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About the solvability of systems of integral equations with different degrees of differences in kernels

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Abstract. The work defines the conditions of solvability of one system of integral convolutional equations with different degrees of differences in kernels. Such the system of the integral convolutional equations has not been studied earlier, and it turned out that all the methods used for the investigation of such a system with the help of Riemann boundary problem at the real axis can not be applied there. The investigation of such a type of the system of equations is based on the investigation of the equivalent system of singular integral equations with the Cauchy type kernels at the real axis. It is determined that the system of the equations is not a Noetherian one. Besides, we have shown the number of the linear independent solutions of the homogeneous system of equations and the number of conditions of solvability for the system of heterogeneous equations. The general form of these conditions is also shown and the spaces of solutions of that system of equations are determined. Thus the system of the convolutional equations that hasn't been studied earlier is presented in that work and the theory of its solvability is built here. So some new and interesting theoretical results are got in the paper.

Mathematics subject classification: 45E05, 45E10.
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The present work is devoted to determining conditions of solvability and some properties of solutions of the next system of Winer-Hoph's type integral equations

$$P_1(x)\varphi(x) + \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} k(x-t)P_2(t)\varphi(t)\,dt = h(x), \quad x > 0, \tag{1}$$

where $h(x) \in \mathbf{L}_2$ is a known vector-function which is an n-dimensional one, $k(x) \in \mathbf{L}$, is a known matrix-function, which is an n-dimensional one, too. $\varphi(x)$ is an unknown vector-function and it is an n-dimensional one.

$$P_1(x) = \sum_{k=0}^{m} a_k x^k, \quad P_2(x) = \sum_{\nu=0}^{s} b_\nu x^\nu$$

are the known polynomials with the degrees m, s respectively. We will note that the belonging of vector-functions and matrix-functions to any space means their elements' belonging to it. The norms of vector-functions and matrix-functions are compatible with each other.

Let $D^+ = \{z \in \mathbf{C} : \mathbf{Imz} > \mathbf{0}\}$ be an upper half plane and $D^- = \{z \in \mathbf{C} : \mathbf{Imz} < \mathbf{0}\}$ be a lower half plane of the complex plane $\mathbf{C}$; $\mathbf{R}$ is the real axis. According to the properties of Fourier transformation [2, p. 16] the investigation of the system of equations (1) reduces to the investigation of the following matrix differential boundary problem

$$\sum_{k=0}^{m}(-1)^k A_k \Phi^{+(k)}(x) + \sum_{\nu=0}^{s}(-1)^\nu B_\nu K(x)\Phi^{+(\nu)}(x) = H(x) + \Phi^-(x), \quad x \in \mathbf{R}.$$

Here $K(x), H(x)$ are the Fourier transformations of the matrix-function $k(x)$ and the vector-function $h(x)$ accordingly. $\Phi^{+(p)}(x)$, $\Phi^-(x)$ are the boundary values at $\mathbf{R}$ of the unknown vector-functions $\Phi^{+(p)}(z)$ and $\Phi^-(z)$ accordingly, where $\Phi^{+(p)}(z)$, $\Phi^-(z)$ are unknown vector-functions, which are analytical in the domains D^+ and D^- accordingly. Let's rewrite this differential boundary problem as the following one

$$\left[\sum_{k=0}^{m}(-1)^k A_k \Phi^{+(k)}(x) + \sum_{\nu=0}^{s}(-1)^\nu B_\nu K(x)\Phi^{+(\nu)}(x)\right] - \Phi^-(x) = H(x), \quad x \in \mathbf{R}. \quad (2)$$

As all the transformations of the system (2) and the system (1) are identical, then they are equivalent in such a sense that they are solvable or unsolvable at the same time, and there is one and only one solution $\Phi^{\pm}(x)$ of the system (2) for every solution $\varphi(x)$ of the system (1) and vice versa. Further the systems with these properties we will name by the equivalent systems. The solutions of the system of equations (1) are expressed via solutions of the system (2) according to the formula

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} \int_{\mathbf{R}} \Phi^+(t) e^{-ixt}\, dt, \quad x > 0. \quad (3)$$

