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Some extensions of the Bühlmann-Straub credibility formulae

Virginia Atanasiu

Abstract. The paper presents some extensions of the Bühlmann-Straub credibility model. In the sequel we describe covariance structures leading to credibility formulae of the updating type, where the new credibility adjusted premium can be computed as a weighted average of the premium quoted in the previous period and the claims in this period. The credibility formula of the updating type is introduced for a wider class of models from the credibility theory, where the risk parameter does not remain the same ever time, and its properties are studied. Also, the expected values (the means) and credibility formulae of the updating type are emphasized. Finally we establish an application which shows that these formulae are attractive from practical point of view, because easy recursive formulae for the computation of the credibility weights (factors) from the Bühlmann-Straub model, can be derived.

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1 Introduction

In most models considered in the credibility theory we assume that the risk parameter remains the same over time. If this is not the case, one considers recursive procedures, the formulae of the *updating type*. These are closely related to the theory of Kalman filtering, where it is assumed that the parameters in a linear model themselves arise from a linear process. Because in some models, the covariances between claim sizes are such that credibility formulae arise of the *updating type*, expressing the premium as a mixture of the claims and the credibility premium of the previous observation period, the article presents these formulae and gives an application which characterizes expected values and covariances leading to credibility formulae of the *updating type*. The examples considered show special cases of credibility formulae of the updating type. Finally, is presented an application which shows that there are easy recursive formulae for the computation of the credibility weights from the Bühlmann-Straub model.

2 Theory

One of the Bühlmann-Straub assumptions is that (for this model, each contract $j = 1, \ldots, k$ of the portfolio is the average of a group of contracts, where the weight (size) $w_{j1}, \ldots, w_{jt}$ of the group j is now changing in time; we assume that all contracts have common expectation of the claim size as a function of the risk parameter θ; in addition, apart from the weighting factor w, the variance is also the same function of the risk parameter; these assumptions express the common characteristics of the risk under consideration; so the Bühlmann-Straub assumptions can be formulated as follows:

($\mathbf{BS}_1$): $E[X_{jq}|\theta_j] = \mu(\theta_j)$, $j = 1, \ldots, k$, $q = 1, \ldots, t$; $Var[X_{jr}|\theta_j] = \sigma^2(\theta_j)/w_{jr}$, $r = 1, \ldots, t$, $j = 1, \ldots, k$, where all $w_{jr} > 0$; $Cov[X_{jr}, X_{jq}|\theta_j] = 0$, $j = 1, \ldots, k$, $r, q = 1, \ldots, t$, $r \neq q$;

($\mathbf{BS}_2$): the contracts $j = 1, \ldots, k$ (i.e. the couples $(\theta_j, \underline{X}_j)$) are independent; the variables $\theta_1, \ldots, \theta_k$ are identically distributed; the observations X_{jr} have finite variance), conditionally given the risk parameter θ_j, claim sizes in different time periods are uncorrelated, that is: $Cov(X_{jr}, X_{jr'}|\theta_j) = 0$, $\forall r, r' = \overline{1,t}$, $r < r'$. The obvious advantage of this assumption is that only two parameters:

$$a \overset{def}{=} Var[\mu(\theta_j)] \overset{def}{=} Var[E(X_{jr}|\theta_j)] \quad \text{and} \quad s^2 = E[\sigma^2(\theta_j)] \overset{def}{=} E[Var(X_{jr}|\theta_j)],$$

$(r = \overline{1,t})$ have to be estimated to determine the whole covariance matrix $Cov[\underline{X}_j]$, because:

$$Cov[\underline{X}_j] \overset{def}{=} [Cov(X_{jr}, X_{jr'})]_{\substack{r,r'=\overline{1,t} \\ r<r'}} =$$

$$= \begin{pmatrix} Cov(X_{j1}, X_{j1}) & Cov(X_{j1}, X_{j2}) & \ldots & Cov(X_{j1}, X_{jt}) \\ Cov(X_{j1}, X_{j2}) & Cov(X_{j2}, X_{j2}) & \ldots & Cov(X_{j2}, X_{jt}) \\ \vdots & \vdots & \vdots & \vdots \\ Cov(X_{j1}, X_{jt}) & Cov(X_{j2}, X_{jt}) & \ldots & Cov(X_{jt}, X_{jt}) \end{pmatrix}$$

