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Academician Vladimir Arnautov – 70th anniversary

Academician Vladimir Arnautov is a Moldavian mathematician and an irrefutable leader of the Moldavian school of topological algebra, which made an important contribution to the topological algebra and to the education of new generations of highly-qualified specialists. In the middle of the summer of 2009 professor Vladimir Arnautov turned 70. This was an excellent opportunity for us, his colleagues and friends, to stop on our own mathematical and personal path and to bring once more into light his life and activity up to this moment.

Vladimir Arnautov was born July 30, 1939 in Bolgrad (Romania, now Ukraine), being the second son (of six children) in a Bulgarian family. His father Ivan Stepanovich Arnautov was a technician at the local communication service. His mother Vera Simionovna Arnautova (Stadnitskaya) was a housewife and her principal occupation was the education of children.

Bolgrad (abbreviated from of Bolgarian town, which was called, until 1818, Tabacu) a little town in the south part of Bassarabia was in the 19th century the residence of the Bassarabian Bulgarians evacuated from Bulgaria who had been subjugated by Ottoman Empire until the 19th century.

In 1956 he successfully finished the local secondary (ten-year) school. The mathematical form and formulas, logical deduction charmed him, and without doubt he decided to continue the mathematical studies. In 1956 he started his university education at the Faculty of Physics and Mathematics (now Faculty of Mathematics and Computer Sciences) of Chişinău State University (now State University of Moldova).

During the student days he was influenced by the talented teachers and schoolarls as Professors C. Sibirschi, A. Zamorzaev, B. Shcherbacov, I. Parovicenco, C. Shchukin and other. Soon he became one of the best students. At the same time he actively contributed to various public organizations. In particular, 1959-1961 he was the Chairman of the Council of the Student Scientific Society of the University. In 1961 V. Arnautov successfully graduated from the Chişinău University. His Master Thesis *"About some classes of completely regular spaces"* was published in the Scientific Notes of the Chişinău University, Mathematics, vol. 1, 1962, p.13-18. His scientific and active public involvement made possible the obtaining of the University Academic Council recommendation for continuing his post-graduate advanced studies in Mathematics at the newly-created Institute of Mathematics and Physics (now Institute of Mathematics and Computer Sciences and Institute of Applied Physics). At the Institute a good fortune has brought him together with the Academician Vladimir Andrunachievici, one of the founders of the Academy of Sciences of Moldova, the first director of the Institute of Mathematics and Physics and the founder of the Moldavian algebraical school. V. Andrunachievici became scientific supervisor of the young scientist.

Academician V. Andrunachievici was one of the best specialists in the abstract theory of radicals. The scientific interest of the supervisor and his own topological knowledge determined the direction of Arnautov's further mathematical investigations: the theory of radicals of the topological rings.

The abstract theory of radicals had already been applied in the theory of topological rings in some works of I. Kaplansky, H. Leptin, D. Zelinsky and other mathematicians. However, they left out of account that these radicals, as a rule, are not closed in the topological rings.

First of all, V. Arnautov proposed the concept of the topological radical.

Let Φ be an associative and commutative topological ring and $\mathcal{K}$ be a class of topological algebras over the topological ring Φ with the following properties:

– if A is an ideal of some algebra $B \in \mathcal{K}$, then $A \in \mathcal{K}$;

– if $B \in K$ and A is a closed ideal of B, then the factor-algebra $B/A \in \mathcal{K}$.

If Φ is the discrete ring of integers $\mathbb{Z}$, then $\mathcal{K}$ is a class of topological rings. We assume that any ideal of the algebra B is a subalgebra of the algebra B.

We say that a radical ρ is defined over the class $\mathcal{K}$ if ρ is a correspondence of $\mathcal{K}$ into $\mathcal{K}$ for which:

1R. $\rho(A)$ is a closed ideal of the algebra $A \in \mathcal{K}$ (it is called the ρ-radical of A).

2R. $\rho(\rho(A)) = \rho(A)$ for any $A \in \mathcal{K}$.

3R. If $A, B \in \mathcal{K}$ and $\varphi : A \to B$ is a continuous homomorphism, then $\varphi(\rho(A)) \subseteq \rho(B)$.

4R. If $A \in \mathcal{K}$, then $\rho(A/\rho(A)) = \{0\}$.