Later on we will consider that $K(x) \in \mathbf{H}_\alpha^{(r)}$, $r \geq 0, 0 < \alpha \leq 1$, where $\mathbf{H}_\alpha^{(r)}$ is a space of functions $f(x) \in C^{(r)}$, the derivatives with the order r of which satisfy the next condition at the real axis $\mathbf{R}$:

$$\left| f^{(r)}(x+h) - f^{(r)}(x) \right| (1+|x|)^\alpha (1+|x+h|)^\alpha \leq A_r h^\alpha, x \in \mathbb{R}, \quad h > 0,$$

where A_r is the Holder's constant of the function $f^{(r)}(x)$ and α is its Holder's exponent; $H(x) \in \mathbf{L}_2^{(r)}$, $r \geq 0$. As the matrix function $k(x) \in \mathbf{L}$, then according to Riemann-Lebesgue theorem $\lim\limits_{x\to\infty} K_{ij}(x) = 0$, $i, j = \overline{1,n}$, thus $\det K(x) = 0$ when $x \to \infty$, where $K_{ij}(x)$ is the Fourier transformation of the elements $k_{ij}(x)$ of the matrix function $k(x)$.

The theory of the solvability of systems of Winer-Hoph type equations with different degrees of differences in kernels such as

$$\varphi(x) + \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} k(x-t)\varphi(t)\, dt = h(x), \quad x > 0,$$

was built in the papers [1, 5, 6] with rather wide assumptions concerning their kernels and right parts. The investigation of systems of such a type of equations was based on the investigation of the corresponding Riemann boundary problem at the real axis, which appears after the Fourier transformation of the every system. But the methods used in the papers [2, 7, 10] can't be applied to the investigation of the system of equations (1), as this system is transformed into the corresponding system of differential boundary problems at the real axis (2) with the help of the properties of Fourier transformation. It is necessary to mention that the attempt of studying the case of an equation as (1) was made in papers [5, 6], where integral representations for functions and their derivatives analytical in domains D^+, D^- were applied for the investigation of the corresponding differential boundary problem. But the kernels of these integral representations had the additional branch points in these domains and it led to appearing multivalued unknown functions which were analytical in domains D^+, D^-. We must also mention that the integral Winer-Hoph equation (or the scalar case) was studied in details in the paper [12]. Thus we will study the system of equations (1) basing on the investigation of the system of differential boundary problems (2). We will transform the system of differential boundary problems (2) into the system of singular integral equations with the kernel of Cauchy using integral representations for the vector functions and derivatives of them which are analytical in domains D^+, D^-. Let construct vector functions $\Phi^+(z)$ and $\Phi^-(z)$ such that they are analytical in the domains D^+, D^- accordingly and disappear at infinity. Besides, the boundary values at $\mathbf{R}$ of vector functions $\Phi^{+(p)}(z)$ and $\Phi^-(z)$ satisfy the following condition $\Phi^{+(p)}(x) \in \mathbf{L}_2^{(r)}$, $\Phi^-(x) \in \mathbf{L}_2^{(r)}$, $r \geq 0, p \geq 0$. According to the papers [4, 11] such vector functions as:

$$\Phi^{\pm}(z) = (2\pi\imath)^{-1} \int\limits_{\mathbf{R}} P^{\pm}(x,z)\rho(x)\,dx, \quad z \in D^{\pm}, \tag{4}$$

where

$$P^-(x,z) = \frac{1}{x-z}, \quad z \in D^-;$$

$$P^+(x,z) = \frac{(-1)^p (x+\imath)^{-p}}{(p-1)!} \times$$

$$\times \left[(x-z)^{p-1} \ln\left(1 - \frac{x+\imath}{z+\imath}\right) - \sum_{k=0}^{p-2} d_{p-k-2}(x+\imath)^{k+1}(z+\imath)^{p-k-2} \right], \quad z \in D^+;$$

$$d_{p-k-2} = (-1)^{k+1} \sum_{j=0}^{k} C_{p-1}^{p-1-j}(k-j+1)^{-1}, \quad k = \overline{0, m-2}$$

satisfy these conditions, and here C_n^m are binomial coefficients; the function $\ln\left[1 - \dfrac{x+\imath}{z+\imath}\right]$ is the main branch ($\ln 1 = 0$) of the logarithmic function in the complex plane with the cut connecting such points as $z = -\imath$ and $z = \infty$, following the negative direction of the axis of ordinate.