But

$$Cov(X_{jr}, X_{jr}) = E[Cov(X_{jr}, X_{jr}|\theta_j)] + Cov[E(X_{jr}|\theta_j), E(X_{jr}|\theta_j)] =$$
$$= E[Var(X_{jr}|\theta_j)] + Cov[\mu(\theta_j), \mu(\theta_j)] = s^2 + Var[\mu(\theta_j)] = s^2 + a, \quad \forall r = \overline{1,t}$$

and

$$Cov(X_{jr}, X_{jr'}) = E[Cov(X_{jr}, X_{jr'}|\theta_j)] + Cov[E(X_{jr}|\theta_j), E(X_{jr'}|\theta_j)] =$$
$$= E(0) + Cov[\mu(\theta_j), \mu(\theta_j)] = 0 + Var[\mu(\theta_j)] = 0 + a = a, \quad \forall r, r' = \overline{1,t},\ r < r',$$

such that we get

$$Cov[\underline{X}_j] = \begin{pmatrix} s^2 + a & a & \ldots & a \\ a & s^2 + a & \ldots & a \\ \vdots & \vdots & \vdots & \vdots \\ a & a & \ldots & s^2 + a \end{pmatrix},$$

where $\underline{X}'_j = (X_{j1}, X_{j2}, \ldots, X_{jt})$ with $j = \overline{1,k}$. But in practice it is quite conceivable that these claim sizes are correlated, such that estimates for their covariances have to be given. For the situation of one contract j to be embedded in a collective of contracts, the classical credibility results have the intuitively appealing form:

$$M^a_{t+1} = (1 - z_j)M_0 + z_j M_j,$$

expressing the premium for contract j and period $(t+1)$ as a mixture of collective and individual experience. In some models the co-variances between the claims sizes are such that credibility formulae arise of the updating type, expressing the premium as a mixture of the claims and the credibility premium of the previous period. These formulae are also attractive because easy recursive formulae for the credibility factors can be derived.

Definition 1 (*Credibility formulae of the updating type)***.** A linear credibility formula is said to be of the *updating type* if there is a sequence $z_1, z_2, \ldots$ of real numbers such that:

$$M^a_{t+1} = (1 - z_t)M^a_t + z_t X_t, \quad t = 1, 2, \ldots \tag{1}$$

with M^a_t the linearized credibility premium for X_t given $X_1, X_2, \ldots, X_{t-1}$.

Remark. A condition equivalent to (1) is that:

$$M^a_{t+1} - M^a_t = z_t(X_t - M^a_t),$$

which shows that the premium adjustment from year t to year $(t+1)$ is proportional to the excess, positive or negative, of claims over premiums in year t.

3 Results and discussion

The following *application* characterizes *expected values* and *covariances* leading to *credibility formulae of the updating type.*

Application 1 (*Means and covariances leading to credibility formulae of the updating type*). Let the numbers c_{tq}, $q = \overline{1,t}$ denote the weights of the claim experience in year q for the (linearized) credibility premium M^a_t in year t, $t = 1, 2, \ldots$, and c_{t0} the constant term, such that:

$$M^a_{t+1} = c_{t0} + \sum_{q=1}^{t} c_{tq} X_q. \tag{2}$$

Then the credibility formulae M^a_t are of the updating type, if and only if there exists a number m and sequences $a_1, a_2, \ldots$ and $b_1, b_2, \ldots$ with $b_q > 0$ such that for all $q, r = 1, 2, \ldots,$

$$E(X_r) = m, \tag{3}$$

$$Cov(X_r, X_q) = \begin{cases} a_r, \ r < q \quad (r = \overline{1, q-1}), \\ b_r, \ r = q, \\ a_q, \ r > q \quad (r = \overline{q+1, t}). \end{cases} \tag{4}$$

Proof. Using the system of equations:

$$E[\mu(\theta)] - c_0 - \sum_{q=1}^{t} c_q E[X_q] = 0 \tag{5}$$

and

$$Cov[\mu(\theta), X_r] = \sum_{q=1}^{t} c_q Cov[X_r, X_q], \quad r = \overline{1,t} \tag{6}$$

from *the original credibility model of Bühlmann* (see *the observation*, which we end Application 1) determining *the optimal credibility estimator* in the proof of *Bühlmann's optimal credibility estimator*, applied to X_{t+1} rather than $\mu(\theta)$, we see that the weights c_{tq} and the means / covariances must obey the following relations:

$$E[X_{t+1}] = c_{t0} + \sum_{q=1}^{t} c_{tq} E[X_q] \tag{7}$$

and

$$Cov[X_{t+1}, X_r] = \sum_{q=1}^{t} c_{tq} Cov[X_r, X_q], \quad r = 1, t. \tag{8}$$