Let ρ be a radical over the given class $\mathcal{K}$ of topological algebras. If $A \in \mathcal{K}$ and $\rho(A) = \{0\}$, then the algebra A is called ρ-semisimple. If $\rho(A) = A$, then A is called a ρ-radical algebra. Thus the factor-algebra $A/\rho(A)$ is the replica of the algebra $A \in \mathcal{K}$ in the class $\mathcal{K}_\rho$ of all ρ-semisimple algebras from $\mathcal{K}$.

If $\mathcal{K}$ is a class of discrete algebras, then each topological radical over the class $\mathcal{K}$ is an abstract radical and vice versa.

Consequently, many facts and notions of the abstract theory are their analogues for the topological case. However:

– for some abstracts radicals there exist a more than one topological variants;

– in some classes of topological algebras distinct radicals can coincide.

Therefore, the topological theory of radicals is a new fundamental area of mathematics with new concepts and techniques which have important and fruitful applications in other branches of algebra, topology and mathematics, in general. V. Arnautov has introduced a number of new notions and gave a number of original and deep results. The following problem was arisen one of first.

Problem 1. *Find the radical properties of distinct classes of topological rings.*

A property $\mathcal{P}$ is a radical property in the class $\mathcal{K}$ of topological rings if there exists a radical ρ such that $\rho(A) = A$ if and only if $A \in \mathcal{K}$ is a topological ring with the property $\mathcal{P}$.

One of the first remarkable results was: the property of a topological ring to contain a non-zero topological nilpotent ideal in any non-trivial continuous homomorphic image is a radical property. This radical was named the Baer-McCoy or the Baer radical. That radical was comprehensively studied in the thesis *"On the theory of topological rings"* for a doctor's degree which was defended in March 1965 at the Institute of Mathematics of the Siberian Branch of the Academy of Sciences of USSR.

For a long time that property was the only non-trivial radical property in the class of all topological rings. Then various radical properties were found in the classe of all topological rings, including:

– the property of a topological ring to be a locally nilpotent ring generates the Levitzky radical;

– to be a nil ring is a radical property which generates the Koethe radical;

– the property to be a quasi-regular ring is a radical property and generates the Jacobson radical;

– the property to be a Boolean ring generates the Boolean radical, etc.

The analogues of some topological radicals in the concrete classes of topological rings were well-known earlier. V. Arnautov has constructed them in the class of all topological rings.

The coincidence of some topological radicals in special classes of spaces was an unexpected fact. For example, in the class of compact rings the topological radicals of Baer-McCoy and Jacobson coincide with the topological quasi-regular radical. This fact confirms the initial assumption of interdependence of algebraical properties of topological radicals and topological properties of classes of topological rings. Moreover, as applications the structural descriptions of some classes of topological rings were obtained.

The methods of the theory of radicals are important for the study of the algebraical properties of the completion $\breve{R}$ of a topological ring R. V. Andrunachievici and V. Arnautov established that for any topological ring R the following assertions are equivalent:

– in the topological ring R any one-sided ideal is trivial and only zero is a generalized zero divisor;

– R is a ring with unity and any element of R is invertible in $\breve{R}$.

The next curious fact immediately follows from this result: a locally compact topological ring without non-trivial one-sided ideals is a topological field.

Some fundamental investigations of V. Arnautov were made jointly with his scientific tutor, Academician V. Andrunachievici, others with his gifted post-graduate student, Mihail Ursul, who then created new interesting directions of research and educated new generations of highly-qualified mathematicians. Interesting results about topological radicals where proved by Mihail Vodinchar and Trinh Dang Khoi, when they were post-graduate students of Professor V. Arnautov.

A voluminous outline of the investigations of the topological radicals and of the radical properties was presented in the review: V. I. Arnautov, *The Theory of Radicals of Topological Rings*, Mathematica Japonica 47:3(1998), 439–544.

In 1946 Professor A. A. Markov, in one of his articles, arose the next question: is it true that on each infinite group there exists a non-discrete Hausdorff topology?

Markov's question generates in the more general aspect the following problems.

