \cdot m = 0 \;\; (m \in M)$ it follows $m = 0$. Suppose that $V \otimes_R m = 0$ in $V \otimes_R M$. Then as in the preceding proof, for every $u \in U$ we have the morphism $\varphi_u : V \otimes_R M \to M$ such that

$$\varphi_u(V \otimes_R m) = [(u)V] \cdot m = 0.$$

Therefore

$$\sum_{u \in U} \varphi_u(V \otimes_R m) = \sum_{u \in U} [(u)V] \cdot m = I \cdot m = 0$$

and from the assumption $M \in {}_I\mathcal{F}$ it follows $m = 0$. So $M \in \mathcal{F}(V_R)$. □

We remark that from Proposition 1.3 we have also the inclusion:

$$\mathcal{F}(V_R) \subseteq (KerT^V)^{\downarrow}.$$

Corollary 3.3. $\overline{t}_V \leq r_{(I)}$ *and* $r_I \geq t_V$.

Proof. Since $Ker\,T^V = \mathcal{R}(\overline{t}_V)$ and $\mathcal{A}(I) = \mathcal{R}\big(r_{(I)}\big)$, from Proposition 3.1 we have $\mathcal{R}(\overline{t}_V) \subseteq \mathcal{R}\big(r_{(I)}\big)$, thus $\overline{t}_V \leq r_{(I)}$.

Similarly, since ${}_I\mathcal{F} = \mathcal{P}(r_I)$ and $\mathcal{F}(V_R) = \mathcal{P}(t_V)$, from Proposition 3.2 it follows $\mathcal{P}(r_I) \subseteq \mathcal{P}(t_V)$, therefore $r_I \geq t_V$. □

In this way, for the functor H we have the following relations between the associated preradicals:

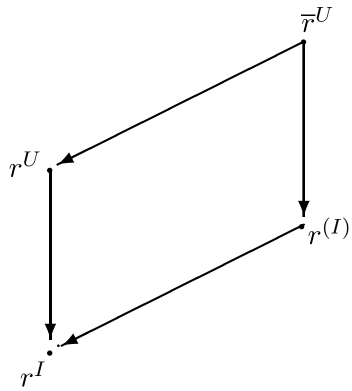

Figure 3.

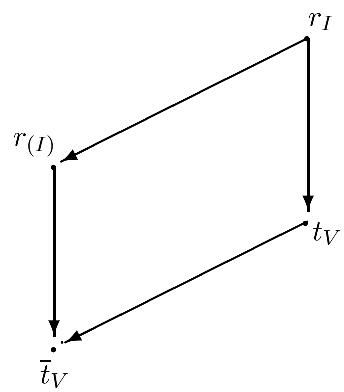

Figure 4.

The conditions of coincidence of preradicals from Figure 3 are shown in part I ([1], Proposition 4.4). A similar result is true for preradicals from Figure 4.

Proposition 3.4. *The following conditions are equivalent*:

1) $t_V = r_I$;
2) $\overline{t}_V = r_I$;
3) $\overline{t}_V = r_{(I)}$;
4) $t_V = r_{(I)}$;
5) $VI = V$.

Proof. The equivalence of conditions 1)–4) can be verified similarly to the proof of Proposition 4.4 of part I [1].

1) ⇒ 5). Let $t_V = r_I$. Then $\mathcal{P}(t_V) = \mathcal{P}(r_I)$, i.e. $\mathcal{F}(V_R) = {}_I\mathcal{F}$. Therefore $\big(\mathcal{F}(V_R)\big)^{\uparrow} = {}_I\mathcal{F}^{\uparrow}$ where $\big(\mathcal{F}(V_R)\big)^{\uparrow} = KerT^V$, thus $KerT^V = {}_I\mathcal{F}^{\uparrow}$. From the relations

$$KerT^V \subseteq \mathcal{A}(I) \subseteq {}_I\mathcal{F}^{\uparrow}$$

we obtain $\mathcal{A}(I) = KerT^V$. Since $R/I \in \mathcal{A}(I)$, we have $R/I \in KerT^V$, i.e. $V \otimes_R (R/I) \cong V/VI = 0$, thus $V = VI$.

5) ⇒ 1). Let $V\,I = V$. It is sufficient to show that $\mathcal{F}(V_R) = {}_I\mathcal{F}$, i.e. the inclusion $\mathcal{F}(V_R) \subseteq {}_I\mathcal{F}$. If $M \in \mathcal{F}(V_R)$ and $I \cdot m = 0$ for some $m \in M$, then:

$$V \otimes_R m = VI \otimes_R m = V \otimes_R (Im) = 0 \ \text{ in } \ V \otimes_R M.$$

From the assumption $M \in \mathcal{F}(V_R)$ now it follows $m = 0$. So $M \in {}_I\mathcal{F}$. □

References

[1] Kashu A. I., *On preradicals associated to principal functors of modulle categories. I.* Bul. A.Ş,R.M. Matematica, 2009, No. 2(60), 62–72.

[2] Kashu A. I., *Functors and torsions in categories of modules.* Acad. of Sciences of RM, Inst. of Math., Chişinău, 1997 (in Russian).

[3] Bican L., Kepka P., Nemec P. *Rings, modules and preradicals.* Marcel Dekker, New York, 1982.

[4] Golan J. S. *Torsion theories*. Longman Sci. Techn., New York, 1986.

[5] Kashu A. I. *Radicals and torsions in modules.* Chişinău, Ştiinţa, 1983 (In Russian).

[6] Stenström B. *Rings of quotients.* Springer Verlag, Berlin, 1975.

[7] Kashu A. I. *On some bijections between ideals, classes of modules and preradicals of R-Mod.* Bul. A.Ş,R.M. Matematica, 2001, No. 2(36), 101–110.

A. I. Kashu
Institute of Mathematics and Computer Science
Academy of Sciences of Moldova
5 Academiei str., Chişinău, MD−2028
Moldova
E-mail: kashuai@math.md

Received April 7, 2009