It's easy to verify that defined by (4) vector functions $\Phi^+(z)$ and $\Phi^-(z)$ according to the structure of $P^\pm(x,z)$ and due to the papers [4, 11] are unique analytical functions in the domains D^+, D^- accordingly. The next vector function $\rho(x) \in \mathbf{L}_2$ or the density of the integral representations (4), is defined uniquely by the vector functions $\Phi^+(z)$ and $\Phi^-(z)$ and vice versa, so with the help of the given vector function $\rho(x) \in \mathbf{L}_2$ both vector functions $\Phi^+(z)$ and $\Phi^-(z)$ can be constructed uniquely. The following representations take place at the same time:

$$\Phi^{+(p)}(z) = (2\pi\imath)^{-1} \int_{\mathbf{R}} (z+\imath)^{-p}(x-z)^{-1}\rho(x)\,dx, \quad z \in D^+,$$

$$\Phi^-(z) = (2\pi\imath)^{-1} \int_{\mathbf{R}} (x-z)^{-1}\rho(x)\,dx, \quad z \in D^-. \tag{5}$$

We consider the case when $m = s$. Using the properties of partial derivatives of function $P^+(x,z)$ with respect to z and Sohotski formulas for derivatives from [7, p. 42], with the help of the representations (4), (5), we will transform the system of differential boundary problems (2) into the following system of singular integral equations and later on investigate it. The system of singular integral equations is

$$A(x)\rho(x) + B(x)(\pi\imath)^{-1} \int_{\mathbf{R}} (t-x)^{-1}\rho(t)\,dt + (T\rho)(x) = H(x), \quad x \in \mathbf{R}, \tag{6}$$

where

$$A(x) = 0,5\{(-1)^m\,[A_m + B_m K(x)]\,(x+\imath)^{-m} + E\},$$
$$B(x) = 0,5\{(-1)^m\,[A_m + B_m K(x)]\,(x+\imath)^{-m} - E\}, \tag{7}$$

$$(T\rho)(x) = \int_{\mathbf{R}} K(x,t)\rho(t)\,dt, \tag{8}$$

$$K(x,t) = \frac{1}{2\pi\imath}\sum_{k=0}^{m-1}(-1)^k\,[A_k + B_k K(x)]\,\frac{\partial^k P^+(t,x)}{\partial x^k},$$

where E is unity matrix and $\dfrac{\partial^k P^+(t,x)}{\partial x^k}$ is a limiting value at $\mathbf{R}$ of the function $\dfrac{\partial^k P^+(t,z)}{\partial z^k}$, $k = \overline{0, m-1}$.

Lemma 1. *If the matrix function* $K(x) \in \mathbf{H}_\alpha^{(r)}$, $r \geq 0$, $0 < \alpha \leq 1$, *then the operator*

$$T : \mathbf{L}_2^{(r)} \to \mathbf{L}_2^{(r)},$$

$r \geq 0$, *defined by the formula (8), is a compact one.*

The proof of this lemma follows from Frechet-Kolmogorov-Riesz criterion of compactness vector functions' sets in the spaces $\mathbf{L}_p$, $p > 1$ and integral operators at the real axis in the spaces $\mathbf{L}_p,\ p > 1$, the properties of function $P^+(x, z)$ and the results of the work [8].

According to the work [10, p. 406], the system of differential boundary problems (2) and the system of singular integral equations (6) are equivalent in such a sense that they are solvable or unsolvable at the same time, and for every solution $\rho(x)$ of the system (6) there exists maybe a nonunique solution $\Phi^\pm(x)$ of the system (2) and vice versa. In order to make this correspondence unique it is necessary to set initial conditions for the system (2). As its solutions $\Phi^\pm(x)$ are found in spaces of functions that disappear at infinity, then according to the properties of Cauchy type integral the solutions of the system (2) are such that $\Phi^{+(k)}(\infty) = 0$, $k = \overline{0, m-1}$, it means that the initial conditions of the system (2) are trivial and set automatically. Thus it follows that the system of differential boundary problems (2) and the system of singular integral equations (6) are equivalent in such a sense that they are solvable or unsolvable at the same time, and there is one and only one solution $\rho(x)$ of the system (6) for every solution $\Phi^\pm(x)$ of the system (2) and vice versa. By the force of formula (4), the solutions of the system (2) are expressed via solutions of the system (6) according to the formula