Problem 2. *Under which conditions on a given universal algebra A there exist some (only one, two) topologies with the given property?*

Problem 3. *Find algebraical properties of a universal algebra A which can be characterized by the properties of the lattice $LT(A)$ of the topologies on the algebra A. In particular, under which conditions the lattice $LT(A)$ has coatoms?*

Problem 4. *Let A be a subalgebra of a universal algebra B and $\mathcal{T}$ be a Hausdorff topology on A. Under which conditions on B there exists a Hausdorff topology $\mathcal{T}'$ such that A is a topological subalgebra of B?*

The Problems 3 and 4 are more difficult if on the space of operations some topology is fixed and, in particular, for modules or for algebras over some topological ring Φ.

The history of solution of Markov's Problem for groups is long and surprising. First, it was established that the positive answer for commutative groups follows from the theory of characters (A. Kertesz and T. Szele, 1956). Then, in 1977, S. Shelah, using forcing method, constructed an uncountable group without non-discrete topologies. In 1980 A. Olšanskii observed that the infinite countable group $A(m,n)/C^m$, where $A(m,n)$ is the infinite countable group constructed by S. Adian in 1975, has not non-discrete Hausdorff topologies.

Markov's Problem for rings was solved by V. Arnautov by 1972. He obtained the following valuable results:

1. On any infinite countable ring there exist non-discrete Hausdorff topologies.

2. On each infinite commutative associative ring there exists a Hausdorff non-discrete topology.

3. There exists an infinite ring on which only the anti-discrete topology $\{0, A\}$ is non-discrete.

Obviously, the Kertesz-Szele's Theorem for commutative groups follows from the Arnautov's results.

For construction on rings the Hausdorff non-discrete topologies with distinct proprieties V. Arnautov developed interesting combinatorial methods. These facts and some structural properties of topological rings constitute the content of his doctor's science degree thesis defended in 1972 at the Institute of Mathematics of the Siberian Branch of the Academy of Sciences of USSR.

The topology constructed by the characters is totally bounded or metrizable and locally totally bounded, but the topologies proposed by V. Arnautov are metrizable only for countable rings and, as a rule, not locally totally bounded. This fact arises the next major problems.

Problem 5. *Under which conditions on an algebra there exists some compact Hausdorff topology?*

Problem 6. *Under which conditions on an algebra there exits a topology generated by a linear ordering?*

Problem 7. *Let τ be an infinite cardinal. Under which conditions on an algebra there exists a Hausdorff non-discrete P_τ-topology?*

The topology is called a P_τ-topology, where τ is an infinite cardinal, if the intersection of τ open sets is an open set.

The Problems 5 - 7 relate to the problem.

Problem 8. *Find the interdependence of algebraical properties of algebra A and properties of topologies on the algebra A.*

Under the guidance of Professor V. Arnautov remarkable results concerning the problems 5 - 8 were obtained by Mihail Ursul, Pavel Chircu, Victor Vizitiu, Elena Marin, Valeriu Popa, Kirill Filippov, Dilfuza Yunusova, Anatolie Topală.

The topological free algebras are important algebraical objects. Interesting results about the properties of topological free rings in concrete classes of topological rings were proposed by V.Arnautov and his post-graduate students Ştefan Alexei, Reli Calistru and Stelian Dumitrashcu (the later was a post-graduate student of M. Cioban too).

A lasting, active and efficient collaboration has been established between Professor V. Arnautov and the Moskow mathematicians Professors Alexander V. Mikhalev and Sergei T. Glavatsky.

In connection with Problem 4 they examined the next two problems.

Problem 9. *Under which conditions on the semigroup ring the topology of the ring and the topology of the semigroup can be simultaneously extended?*

Problem 10. *Under which condition the topology of the ring admits some extension over its ring of quotients?*

The following monographs constitute the final result of this collaboration:

1. Arnautov V. I, Vodinchar M. I., Mikhalev A. V, *Introduction to the Theory of Topological Rings and Modules,* – Ştiinţa: Chişinău, 1981, 175 p. (In Russian).

2. Arnautov V. I, Vodinchar M. I., Glavatsky S. T., Mikhalev A. V, *Constructions of the Topological Rings and Modules,* – Ştiinţa: Chişinău, 1988, 168 p. (In Russian).

3. Arnautov V. I, Glavatsky S. T., Mikhalev A. V, *Introduction to the Theory of Topological Rings and Modules,* – Marcel Dekker: New York-Basel, 1996, 502 p.

