BULETINUL ACADEMIEI DE ŞTIINŢE
A REPUBLICII MOLDOVA. MATEMATICA
Number 3(61), 2009, Pages 30–41
ISSN 1024–7696

Flow of an Unsteady Dusty Visco-Elastic Fluid Between Two Moving Plates in Frenet-Frame Field System

B. J. Gireesha, T. Nirmala, C. S. Vishalakshi, C. S. Bagewadi

Abstract. The present investigation deals with the study of an unsteady motion of a dusty viscoelastic conducting fluid under arbitrary pressure gradient between two infinite moving parallel plates. The influence of time dependent pressure gradients, i.e. impulsive, transition and motion for a finite time is considered along with the effect of the movement of the plates and the presence of uniform magnetic field. Expressions for the velocities of the fluid and particles are obtained by using the Laplace transform technique. Results are presented in graphical form. Finally the skin friction at the boundaries is calculated.

Mathematics subject classification: 76T10, 76T15.
Keywords and phrases: Frenet frame field system; parallel plates, dusty fluid; velocity of dust phase and fluid phase, conducting dusty fluid, magnetic field.

1 Introduction

The presence of dust particles in fluids has certain influence on the motion of the fluids, and such situations arise, for instance in the movement of dust-laden air, in fluidization, in the use of dust in gas cooling systems, and in sedimentation in tidal waves, powder technology, acoustics, performance of solid fuel rocket nozzles, rainerosion, guided missiles, paint spraying, etc.

The stability of the laminar flow of a dusty gas in which the dust particles are uniformly distributed has been discussed by P. G. Saffman [18] and the basic equations for the flow of dusty fluid were formulated. T. M. Nabil [16] studied the effect of couple stresses on pulsatile hydromagnetic Poiseuille flow. N. Datta [5] obtained the solutions for Pulsatile flow of heat transfer of a dusty fluid through an infinitely long annular pipe. Girish Kumar, R. K. S. Chaudhary and K. K. Singh [9] have discussed the unsteady flow of conducting dusty visco-elastic liquid through a channel, and N. C. Ghosh, B. C. Ghosh and L. Debnath [10] obtained the results for the hydromagnetic flow of a dusty visco-elastic fluid between two infinite parallel plates.

Some researchers like Kanwal [12], Truesdell [19], Indrasena [11], Purushotham [17], Bagewadi and Gireesha [1, 2] have applied differential geometry techniques to investigate the kinematical properties of fluid flows in the field of fluid mechanics.

© B. J. Gireesha, T. Nirmala, C. S. Vishalakshi, C. S. Bagewadi, 2009

Further, recently the authors [6–8] have studied dusty fluid flow in Frenet frame field system under varying time dependent pressure gradients.

The present investigation deals with the study of an electrically conducting dusty viscoelastic fluid flow between two infinitely extended non-conducting parallel plates in Frenet frame field system. Initially, the fluid and dust particles are assumed to be at rest. The motion of fluid is due to the influence of time dependent pressure gradient along with movement of the plates and applied uniform magnetic field. The analytical expressions are obtained for velocities of fluid and dust particles in three cases. For each case the skin friction at boundaries is obtained. The changes in the velocity profiles for different Hartmann numbers are shown graphically. section-Frenet Frame Field System

Let $\overrightarrow{s}, \overrightarrow{n}, \overrightarrow{b}$ be triply orthogonal unit vectors tangent, principal normal, binormal respectively to the spatial curves of congruences formed by fluid phase velocity and dusty phase velocity lines respectively as shown in Figure 1.

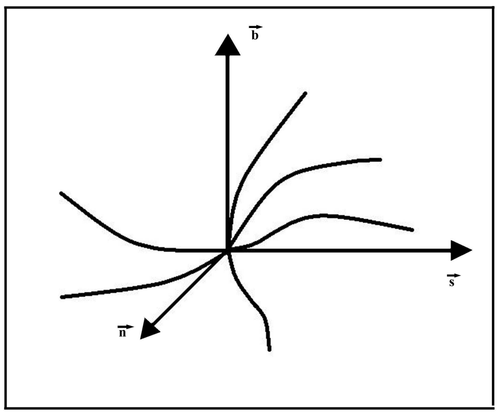


Figure 1. Frenet Frame Field System

Geometrical relations are given by Frenet formulae [3]