$$\Phi^+(x) = (2\pi\imath)^{-1} \int_{\mathbf{R}} P^+(t, x)\rho(t)\, dt, x \in \mathbf{R}, \tag{9}$$

where $p = m$; $P^+(t, x)$ are the boundary values at $\mathbf{R}$ of the vector functions $P^+(t, z)$, and the vector function $\rho(x)$ is the solution of the system (6). As the system of the equations (1) and the system (2) are equivalent, the system (2) and the system of singular integral equations (6) are equivalent, too, it follows that the system (1) and the system (6) are equivalent in such a sense that they are solvable or unsolvable at the same time, and there is one and only one solution $\varphi(x)$ of the system of equations (1) for every solution $\rho(x)$ of the system of the equations (6) and vice versa. Thus the solutions of the system (1) are expressed via solutions of the system (6) according to the formulas (10), (3). That is why we will call the system of the equations (1) a Noetherian if the system of the equations (6) is a Noetherian one.

Theorem 1. *The system (1) is not a Noetherian one.*

Proof. According to the papers [2, 3, 10] the system of the singular integral equations (6) is a Noetherian one if and only if when the following conditions take place:

$$\det[A(x) + B(x)] \neq 0, \quad \det[A(x) - B(x)] \neq 0$$

at $\mathbf{R}$. As $A(x) - B(x) = E$; $A(x) + B(x) = (-1)^m(x + \imath)^{-m}[A_m + B_m K(x)]$, then $\det[A(x) + B(x)]$ has a null at least with order m at infinity. It means that the system of the equations (6) is not a Noetherian one. Then as the systems (1) and (6) are equivalent, the system of the equations (1) is not a Noetherian one, too.

The theorem is proved. □

Let's determine conditions when the system of equations (1) is a Notherian one and it is a solvable one due to it. First we consider the case when $\det[A_m + B_m K(x)] \neq 0$ at the finite points of the real axis $\mathbf{R}$. The following representation [3, p. 329] for the matrix function $A(x) + B(x)$ takes place:

$$A(x) + B(x) = M(x) \cdot D(x) \cdot R(x). \tag{10}$$

Here $M(x)$ is a matrix function of size measure n and $\det M(x) \neq 0$ at $\mathbf{R}$;

$R(x)$ is a matrix function with such a determinant which is constant and different from zero with polynomials of degrees $\dfrac{1}{x+\imath}$ as its elements;

$D(x)$ is a diagonal matrix function such as:

$$D(x) = diag\left\{\frac{1}{(x+\imath)^{\nu_0^{(1)}}}, \cdots, \frac{1}{(x+\imath)^{\nu_0^{(n)}}}\right\},$$

where $\nu_0^{(1)}, \ldots, \nu_0^{(n)}$ are integer non-negative numbers such that

$$\sum_{j=0}^{n} \nu_0^{(j)} = \nu_0 = m. \tag{11}$$

Let denote

$$r_0 = \max\{\nu_0^{(1)}, \ldots, \nu_0^{(n)}\}. \tag{12}$$

We will investigate the matrix function $M(x) \in \mathbf{H}_\alpha^{(r)}$, $r \geq r_0$, $0 < \alpha \leq 1$, where the number r_0 is defined by the formula (12). According to the paper [9, p. 53] it allows the following factorisation

$$M(x) = X^+(x) \cdot \Lambda(x) \cdot X^-(x), \tag{13}$$

where $\det X^{\pm}(x) \neq 0$ at $\mathbf{R}$ and

$$\Lambda(x) = diag\left\{\left(\frac{x-\imath}{x+\imath}\right)^{\chi_1}, \cdots, \left(\frac{x-\imath}{x+\imath}\right)^{\chi_n}\right\}, \tag{14}$$

and χ_j, $j = \overline{1,n}$ are the partial indexes of the matrix function $M(x)$.