In the mentioned books the results of the authors of books and of their former students constitute a significent part and the impact of this works on the development of Topological Algebra is considerable.

The last scientific researches of Professor V. Arnautov are dedicated to the investigation of the lattice of topologies of groups and rings. One general method of construction of neighbour pairs of topologies was found. Two topologies on the algebra A form a neighbour pair if between them other topologies do not exist. In particular, any coatom and the maximal topology of the lattice $LT(A)$ form a neighbour pair. If A is an algebra over a discrete ring or a group, then the maximal topology of the lattice $LT(A)$ is discrete. Mentioned facts confirm the importance of this concept and method. Professor V. Arnautov proved that the lattice $LT(A)$, where A is a linear space over field of reals, does not contain Hausdorff coatoms. In this case the maximal element of the lattice $LT(A)$ is not discrete.

Academician V. Arnautov has published more than 160 research papers and 3 monographs. Having a good prestige in the world of mathematics, Professor Vladimir Arnautov has been invited at more then 40 prestigious international conferences in Algebra and Topology (Russia, Belarus, Ukraine, Poland, Austria, etc). He passionately and skillfully has organized in collaboration with colleagues several (about 20) national and international conferences on Algebra, Topology and Topological Algebra. For instance, in 1984 and 1986 Professor V. Arnautov in collaboration with Professors A. Arhangel'skii, M. Cioban and A. Mikhalev organized in Tiraspol the well-known workshops "Topological Algebra" which had a considerable influence on the development of Topological Algebra and General Topology. In particular, these workshops have established close contacts between many algebraical and topological schools of the former USSR (from Moscow, Saint Petersburg, Novosibirsk, Tomsk, Yekaterinburg, Ukraine, Byelorussia, Moldova, Estonia, etc).

Thus in 1961 Professor V. Arnautov steady and full of energy began the scholarly activity and didactic carrier. During 1964–1967 and 1967–1970 he was respectively scientific worker and superior scientific worker at the Institute of Mathematics and Computer Sciences of the Academy of Sciences of Moldova (IMCS ASM). Between 1970–1978 he was the head of the laboratory of IMCS ASM. In 1978 he becomes the full Professor. Between 1978–1988 and 1990–1993 he was the deputy director of IMI ASM for research problems. In 1984 he was elected the corresponding member of the Academy of Sciences of Moldova. In the period 1990–1993 he was the associate member of the Presidium of the Academy of Sciences of Moldova. Between 1993–1999 he was the principal scientific worker and since 1999 he is the head of Department of Theoretical Mathematics of IMCS of ASM. In 2007 Professor V. Arnautov was elected the full member of the Academy of Sciences of Moldova, the highest scientific forum of the Republic of Moldova and the highest recognition which a scholar may receive in the native country.

The contribution of Professor V. Arnautov to the education of new generations of highly-qualified mathematicians is enormous. He has trained 13 doctors of sciences and Ph.D's. To his colleagues and former students he has an inspiration not only as a mathematician, but as a human being.

Professor V. Arnautov is an active member of many state communities and commissions. He was a member of the Commission of Experts and of the Scientific Council of the Higher Certifying Committee of USSR for the academic degree and rank. During 1973–1977 he was a Chairman of the Council of Young Researchers of the Republic of Moldova. Now he is a Chairman of the Council and a member of the Experts Commission of CNAA (National Council for Accreditation and Attestation).

Professor Vladimir Arnautov was awarded the Prize of the Moldovian Komsomol (1972) for the young researchers, the prize "Academician Constantin Sibirschi" (2001). He is a "Honoured scientist of the Republic of Moldova" and is awarded with the "Honour Diploma of the Presidium of the Supreme Soviet of MSSR", medal "Distinction in Labour" and order "Glory of Labour".

Professor V. Arnautov is a member of the Moldavian Mathematics Society, American Mathematical Society and of the Editorial Board of the Bulletin of the Academy of Science of Moldova, Mathematics.

At the age of 70, full of vigor and optimism, the academician Vladimir Arnautov is a prominent personality and continues an active presence in the academic community of the Republic of Moldova. We wish him a good health, prosperity and new accomplishments in his prodigious scientific and didactic activities: "Happy Birthday to You, Happy returns of the Day".

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