We write the condition (7) as

$$E[X_{t+1}] = E\left[c_{t0} + \sum_{q=1}^{t} c_{tq} X_q\right],$$

that is (see (2)):

$$E[X_{t+1}] = E[M_{t+1}^a], \quad t = 1, 2, \ldots$$

We have

$$E[M_{t+1}^a] = E[X_{t+1}], \quad t = 1, 2, \ldots \tag{9}$$

Condition (9) expresses that M_{t+1}^a is unbiased. For the 'only if' – part of the application, suppose that the credibility formulae M_t^a are of the updating type. Taking expectation in (1) gives:

$$E[M_{t+1}^a] = (1 - z_t)E[M_t^a] + z_t E[X_t] = (1 - z_t)E[X_t] + z_t E[X_t] = E[X_t], \ t = 1, 2, \ldots$$

So

$$E[M_{t+1}^a] = E[X_t], \quad t = 1, 2, \ldots \tag{10}$$

From (9) and (10) it follows that $E[X_{t+1}] = E[X_t]$ for all t which proves (3). Replacing the $M_t^{a'}s$ in (1) with their definition (2), that is:

$$\begin{aligned} M_{t+1}^a &\overset{(1)}{=} (1 - z_t)\left[c_{t-1,0} + \sum_{q=1}^{t-1} c_{t-1,q} X_q\right] + z_t X_t = \\ &= (1 - z_t)c_{t-1,0} + \sum_{q=1}^{t-1} (1 - z_t) \cdot c_{t-1,q} X_q + z_t X_t \end{aligned} \tag{11}$$

and comparing the coefficients of the $X_q's$ (see (11) and (2)) one gets:

$$c_{t0} + \sum_{q=1}^{t-1} c_{tq}X_q + c_{tt}X_t = (1 - z_t)c_{t-1,0} + \sum_{q=1}^{t-1}(1 - z_t)c_{t-1,q}X_q + z_tX_t.$$

We have

$$\begin{cases} c_{t0} = (1 - z_t)c_{t-1,0}, \\ c_{tq} = (1 - z_t)c_{t-1,q}, \quad q = \overline{1, t-1}, \\ c_{tt} = z_t. \end{cases}$$

So

$$c_{tq} = (1 - z_t)c_{t-1,q}, \quad q = \overline{0, t-1} \tag{12}$$

and

$$c_{tt} = z_t. \tag{13}$$

Inserting (12) in (8) and again applying (8) for $(t - 1)$ one obtains:

$$Cov[X_r, X_{t+1}] \overset{(8)}{=} \sum_{q=1}^{t} c_{tq}Cov[X_q, X_r] = \sum_{q=1}^{t-1} c_{tq}Cov[X_q, X_r] + c_{tt}Cov[X_t, X_r] =$$

$$= \sum_{q=1}^{t-1}(1 - z_t)c_{t-1,q}Cov[X_q, X_r] + z_tCov[X_t, X_r] = z_tCov[X_t, X_r] + (1 - z_t)\times$$

$$\times \sum_{q=1}^{t-1} c_{t-1,q}Cov[X_q, X_r] = z_t \sum_{q=1}^{t-1} c_{t-1,q}Cov[X_q, X_r] + \sum_{q=1}^{t-1} c_{t-1,q}Cov[X_q, X_r]-$$

$$-z_t \sum_{q=1}^{t-1} c_{t-1,q} \cdot Cov[X_q, X_r] = Cov[X_t, X_r] = Cov[X_r, X_t], \text{ for } r = \overline{1, t-1}.$$

Therefore we may write

$$Cov[X_r, X_{t+1}] = Cov[X_r, X_t] = a_r, \quad r = \overline{1, r-1},$$
$$Cov[X_r, X_t] = b_r.$$