As there can be positive and negative partial indexes at the same time among all of them, we will define them by the next equalities:

$$\omega = \sum_{\chi_j \geq 0} \chi_j, \quad q = -\sum_{\chi_j < 0} \chi_j, \tag{15}$$

then the summarized index of the matrix function $M(x)$ is defined by the formula

$$\chi = \omega - q. \tag{16}$$

The next theorem takes place.

Theorem 2. *Let the matrix function* $k(x) \in \mathbf{L}$, *vector function* $h(x) \in \mathbf{L}_2$; *the matrix function* $K(x) \in \mathbf{H}_\alpha^{(r)}$, $r \geq r_0$, $0 < \alpha \leq 1$, *the vector function* $H(x) \in \mathbf{L}_2^{(r)}$, $r \geq r_o$, *where the number* r_0 *is defined by the formula (12);* $\det[A_m + B_m K(x)] \neq 0$ *at the finite points of the real axis* $\mathbf{R}$, *the numbers* ω, q *are defined by the formula (15), the number* χ *is defined by the formula (16) and the representation (13) takes place.*

If $q - 2m \geq 0$, *then the homogeneous system (1) has not less than* $q - 2m$ *linear independent solutions; the heterogeneous system(1) is a solvable one if not less than* ω *conditions of solvability*

$$\int_{\mathbf{R}} H(x)\psi_j(x)\,dt = 0, \tag{17}$$

are executed. Here in (17) the vector function $H(x)$ *is a right part of the system of the singular integral equations (6) and the vector functions* $\psi_j(x)$ *are linear independent solutions of the system of homogeneous singular integral equations*

$$\tilde{A}(x)\psi(x) - (\pi\imath)^{-1}\int_{\mathbf{R}}(t-x)^{-1}\tilde{B}(t)\psi(t)\,dt + \int_{\mathbf{R}}\tilde{K}(t,x)\psi(t)\,dt = 0, \tag{18}$$

allied to the equation (6), where the matrices $\tilde{A}(x), \tilde{B}(x), \tilde{K}(x,t)$ *are transposed with respect to matrices* $A(x), B(x), K(x,t)$ *which are the coefficients and the regular kernel in the system of equations (6) respectively.*

If $q - 2m < 0$ *then the heterogeneous system(1) is an unsolvable one. It will become a solvable one if* $\omega + 2m$ *conditions (17) are executed.*

The summarized index of the system (1) is $-(\chi + 2m)$.

According to the paper [2, p. 262] let's denote by $\mathbf{L}_2[-\mu; 0]$ the space of functions $\varphi(x) \in \mathbf{L}_2$ which satisfy the condition $(x+\imath)^\mu \varphi(x) \in \mathbf{L}_2$.

Theorem 3. *Let the matrix function* $k(x) \in \mathbf{L}$, *the vector function* $h(x) \in \mathbf{L}_2$; *the matrix function* $K(x) \in \mathbf{H}_\alpha^{(r)}$, $r \geq r_0$, $0 < \alpha \leq 1$, *where the number* r_0 *is defined by the formula (12), the vector function* $H(x) \in \mathbf{L}_2^{(r)}$, $r \geq r_0$; $\det[A_m + B_m K(x)] \neq 0$ *at the finite points of the real axis* $\mathbf{R}$ *and the system (1) is a solvable one. Then its solutions belong to the space* $\mathbf{L}_2[-r - m + r_0; 0]$, $r \geq r_0$.

Now we will study the singular case.