$$\begin{aligned}
&i) && \frac{\partial \overrightarrow{s}}{\partial s} = k_s \overrightarrow{n}, \ \frac{\partial \overrightarrow{n}}{\partial s} = \tau_s \overrightarrow{b} - k_s \overrightarrow{s}, \ \frac{\partial \overrightarrow{b}}{\partial s} = -\tau_s \overrightarrow{n};\\
&ii) && \frac{\partial \overrightarrow{n}}{\partial n} = k'_n \overrightarrow{s}, \ \frac{\partial \overrightarrow{b}}{\partial n} = -\sigma'_n \overrightarrow{s}, \ \frac{\partial \overrightarrow{s}}{\partial n} = \sigma'_n \overrightarrow{b} - k'_n \overrightarrow{n};\\
&iii) && \frac{\partial \overrightarrow{b}}{\partial b} = k''_b \overrightarrow{s}, \ \frac{\partial \overrightarrow{n}}{\partial b} = -\sigma''_b \overrightarrow{s}, \ \frac{\partial \overrightarrow{s}}{\partial b} = \sigma''_b \overrightarrow{n} - k''_b \overrightarrow{b};\\
&iv) && \nabla . \overrightarrow{s} = \theta_{ns} + \theta_{bs}; \ \nabla . \overrightarrow{n} = \theta_{bn} - k_s; \ \nabla . \overrightarrow{b} = \theta_{nb},
\end{aligned} \tag{1}$$

where $\partial/\partial s$, $\partial/\partial n$ and $\partial/\partial b$ are the intrinsic differential operators along fluid phase velocity (or dust phase velocity) lines, principal normal and binormal. The functions (k_s, k'_n, k''_b) and $(\tau_s, \sigma'_n, \sigma''_b)$ are the curvatures and torsion of the above curves and

θ_{ns} and θ_{bs} are normal deformations of these spatial curves along their principal normal and binormal respectively.

2 Formulation and Solution of the Problem

The present discussion considers a dusty visco-elastic fluid bounded by two infinite flat moving plates separated by a distance h in the absence of body force. Both the fluid and the dust particle clouds are supposed to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size. The number density of the dust particles is taken as a constant throughout the flow. It is assumed that the dust particles are electrically nonconducting and neutral. The flow is due to the influence of time dependent pressure gradient along with motion of plates and due to magnetic field of uniform strength B_0. Under these assumptions the flow will be a parallel flow in which the streamlines are along the tangential direction as shown in Figure 2.

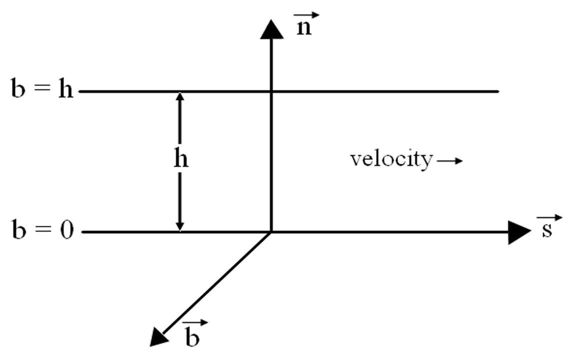


Figure 2. Geometry of the flow

For the above described flow the velocities of fluid and dust are of the form

$$\overrightarrow{u} = u_s \overrightarrow{s}, \qquad \overrightarrow{v} = v_s, \overrightarrow{s} \tag{2}$$

i.e., $u_n = u_b = 0$ and $v_n = v_b = 0$, where (u_s, u_n, u_b) and (v_s, v_n, v_b) denote the velocity components of fluid and dust respectively.

Since the flow is in between two moving plates, we can assume the velocity of both fluid and dust particles do not vary along tangential direction. Suppose the fluid extends to infinity in the principal normal direction, then the velocities of both may be neglected in this direction.

The modified Saffman's [18] equations for the dusty visco-elastic fluid with the help of equation (1) are given by:

$$\frac{\partial u_s}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial s} + \left(\alpha + \beta\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u_s}{\partial b^2} - C_r u_s\right) + \frac{kN}{\rho}(v_s - u_s) - \frac{\sigma B_0^2}{\rho}u_s; \tag{3}$$

$$\frac{\partial v_s}{\partial t} = \frac{k}{m}(u_s - v_s). \tag{4}$$

We have the following nomenclature:

$\rho-$density of the gas, $p-$pressure of the fluid, $N-$number of density of dust particles, $k = 6\pi a\mu-$ Stoke's resistance (drag coefficient), $a-$spherical radius of dust particle, $m-$mass of the dust particle, $B_0-$the intensity of the imposed transverse magnetic field, $\sigma-$electrical conductivity of the fluid, $m/k-$relaxation time of the dust particles, α & β are the kinematic coefficients of visco-elasticity of the fluid, $t-$time, and $C_r = (\sigma_b'^2 + k_n'^2 + k_b'^2 + \sigma_b''^2)$ is called the curvature term [2].

Introducing the nondimensional quantities

$$x' = x/h, \quad y' = y/h, \quad t' = \alpha t/h^2, \quad p' = ph^2/\alpha^2\rho, \quad u_s' = u_s h/\alpha, \quad v_s' = v_s h/\alpha$$

in equations (3) and (4) and dropping the primes one can get

$$\frac{\partial u_s}{\partial t} = -\frac{\partial p}{\partial s} + \left(1 + E\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u_s}{\partial b^2} - C_r u_s\right) + \frac{l}{w}(v_s - u_s) - M^2 u_s; \tag{5}$$

$$\frac{\partial v_s}{\partial t} = \frac{1}{w}(u_s - v_s) \tag{6}$$

where $E = \beta/h^2$ is the elastic parameter, $l = mN/\rho$, $w = m\alpha/kh^2$, $M = B_0 h\sqrt{\sigma/\mu}$ (Hartmann number).