For the 'if'- part of the application, assume that (3) and (4) hold. Then to prove (1), we have to show that (12) and (13) hold again. From (2) we get using (8) for each $r = \overline{1, t-1}$:

$$\sum_{q=1}^{t-1} c_{tq}Cov[X_q, X_r] \overset{(8)}{=} Cov[X_r, X_{t+1}] - c_{tt}Cov[X_r, X_t] = a_r - c_{tt}Cov[X_r, X_t] =$$

$$= Cov[X_r, X_t] - c_{tt}Cov[X_r, X_t] = (1 - c_{tt})Cov[X_r, X_t] = (1 - c_{tt}) \sum_{q=1}^{t-1} c_{t-1,q}\times$$

$$\times Cov[X_q, X_r] = \sum_{q=1}^{t-1}(1 - c_{tt}) \cdot c_{t-1,q} \cdot Cov[X_q, X_r].$$

So

$$c_{tq} = (1 - z_t)C_{t-1,q}; \quad q = \overline{1, t-1}, \tag{14}$$

where $z_t \overset{(not)}{=} c_{tt}$.

This formula also holds because of (7):

$$(7) \Leftrightarrow m = c_{t0} + \sum_{q=1}^{t-1} (1 - z_t)c_{t-1,q} \cdot m + z_t \cdot m \Leftrightarrow c_{t0} =$$

$$= (1 - z_t) \cdot m - (1 - z_t) \cdot \left[\sum_{q=1}^{t-1} c_{t-1,q} \right] \cdot m \Leftrightarrow c_{t0} = (1 - z_t) \cdot c_{t-1,0},$$

because from (7) applied to $(t-1)$ we conclude that: $m = c_{t-1,0} + \sum_{q=1}^{t-1} c_{t-1,q} m$, that is: $c_{t-1,0} = m - \left[\sum_{q=1}^{t-1} c_{t-1,q} \right]$, so we conclude that indeed (14) also holds for $t = 0$. Therefore we may write:

$$M_{t+1}^a = c_{t0} + \sum_{q=1}^{t-1} c_{tq} X_q + c_{tt} X_t = (1 - z_t)c_{t-1,0} + \sum_{q=1}^{t-1} (1 - z_t)c_{t-1,q} X_q + c_{tt} X_t =$$

$$= (1 - z_t) \left[c_{t-1,0} + \sum_{q=1}^{t-1} c_{t-1,q} X_q \right] + c_{tt} X_t = (1 - z_t) M_t^a + z_t X_t.$$

$\square$

A covariance matrix such as in (4) can be depicted as follows:

$$Cov[\underline{X}] = (Cov(X_r, X_q))_{r,q=\overline{1,t}} = \begin{pmatrix} b_1 & a_1 & a_1 & \dots & a_1 & \dots & a_1 \\ a_1 & b_2 & a_2 & \dots & a_2 & \dots & a_2 \\ a_1 & a_2 & b_3 & \dots & a_3 & \dots & a_3 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ a_1 & a_2 & a_3 & \dots & b_q & \dots & a_q \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_1 & a_2 & a_3 & \dots & a_q & \dots & b_r \end{pmatrix}.$$

As a *special case*, when $Cov[X_i, X_j] = a + \delta_{ij} s^2$ like in Bühlmann's models we obtain $z_t = at/(at + s^2)$ leading to *uniform* credibility weights:

$$c_{t1} = c_{t2} = \ldots = c_{tt} = at/(at + s^2).$$

Another *special case* of credibility formulae of the updating type arises when $z_t = z$, $t = 1, 2, \ldots$; then one obtains *geometric* credibility weights:

$$\begin{aligned} c_{tq} &= (1 - z_t)c_{t-1,q} = (1 - z)c_{t-1,q} = (1 - z)(1 - z_{t-1})c_{t-2,q} = \\ &= (1 - z)^2 c_{t-2,q} = \ldots = (1 - z)^{t-q} c_{qq} = (1 - z)^{t-q} z_q = (1 - z)^{t-q} z \end{aligned}$$

If the means and covariances are as in Application 1 there are easy recursive formulae for the computation of the credibility weights z_t.

Observation 1 (*The original credibility model of Bühlmann*). In the original credibility model of Bühlmann, we consider one contract with unknown and fixed risk parameter θ, during a period of t years. The yearly claim amounts are noted by $X_1, \ldots, X_t$. The risk parameter θ is supposed to be taken from some structure distribution $U(\cdot)$. It is assumed that, for given $\theta = \theta$, the claims are conditionally independent and identically distributed with known common distribution function $F_{X|\theta}(x, \theta)$. For this model we want to estimate the net premium $\mu(\theta) = E[X_r|\theta = \theta]$, $r = \overline{1, t}$ as well as X_{t+1} for a contract with risk parameter θ.