Let the condition $\det[A_m + B_m K(x)] \neq 0$ at the finite points of the real axis $\mathbf{R}$ is not executed. Then we suppose that $\det[A_m + B_m K(x)]$ has zeroes at the real axis $\mathbf{R}$ in finite points $a_1, a_2, \ldots, a_u$ with integer orders $\nu_1, \nu_2, \ldots, \nu_u$ respectively. Then in virtue of the work [3, p. 328] the representation (10) for the matrix function $A(x) + B(x)$ takes place. Here the matrix functions $M(x)$, $R(x)$ are the same as in the previous case, and $D(x)$ is the following diagonal matrix

$$D(x) = diag\left\{\frac{1}{(x+\imath)^{\nu_0^{(1)}}}\prod_{j=1}^{u}\left(\frac{x-a_j}{x+\imath}\right)^{\nu_j^{(1)}}, \cdots, \frac{1}{(x+\imath)^{\nu_0^{(n)}}}\prod_{j=1}^{u}\left(\frac{x-a_j}{x+\imath}\right)^{\nu_j^{(n)}}\right\}, \tag{19}$$

where $\nu_0^{(1)},\dots,\nu_0^{(n)},\nu_1^{(1)},\dots,\nu_1^{(n)},\dots,\nu_u^{(1)},\dots,\nu_u^{(n)}$ are integer nonnegative numbers such that

$$\sum_{j=1}^{n}\nu_0^{(j)}=\nu_0=m,$$

$$\nu_k=\sum_{j=1}^{n}\nu_k^{(j)},\quad k=\overline{1,u},\quad \nu=\sum_{k=1}^{u}\nu_k. \tag{20}$$

Let

$$r_0=\max\{\nu_0^{(1)},\dots,\nu_0^{(n)},\nu_1^{(1)},\dots,\nu_1^{(n)},\dots,\nu_u^{(1)},\dots,\nu_u^{(n)}\}. \tag{21}$$

Analogously as in the previous case the matrix function $M(x)\in\mathbf{H}_\alpha^{(r)}$, $r\ge r_0$, admits the factorization (13), where the matrix function $\Lambda(x)$ is defined by the formula (14) and χ_j, $j=\overline{1,n}$ are the partial indexes of the matrix function $M(x)$ that are defined by the formula (15). The summarizing index of the system of singular integral equations (6) is also given by the formula (16).

The next theorem takes place.

Theorem 4. *Let the matrix function $k(x)\in\mathbf{L}$, vector function $h(x)\in\mathbf{L}_2$; the matrix function $K(x)\in\mathbf{H}_\alpha^{(r)}$, $r\ge r_0$, $0<\alpha\le1$, the vector function $H(x)\in\mathbf{L}_2^{(r)}$, $r\ge r_o$, where the number r_0 is defined by the formula (21), $\det[A_m+B_mK(x)]$ has zeroes at the real axis $\mathbf{R}$ at such finite points as $a_1,a_2,\dots,a_u$ with integer orders $\nu_1,\nu_2,\dots,\nu_u$ respectively and the representation (10) takes place.*

Here in that representation the matrix function $D(x)$ is defined by the formula (19); $\det M(x)\ne0$ at $\mathbf{R}$; the numbers ω,q are defined by the formula (15), the number ν is defined by the formula (20) and the number χ is defined by the formula (16) and the representation (13) takes place.

If $q-2m-2\nu\ge0$, then the homogeneous system (1) has not less than $q-2m-2\nu$ linear independent solutions; the heterogeneous system(1) is a solvable one if not less than ω conditions of solvability (17) are executed.

If $q-2m-2\nu<0$ then the heterogeneous system(1) is an unsolvable one. It will become a solvable one if $\omega+2m+2\nu$ conditions (17) are executed.

The summarized index of the system (1) is $-(\chi+2m+2\nu)$.

Theorem 5. *Let the matrix function $k(x)\in\mathbf{L}$, the vector function $h(x)\in\mathbf{L}_2$; the matrix function $K(x)\in\mathbf{H}_\alpha^{(r)}$, $r\ge r_0$, $0<\alpha\le1$, where the number r_0 is defined by the formula (21), the vector function $H(x)\in\mathbf{L}_2^{(r)}$, $r\ge r_0$; $\det[A_m+B_mK(x)]$ has zeroes at the real axis $\mathbf{R}$ at finite points $a_1,a_2,\dots,a_u$ with integer orders $\nu_1,\nu_2,\dots,\nu_u$ respectively, the representation (10) takes place and the system (1) is a solvable one. Then its solutions belong to the space $\mathbf{L}_2[-r-m+r_0;0]$, $r\ge r_0$.*

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