Equations (5) and (6) are to be solved subject to the initial and boundary conditions in nondimensional form as:

$$\begin{aligned} \text{Initial condition; at } t = 0;\ u_s &= 0, v_s = 0 \\ \text{Boundary condition; for } t > 0; u_s &= f(t), \text{ at } b = 0 \\ \text{and } u_s &= g(t) \text{ at } b = 1 \end{aligned} \tag{7}$$

Let $P(t)$ be the time dependent pressure gradient to be impressed on the system for $t > 0$. So we can write

$$-\frac{\partial p}{\partial s} = P(t).$$

We define Laplace transformations of u_s and v_s as

$$U = \int_0^\infty e^{-xt} u_s dt \quad \text{and} \quad V = \int_0^\infty e^{-xt} v_s dt. \tag{8}$$

Applying the Laplace transform to equations (5) and (6) and to boundary conditions, then by using initial conditions one obtains

$$xU = P(x) + (1 + xE)\left(\frac{\partial^2 U}{\partial b^2} - C_r U\right) + \frac{l}{w}(V - U) - M^2 U; \tag{9}$$

$$xV = \frac{1}{w}(U - V); \tag{10}$$

$$U = F(x), \text{ at } b=0 \text{ and } U=G(x) \text{ at } b=1, \tag{11}$$

where $F(x)$, $G(x)$ and $P(x)$ are Laplace transforms of $f(t)$, $g(t)$ and $P(t)$ respectively.

Eliminating V from (9) and (10) we obtain the following equation

$$\frac{d^2U}{db^2} - Q^2U = -\frac{P(x)}{1+xE}, \tag{12}$$

where $Q^2 = \left(C_r + \frac{x}{1+xE} + \frac{M^2}{1+xE} + \frac{xl}{(1+xE)(1+xw)}\right)$.

CASE 1. Impulsive Motion: Consider the case of impulsive motion, in which

$$\begin{aligned} f(t) &= u_0\delta(t) \text{ at } b=0, \\ g(t) &= u_1\delta(t) \text{ at } b=1, \\ P(t) &= p_0\delta(t), \end{aligned}$$

where $\delta(t)$ is the Dirac delta function and u_0, u_1 & p_0 are constants.

The velocities of fluid and dust particle are obtained by solving the equation (12) subjected to the boundary conditions (11) as follows:

$$\begin{aligned} U &= \left[\frac{u_1\sinh(Qb) - u_0\sinh(Q(b-1))}{sinh(Q)}\right] + \\ &+ \frac{p_0}{Q^2(1+xE)}\left[\frac{\sinh(Q(b-1)) - \sinh(Qb)}{sinh(Q)} + 1\right]. \end{aligned}$$

Using U in (10) we obtain V as

$$\begin{aligned} V &= \frac{1}{(1+xw)}\left[\frac{u_1\sinh(Qb) - u_0\sinh(Q(b-1))}{sinh(Q)}\right] + \\ &+ \frac{p_0}{Q^2(1+xE)(1+xw)}\left[\frac{\sinh(Q(b-1)) - \sinh(Qb)}{sinh(Q)} + 1\right]. \end{aligned}$$

By taking the inverse Laplace transform to U and V, one can obtain

$$\begin{aligned} u_s &= 2\pi\sum_{r=0}^{\infty} r[u_0 - u_1(-1)^r]\sin(r\pi b) \times \\ &\times \left[\frac{e^{\alpha_1 t}(1+\alpha_1 E)^2(1+\alpha_1 w)^2}{\delta_1} + \frac{e^{\alpha_2 t}(1+\alpha_2 E)^2(1+\alpha_2 w)^2}{\delta_2}\right] + \\ &+ \frac{2p_0}{\pi}\sum_{r=0}^{\infty}\frac{[(-1)^r - 1]}{r}\sin(r\pi b) \times \\ &\times \left[\frac{e^{\alpha_1 t}(1+E\alpha_1)(1+w\alpha_1)^2}{\delta_1} + \frac{e^{\alpha_2 t}(1+E\alpha_2)(1+w\alpha_2)^2}{\delta_2}\right] + \end{aligned}$$

$$
\begin{aligned}
&+ \quad \left[\frac{u_1 \sinh(Xb) - u_0 \sinh(X(b-1))}{\sinh(X)}\right] + \\
&+ \quad \frac{p_0}{X^2}\left[\frac{\sinh(X(b-1)) - \sinh(Xb)}{\sinh(X)} + 1\right];
\end{aligned}
$$