We present the following result:

Bühlmann's optimal credibility estimator. Suppose $X_1, \ldots, X_t$ are random variables with finite variation, which are, for given $\theta = \theta$, conditionally independent and identically distributed with already known common distribution function $F_{X|\theta}(x, \theta)$. The structure distribution function is $U(\theta) = P[\theta \leq \theta]$. Let D represent the set of non-homogeneous linear combinations $g(\cdot)$ of the observable random variables $X_1, \ldots, X_t$: $g(\underline{X}') = c_0 + c_1X_1 + \ldots + c_tX_t$. Then the solution of the problem: $\underset{g \in D}{\text{Min}}\ E\{[\mu(\theta) - g(X_1, \ldots, X_t)]^2\}$ is: $g(X_1, \ldots, X_t) = M^a = z\overline{Z} + (1 - z)m$, where $\underline{X}' = (X_1, \ldots, X_t)$ is the vector of observations, $z = at/(s^2 + at)$, is the resulting credibility factor, $\overline{X} = \dfrac{1}{t}\displaystyle\sum_{i=1}^{t} X_i$ is the individual estimator, and a, s^2 and m are the structural parameters as defined by the following formulae: $m = E[X_r] = E[\mu(\theta)]$, $r = \overline{1, t}$, $a = Var\{E[X_r|\theta]\} = Var[\mu(\theta)]$, $r = \overline{1, t}$, $\sigma^2(\theta) = Var[X_r|\theta]$, $r = \overline{1, t}$, $s^2 = E\{Var[X_r|\theta]\} = E[\sigma^2(\theta)]$, $r = \overline{1, t}$. If $\mu(\theta)$ is replaced by X_{t+1} in the above minimization problem, exactly the same solution M^a is obtained, since the co-variations with $\underline{X}$ are the same.

Proof. We have to solve the following minimization problem:

$$\underset{c_0, \ldots, c_t}{\text{Min}}\ E\left\{\left[\mu(\theta) - c_0 - \sum_{r=1}^{t} c_r X_r\right]^2\right\}.$$

Since the above problem is the minimum of a positive definite quadratic form, it suffices to find a solution with all partial derivatives equal to zero. Taking the partial derivative with respect to c_0 we get the equation: $E\left[\mu(\theta) - c_0 - \displaystyle\sum_{r=1}^{t} c_r X_r\right] = 0$

(see (5)). Using $m = E[X_r] = E[\mu(\theta)]$, we may solve this equation for c_0 and insert the result in the minimization problem. We get:

$$\underset{c_1,\ldots,c_t}{\text{Min}} \; E\left\{\left[\mu(\theta) - m - \sum_{r=1}^{t} c_r(X_r - m)\right]^2\right\}.$$

Taking the derivative with respect to c_q, $q = 1, \ldots, t$ leads to the equation: $E\left\{-2\left[\mu(\theta) - m - \sum_{r=1}^{t} c_r(X_r - m)\right] \cdot (X_q - m)\right\} = 0$, $q = 1, \ldots, t$. This is equivalent to: $Cov[\mu(\theta), X_q] = \sum_{r=1}^{t} c_r Cov(X_q, X_r)$, $q = 1, \ldots, t$ (see (6)) . Since $Cov(X_q, X_r) = a + \delta_{rq} s^2$ and $Cov[\mu(\theta), X_q] = a$ and since the system of equations is symmetrical in $c_1, \ldots, c_t$ one finds from: $Cov[\mu(\theta), X_q] = \sum_{r=1}^{t} c_r Cov(X_q, X_r)$, $q = 1, \ldots, t$ that: $c_1 = c_2 = \ldots = c_t = a/(s^2 + at)$. Now introducing $z = at/(s^2 + at)$, from $E\left[\mu(\theta) - c_0 - \sum_{r=1}^{t} c_r X_r\right] = 0$ we see that $c_0 = (1 - z) \cdot m$, so M^a is optimal.