$$
\begin{aligned}
v_s \quad &= \quad 2\pi \sum_{r=0}^{\infty} r[u_0 - u_1(-1)^r] \sin(r\pi b) \times \\
&\times \quad \left[\frac{e^{\alpha_1 t}(1+\alpha_1 E)^2(1+\alpha_1 w)}{\delta_1} + \frac{e^{\alpha_2 t}(1+\alpha_2 E)^2(1+\alpha_2 w)}{\delta_2}\right] + \\
&+ \quad \frac{2p_0}{\pi} \sum_{r=0}^{\infty} \frac{[(-1)^r - 1]}{r} \sin(r\pi b) \times \\
&\times \quad \left[\frac{e^{\alpha_1 t}(1+E\alpha_1)(1+w\alpha_1)}{\delta_1} + \frac{e^{\alpha_2 t}(1+E\alpha_2)(1+w\alpha_2)}{\delta_2}\right] + \\
&+ \quad \left[\frac{u_1 \sinh(Xb) - u_0 \sinh(X(b-1))}{\sinh(X)}\right] + \\
&+ \quad \frac{p_0}{X^2}\left[\frac{\sinh(X(b-1)) - \sinh(Xb)}{\sinh(X)} + 1\right].
\end{aligned}
$$

Shear stress (Skin friction): The expression for shear stress at the plates $b = 0$ and $b = 1$ are respectively given by:

$$
\begin{aligned}
D_0 \quad &= \quad 2\pi^2 \mu \sum_{r=0}^{\infty} r^2[u_0 - u_1(-1)^r] \times \\
&\times \quad \left[\frac{e^{\alpha_1 t}(1+\alpha_1 E)^2(1+\alpha_1 w)^2}{\delta_1} + \frac{e^{\alpha_2 t}(1+\alpha_2 E)^2(1+\alpha_2 w)^2}{\delta_2}\right] + \\
&+ \quad 2p_0\mu \sum_{r=0}^{\infty} [(-1)^r - 1] \left[\frac{e^{\alpha_1 t}(1+E\alpha_1)(1+w\alpha_1)^2}{\delta_1} + \frac{e^{\alpha_2 t}(1+E\alpha_2)(1+w\alpha_2)^2}{\delta_2}\right] \times \\
&\times \quad \mu X \left[\frac{u_1 - u_0 \cosh(X)}{\sinh(X)}\right] + \frac{\mu p_0}{X}\left[\frac{cosh(X) - 1}{\sinh(X)}\right];
\end{aligned}
$$

$$
\begin{aligned}
D_1 \quad &= \quad 2\pi^2 \mu \sum_{r=0}^{\infty} r^2[u_0(-1)^r - u_1] \times \\
&\times \quad \left[\frac{e^{\alpha_1 t}(1+\alpha_1 E)^2(1+\alpha_1 w)^2}{\delta_1} + \frac{e^{\alpha_2 t}(1+\alpha_2 E)^2(1+\alpha_2 w)^2}{\delta_2}\right] + \\
&+ \quad 2p_0\mu \sum_{r=0}^{\infty} [1 - (-1)^r] \left[\frac{e^{\alpha_1 t}(1+E\alpha_1)(1+w\alpha_1)^2}{\delta_1} + \frac{e^{\alpha_2 t}(1+E\alpha_2)(1+w\alpha_2)^2}{\delta_2}\right] \times \\
&\times \quad \mu X \left[\frac{u_1 \cosh(X) - u_0}{\sinh(X)}\right] + \frac{\mu p_0}{X}\left[\frac{1 - \cosh(X)}{\sinh(X)}\right].
\end{aligned}
$$

CASE 2. Transition Motion: We consider the case of transition motion in which

$$\begin{aligned} f(t) &= u_0 H(t) e^{-\lambda t} \quad \text{at} \;\; b=0, \\ g(t) &= u_1 H(t) e^{-\lambda t} \quad \text{at} \;\; b=1, \\ P(t) &= p_0 H(t) e^{-\lambda t} \quad \lambda > 0, \end{aligned}$$

where $H(t)$ is the Heaviside unit step function.

Now we obtain the expressions for velocities of both fluid and dust phase as

$$\begin{aligned} u_s &= 2\pi \sum_{r=0}^{\infty} r[u_0 - u_1(-1)^r] \sin(r\pi b) \times \\ &\times \left[\frac{e^{\alpha_1 t}(1+\alpha_1 E)^2(1+\alpha_1 w)^2}{\delta_1(\alpha_1+\lambda)} + \frac{e^{\alpha_2 t}(1+\alpha_2 E)^2(1+\alpha_2 w)^2}{\delta_2(\alpha_2+\lambda)} \right] + \\ &+ e^{-\lambda t} \left[\frac{u_1 \sinh(Yb) - u_0 \sinh(Y(b-1))}{\sinh(Y)} \right] + \\ &+ \frac{2p_0}{\pi} \sum_{r=0}^{\infty} \frac{[(-1)^r - 1]}{r} \sin(r\pi b) \times \\ &\times \left[\frac{e^{\alpha_1 t}(1+E\alpha_1)(1+w\alpha_1)^2}{\delta_1(\alpha_1+\lambda)} + \frac{e^{\alpha_2 t}(1+E\alpha_2)(1+w\alpha_2)^2}{\delta_2(\alpha_2+\lambda)} \right] + \\ &+ \frac{p_0 e^{-\lambda t}}{(1-\lambda E)Y^2} \left[\frac{\sinh(Y(b-1)) - \sinh(Yb) + \sinh(Y)}{\sinh(Y)} \right]; \end{aligned}$$

$$\begin{aligned} v_s &= 2\pi \sum_{r=0}^{\infty} r[u_0 - u_1(-1)^r] \sin(r\pi b) \times \\ &\times \left[\frac{e^{\alpha_1 t}(1+\alpha_1 E)^2(1+\alpha_1 w)}{\delta_1(\alpha_1+\lambda)} + \frac{e^{\alpha_2 t}(1+\alpha_2 E)^2(1+\alpha_2 w)}{\delta_2(\alpha_2+\lambda)} \right] + \\ &+ e^{-\lambda t} \left[\frac{u_1 \sinh(Yb) - u_0 \sinh(Y(b-1))}{\sinh(Y)(1-\lambda w)} \right] + \\ &+ \frac{2p_0}{\pi} \sum_{r=0}^{\infty} \frac{[(-1)^r - 1]}{r} \sin(r\pi b) \times \\ &\times \left[\frac{e^{\alpha_1 t}(1+E\alpha_1)(1+w\alpha_1)}{\delta_1(\alpha_1+\lambda)} + \frac{e^{\alpha_2 t}(1+E\alpha_2)(1+w\alpha_2)}{\delta_2(\alpha_2+\lambda)} \right] + \\ &+ \frac{p_0 e^{-\lambda t}}{(1-\lambda E)Y^2} \left[\frac{\sinh(Y(b-1)) - \sinh(Yb) + \sinh(Y)}{\sinh(y)(1-\lambda w)} \right]. \end{aligned}$$

Shear stress (Skin friction): The shear stress at the plates $b=0$ and $b=1$ for transition motion are, respectively, given by:

$$D_0 = 2\pi^2 \mu \sum_{r=0}^{\infty} r^2[u_0 - u_1(-1)^r] \times$$

$$\begin{aligned}
&\times \left[\frac{e^{\alpha_1 t}(1+\alpha_1 E)^2(1+\alpha_1 w)^2}{\delta_1(\alpha_1+\lambda)} + \frac{e^{\alpha_2 t}(1+\alpha_2 E)^2(1+\alpha_2 w)^2}{\delta_2(\alpha_2+\lambda)}\right] + \\
&+ \mu Y e^{-\lambda t}\left[\frac{u_1 - u_0\cosh(Y)}{\sinh(Y)}\right] + \frac{p_0\mu e^{-\lambda t}}{(1-\lambda E)Y}\left[\frac{\cosh(Y)-1}{\sinh(Y)}\right] + \\
&+ 2\mu p_0 \sum_{r=0}^{\infty}[(-1)^r - 1]\left[\frac{e^{\alpha_1 t}(1+E\alpha_1)(1+w\alpha_1)^2}{\delta_1(\alpha_1+\lambda)} + \frac{e^{\alpha_2 t}(1+E\alpha_2)(1+w\alpha_2)^2}{\delta_2(\alpha_2+\lambda)}\right];
\end{aligned}$$

$$\begin{aligned}
D_1 &= 2\pi^2\mu\sum_{r=0}^{\infty} r^2[u_0(-1)^r - u_1]\times \\
&\times \left[\frac{e^{\alpha_1 t}(1+\alpha_1 E)^2(1+\alpha_1 w)^2}{\delta_1(\alpha_1+\lambda)} + \frac{e^{\alpha_2 t}(1+\alpha_2 E)^2(1+\alpha_2 w)^2}{\delta_2(\alpha_2+\lambda)}\right] + \\
&+ Y\mu e^{-\lambda t}\left[\frac{u_1\cosh(Y) - u_0}{\sinh(Y)}\right] + \frac{p_0\mu e^{-\lambda t}}{(1-\lambda E)Y}\left[\frac{1-\cosh(Y)}{\sinh(Y)}\right] + \\
&+ 2\mu p_0 \sum_{r=0}^{\infty}[1-(-1)^r]\left[\frac{e^{\alpha_1 t}(1+E\alpha_1)(1+w\alpha_1)^2}{\delta_1(\alpha_1+\lambda)} + \frac{e^{\alpha_2 t}(1+E\alpha_2)(1+w\alpha_2)^2}{\delta_2(\alpha_2+\lambda)}\right].
\end{aligned}$$