Application 2 (*Expressions for credibility weights*). Under the conditions of the previous application, and writing $s_t = b_t - a_t$ the credibility weights z_t can be calculated by means of:

$$\begin{cases} z_1 = a_1/(a_1 + s_1), \\ z_t = (a_t - a_{t-1} + z_{t-1}s_{t-1})/(a_t - a_{t-1} + z_{t-1}s_{t-1}), \ t = 2, 3, \ldots \end{cases}$$

Proof. Equation (12) for $t = q = 1$, together with (8) and (4), gives the expression for z_1:

$$z_1 = c_{11} = \frac{Cov(X_2, X_1)}{Cov(X_1, X_1)} = \frac{a_1}{b_1} = \frac{a_1}{s_1 + a_1}.$$

Equation (8) for $r = t$, together with (12) gives:

$$\begin{aligned} Cov[X_t, X_{t+1}] &= c_{tt} Cov[X_t, X_t] + \sum_{q=1}^{t-1} c_{tq} Cov[X_q, X_t] = \\ &= z_t Var[X_t] + (1 - z_t) \cdot \sum_{q=1}^{t-1} c_{t-1,q} \cdot Cov[X_q, X_t]. \end{aligned} \tag{15}$$

The summation in (15) can be rewritten as:

$$c_{t-1,t-1}Cov[X_{t-1},X_t]+\sum_{q=1}^{t-2}c_{t-1,q}Cov[X_q,X_t]=z_{t-1}Cov[X_{t-1},X_t]+$$
$$+\sum_{q=1}^{t-2}c_{t-1,q}a_q=z_{t-1}Cov[X_{t-1},X_t]+\sum_{q=1}^{t-2}c_{t-1,q}Cov[X_q,X_{t-1}]=$$
$$=z_{t-1}Cov[X_{t-1},X_t]-c_{t-1,t-1}\cdot Cov[X_{t-1},X_{t-1}]+\sum_{q=1}^{t-1}c_{t-1,q}Cov[X_q,X_{t-1}]=$$
$$\overset{(8)}{=}z_{t-1}\{Cov[X_{t-1},X_t]-Var[X_{t-1}]\}+Cov[X_t,X_{t-1}],\ t\geq 2 \tag{16}$$

Inserting (16) in (15) and again because of (4) one gets for (15):

$$a_t=Cov[X_t,X_{t+1}]=z_tb_t+(1-z_t)\{z_{t-1}(a_{t-1}-b_{t-1})+a_{t-1}\}=$$
$$=z_t(s_t+a_t)+(1-z_t)(a_{t-1}-z_{t-1}s_{t-1}),\ t\geq 2.$$

We have

$$a_t=z_t(s_t+a_t)+(1-z_t)(a_{t-1}-z_{t-1}s_{t-1}),$$

that is

$$z_t=(a_t-a_{t-1}+z_{t-1}s_{t-1})/(a_t-a_{t-1}+z_{t-1}s_{t-1}+s_t),$$

□

4 Conclusions

The paper describes covariance structures leading to credibility formulae of the updating type, where the new credibility adjusted premium can be computed as a weighted average of the premium quoted in the previous period and the claims in this period. So, the credibility formulae of the updating type for the credibility factors from the Bühlmann-Straub model can be derived. In other models from the credibility theory, the covariances between the claims sizes are such that credibility formulae arise of the *updating type*, expressing the premium as a mixture of the claims and the credibility premium of the previous observation period.

References

[1] Atanasiu V. *A credibility model.* Economic Computation and Economic Cybernetics Studies and Research 3/1998, XXXII, Academy of Economic Studies, Bucharest, Romania.

[2] De Vylder F., Goovaerts M. J. *Semilinear credibility with several approximating fuctions.* Insurance: Mathematics and Economics, 1985, **4**, 155–162. (Zbl. No. 0167–6687).

[3] Gerber H. U., Jones M. J. *Credibility formulae of the updating type.* "Credibility theory and application", Proceedings of the Berkeley Actuarial Research Conference on credibility, Academic Press, New York, 1975, 89–109.

[4] Gerber H. U. *Credibility for Esscher premiums*. Mitteilungen der VSVM, 1980, **80**, 307–312.

[5] Goovaerts M. J., Kaas R., Van Heerwaarden A. E., Bauwelinckx T. *Effective Actuarial Methods*. Elsevier Science Publishers B. V., 1990, **3**.

[6] Pentikäinen T., Daykin C. D., Pedonen M. *Practical Risk Theory for Actuaries*. Chapman & Hall, 1993.

[7] Sundt B. *An Introduction to Non-Life*, Insurance Mathematics, volume of the "Mannheim Series", 1984, 22–54.

Virginia Atanasiu
Academy of Economic Studies
Department of Mathematics
Calea Dorobanţilor nr. 15-17
Sector 1 Bucharest, 010552, Romania
E-mail: *virginia_atanasiu@yahoo.com*

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