CASE 3. Motion for a finite time. This case considers the motion of the plates and the pressure gradient get ceased after a finite time, Hence it can be taken as

$$\begin{aligned}
f(t) &= u_0[H(t) - H(t-T)] \quad \text{at } b = 0, \\
g(t) &= u_1[H(t) - H(t-T)] \quad \text{at } b = 1, \\
P(t) &= p_0[H(t) - H(t-T)] \quad \lambda > 0,
\end{aligned}$$

where $H(t)$ is the Heaviside unit step function. For this case the expressions for velocities of both fluid and dust phase are obtained as

$$\begin{aligned}
u_s &= 2\pi\sum_{r=0}^{\infty} r[u_0 - u_1(-1)^r]\sin(r\pi b)\times \\
&\times \left[\frac{e^{\alpha_1 t}(1+\alpha_1 E)^2(1+\alpha_1 w)^2(1-e^{-\alpha_1 T})}{\delta_1\alpha_1} + \right. \\
&+ \left.\frac{e^{\alpha_2 t}(1+\alpha_2 E)^2(1+\alpha_2 w)^2(1-e^{-\alpha_2 T})}{\delta_2\alpha_2}\right] + \\
&+ \frac{2p_0}{\pi}\sum_{r=0}^{\infty}\frac{[(-1)^r - 1]}{r}\sin(r\pi b)\left[\frac{e^{\alpha_1 t}(1+E\alpha_1)(1+w\alpha_1)^2(1-e^{-\alpha_1 T})}{\delta_1\alpha_1} + \right. \\
&+ \left.\frac{e^{\alpha_2 t}(1+E\alpha_2)(1+w\alpha_2)^2(1-e^{-\alpha_2 T})}{\delta_2\alpha_2}\right];
\end{aligned}$$

$$v_s = 2\pi\sum_{r=0}^{\infty} r([u_0 - u_1(-1)^r]\sin(r\pi b)\times$$

$$
\begin{aligned}
&\times \left[\frac{e^{\alpha_1 t}(1+\alpha_1 E)^2(1+\alpha_1 w)(1-e^{-\alpha_1 T})}{\delta_1\alpha_1}+\right.\\
&+ \left.\frac{e^{\alpha_2 t}(1+\alpha_2 E)^2(1+\alpha_2 w)(1-e^{-\alpha_2 T})}{\delta_2\alpha_2}\right]+\\
&+ \frac{2p_0}{\pi}\sum_{r=0}^{\infty}\frac{[(-1)^r-1]}{r}\sin(r\pi b)\left[\frac{e^{\alpha_1 t}(1+E\alpha_1)(1+w\alpha_1)(1-e^{-\alpha_1 T})}{\delta_1\alpha_1}+\right.\\
&+ \left.\frac{e^{\alpha_2 t}(1+E\alpha_2)(1+w\alpha_2)(1-e^{-\alpha_2 T})}{\delta_2\alpha_2}\right].
\end{aligned}
$$

Shear stress (Skin friction): The shear stress at the plates $b=0$ and $b=1$ for this flow are, respectively, given by:

$$
\begin{aligned}
D_0 &= 2\mu\pi^2\sum_{r=0}^{\infty}r^2[u_0-u_1(-1)^r]\times\\
&\times \left[\frac{e^{\alpha_1 t}(1+\alpha_1 E)^2(1+\alpha_1 w)^2(1-e^{-\alpha_1 T})}{\delta_1\alpha_1}+\right.\\
&+ \left.\frac{e^{\alpha_2 t}(1+\alpha_2 E)^2(1+\alpha_2 w)^2(1-e^{-\alpha_2 T})}{\delta_2\alpha_2}\right]+\\
&+ 2\mu p_0\sum_{r=0}^{\infty}[(-1)^r-1]\left[\frac{e^{\alpha_1 t}(1+E\alpha_1)(1+w\alpha_1)^2(1-e^{-\alpha_1 T})}{\delta_1\alpha_1}+\right.\\
&+ \left.\frac{e^{\alpha_2 t}(1+E\alpha_2)(1+w\alpha_2)^2(1-e^{-\alpha_2 T})}{\delta_2\alpha_2}\right];\\
D_1 &= 2\mu\pi^2\sum_{r=0}^{\infty}r^2[u_0(-1)^r-u_1]\times\\
&\times \left[\frac{e^{\alpha_1 t}(1+\alpha_1 E)^2(1+\alpha_1 w)^2(1-e^{-\alpha_1 T})}{\delta_1\alpha_1}+\right.\\
&+ \left.\frac{e^{\alpha_2 t}(1+\alpha_2 E)^2(1+\alpha_2 w)^2(1-e^{-\alpha_2 T})}{\delta_2\alpha_2}\right]+\\
&+ 2\mu p_0\sum_{r=0}^{\infty}[1-(-1)^r]\left[\frac{e^{\alpha_1 t}(1+E\alpha_1)(1+w\alpha_1)^2(1-e^{-\alpha_1 T})}{\delta_1\alpha_1}+\right.\\
&+ \left.\frac{e^{\alpha_2 t}(1+E\alpha_2)(1+w\alpha_2)^2(1-e^{-\alpha_2 T})}{\delta_2\alpha_2}\right],
\end{aligned}
$$

where

$$
\begin{aligned}
a_1 &= \left[(C_r+r^2\pi^2)E+1\right]w, \quad b_1 = C_r(w+E)+1+M^2w+l+r^2\pi^2(w+E);\\
c_1 &= C_r+M^2+r^2\pi^2, \quad \alpha_1=\frac{-b1+\sqrt{b_1^2-4a_1c_1}}{2a_1}, \quad \alpha_2=\frac{-b1-\sqrt{b_1^2-4a_1c_1}}{2a_1};\\
Y &= \sqrt{\frac{C_r(1-E\lambda)(1-\lambda w)+(M^2-\lambda)(1-\lambda w)-l\lambda}{(1-E\lambda)(1-\lambda w)}}, \quad X=C_r+M^2;
\end{aligned}
$$

$$
\begin{aligned}
\delta_1 &= (1 - M^2 E)(1 + \alpha_1 w)^2 + l(1 - \alpha_1^2 E w); \\
\delta_2 &= (1 - M^2 E)(1 + \alpha_2 w)^2 + l(1 - \alpha_2^2 E w).
\end{aligned}
$$

3 Conclusions

Figures 3 to 5 show the parabolic nature of velocity profiles for the fluid and dust particles for all three cases.

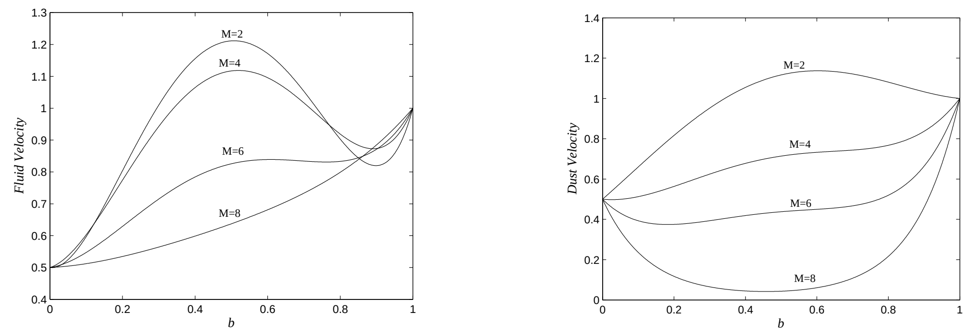


Figure 3. Variation of fluid and dust phase velocity with b (for Case 1)

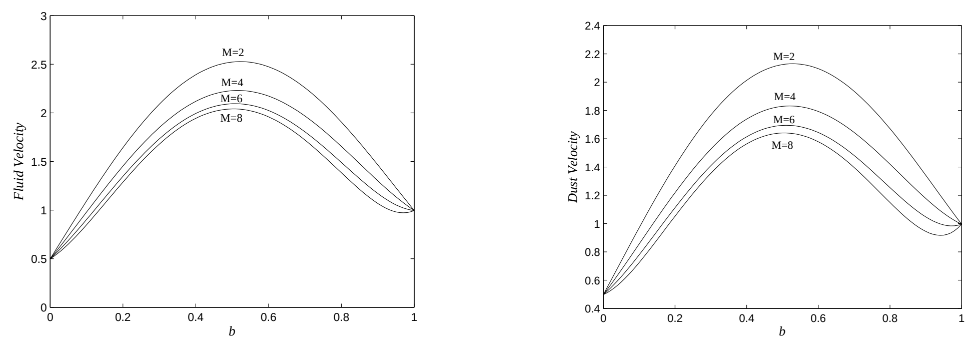
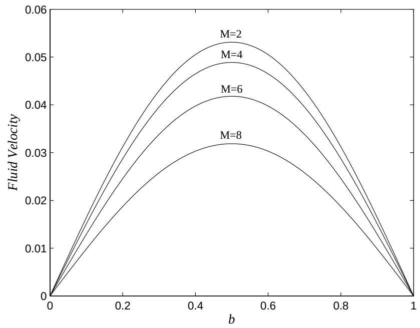


Figure 4. Variation of fluid and dust phase velocity with b (for Case 2)

According to Frenet approximation of a curve in the osculating plane the path of the curve near origin is parabolic. Hence the results obtained here are analogous to [3]. It is concluded that the velocity of fluid particles is parallel to velocity of dust particles. Also it is evident from the graphs that, as we increase the strength of the magnetic field, it has an appreciable effect on the velocities of fluid and dust particles. Further one can observe that if the magnetic field is zero then the results are in agreement with the plane Couette flow. The velocities for fluid and dust particles decreases for large values of t. We observe that if the dust is very fine then the velocities of both fluid and dust particles will be the same.



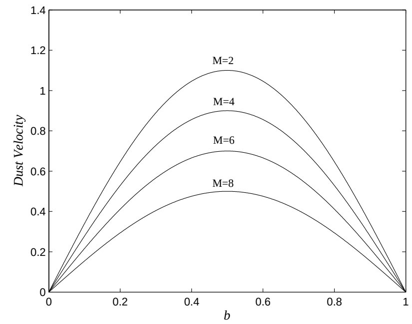


Figure 5. Variation of fluid and dust phase velocity with b (for Case 3)

References

[1] Bagewadi C. S, Gireesha B. J. *A study of two-dimensional steady dusty fluid flow under varying temperature.* Int. Journal of Appl. Mech. and Eng., 2004, **09**, 647–653.

[2] Bagewadi C. S, Gireesha B. J. *A study of two-dimensional unsteady dusty fluid flow under varying pressure gradient.* Tensor. N.S., 2003, **64**, 232–240.

[3] Barret O' Nell. *Elementary Differential Geometry.* Academic Press, New York–London, 1966.

[4] Baral M. C. *Plane parallel flow of conducting dusty gas.* Jour. Phys. Soc. of Japan, 1968, **25**, 1701–1702.

[5] Datta N, Dalal D. C. *Pulsatile flow of heat transfer of a dusty fluid through an infinitly long annular pipe.* Int. J. Multiphase flow, 1995, **21(3)**, 515–528.

[6] Gireesha B. J., Bagewadi C. S., Prasannakumara B. C. *Flow of unsteady dusty fluid between two parallel plates under constant pressure gradient.* Tensor.N.S., 2007, **68**, 1701–1702.

[7] Gireesha B. J., Bagewadi C. S., Prasannakumara B. C. *Unsteady dusty fluid Flow through Rectangular Channel in Frenet Frame Field System.* Int. Jour. Pure and Appl. Math., 2007, **34(4)**, 525–535.

[8] Gireesha B. J., Bagewadi C. S., Prasannakumara B. C. *A study of unsteady dusty gas flow in Frenet Frame Field.* Indian Journal Pure Appl. Math., 2000, **31**, 1405–1420.

[9] Girish Kumar, Chaudhary R. K. S., Singh K. K. *Unsteady flow of conducting dusty visco-elastic liquid through a channel.* Proc. Nat. Acad. Sci. India., 1990, **60(A)**, IV, 393–400.

[10] Ghosh N. C., Ghosh B. C., Debnath L. *The hydromagnetic flow of a dusty visco-elastic fluid between two infinite parallel plates.* Comp. Math. App., 2000, **39**, 103–116.

[11] Indrasena. *Steady rotating hydrodynamic-flows.* Tensor, N.S., 1978, **32**, 350–354.

[12] Kanwal R. P. *Variation of flow quantities along streamlines, principal normals and bi-normals in three-dimensional gas flow.* J.Math., 1957, **6**, 621–628.

[13] Liu J. T. C. *Flow induced by an oscillating infinite plat plate in a dusty gas* Phys. Fluids, 1966, **9**, 1716–1720.

[14] Lokenath Debnath, Ghosh A. K. *Unsteady hydromagnetic flows of a study fluid between two oscillating plates.* Journal of Appl. Scientific Research, 1988, **45**, 353–365.

[15] Michael D. H, Miller D. A. *Plane parallel flow of a dusty gas.* Mathematika, 1966, **13**, 97–109.

[16] Nabil T. M., EL-Dabe., Salwa M. G., EL-Mohandis. *Effect of couple stresses on pulsatile hydromagnetic poiseuille flow.* Fluid Dynamic Research., 1995, **15**, 313–324.

[17] Purushotham G, Indrasena. *On intrinsic properties of steady gas flows.* Appl.Sci. Res., 1965, **A15**, 196–202.

[18] Saffman P. G. *On the stability of laminar flow of a dusty gas.* Journal of Fluid Mechanics, 1962, **13**, 120–128.

[19] Truesdell C. *Intrinsic equations of spatial gas flows.* Z. Angew. Math. Mech., 1960, **40**, 9–14.

Received September 21, 2009

B. J. Gireesha, C. S. Vishalakshi, C. S. Bagewadi
Department of P.G. Studies and Research in Mathematics
Kuvempu University, Shankaraghatta-577451
Shimoga, Karnataka, India

E-mail: *bjgireesu@rediffmail.com*
vishalasen@gmail.com
prof_bagewadi@yahoo.co.in

T. Nirmala
Department of Mathematics
JNN College of Engineering, Shimoga
Karnataka, India

E-mail: *dr.nirmala29@rediffmail.com*