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# Properties of one-sided ideals of pseudonormed rings when taking the quotient rings

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**Abstract.** Let $\varphi : (R, \xi) \to \left(\widehat{R}, \widehat{\xi}\right)$ be an isomorphism of pseudonormed rings. The inequalities $\frac{\xi(a \cdot b)}{\xi(b)} \leq \widehat{\xi}(\varphi(a)) \leq \xi(a)$ are fulfilled for any $a, b \in R \setminus \{0\}$ iff there exists a pseudonormed ring $\left(\widetilde{R}, \widetilde{\xi}\right)$ such that $(R, \xi)$ is a left ideal in $\left(\widetilde{R}, \widetilde{\xi}\right)$ and the isomorphism $\varphi$ can be extended up to an isometric homomorphism $\widetilde{\varphi} : \left(\widetilde{R}, \widetilde{\xi}\right) \to \left(\widehat{R}, \widehat{\xi}\right)$.

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We will think that a pseudonormed ring is a ring $R$ which may be non-associative and has a pseudonorm, i.e. a real function $\xi(r)$ such that the following conditions are satisfied: $\xi(-r) = \xi(r) \geq 0$; $\xi(r) = 0$ iff $r = 0$; $\xi(r_1 + r_2) \leq \xi(r_1) + \xi(r_2)$ and $\xi(r_1 \cdot r_2) \leq \xi(r_1) \cdot \xi(r_2)$ for any $r_1, r_2 \in R$.

The following isomorphism theorem is often applied in algebra and, in particular, in the ring theory:

If $A$ is a subring of a ring $R$ and $I$ is an ideal of the ring $R$ then the quotient rings $A/(A \cap I)$ and $(A + I)/I$ are isomorphic rings. In particular, if $A \cap I = 0$ then the ring $A$ is isomorphic to the ring $(A + I)/I$, i.e. the rings $A$ and $(A + I)/I$ possess identical algebraic properties.

Since when studying the pseudonormed rings it is necessary to take into account properties of pseudonorms besides algebraic properties there is a need to consider isomorphisms which keep pseudonorms instead of ring isomorphism. Such isomorphisms are called isometric isomorphisms.

Taking into consideration this fact the above specified isomorphism theorem not always takes place for pseudonormed rings. As it is shown in Theorem 2.1 from [1] it is impossible to tell anything more than performance of an inequality in case $A \cap I = 0$.

Therefore it is necessary to impose additional conditions on the ring $A$. For example, the cases when $A$ is an ideal or one-sided ideal of the pseudonormed ring $(R, \xi)$ are considered.

The case when $A$ is an ideal of pseudonormed ring $(R, \xi)$ was investigated in [1].

The present article is a continuation of the article [1]. The case when $A$ is an one-side ideal of the pseudonormed ring $(R, \xi)$ is investigated in the present article.

**Definition 1.** A homomorphism $\varphi : (R, \xi) \to \left(\widehat{R}, \widehat{\xi}\right)$ of pseudonormed rings is called an isometric homomorphism if $\widehat{\xi}\,(\varphi\,(r)) = \inf\{\,\xi\,(r + a) \mid a \in ker\varphi\}$ for all $r \in R$.

*Remark* 1. It is clear that if an isometric homomorphism is an isomorphism then it is an isometric isomorphism in usual sense.

*Remark* 2. If $I$ is a closed ideal of a pseudonormed ring $(R, \xi)$ then the canonical homomorphism[1] $\varepsilon : (R, \xi) \to (R, \xi)\,/I$ is an isometric homomorphism, and if $\varphi : (R, \xi) \to \left(\widehat{R}, \widehat{\xi}\right)$ is an isometric homomorphism of pseudonormed rings and $I = \ker\varphi$ then the pseudonormed rings $\left(\widehat{R}, \widehat{\xi}\right)$ and $(R, \xi)\,/I$ are isometrically isomorphic.

**Definition 2.** Let $(R, \xi)$ and $\left(\widehat{R}, \widehat{\xi}\right)$ be pseudonormed rings. By analogy with the definition in [1], an isomorphism $\varphi : (R, \xi) \to \left(\widehat{R}, \widehat{\xi}\right)$ is said to be a semi-isometric isomorphism on the left (on the right) if there exists a pseudonormed ring $\left(\widetilde{R}, \widetilde{\xi}\right)$ such that the pseudonormed ring $(R, \xi)$ is a left (right) ideal of the pseudonormed ring $\left(\widetilde{R}, \widetilde{\xi}\right)$ and the isomorphism $\varphi$ can be extended up to an isometric homomorphism $\widetilde{\varphi} : \left(\widetilde{R}, \widetilde{\xi}\right) \to \left(\widehat{R}, \widehat{\xi}\right)$.

**Theorem 1.** *Let $(R, \xi)$ and $\left(\widehat{R}, \widehat{\xi}\right)$ be pseudonormed rings and $\varphi : (R, \xi) \to \left(\widehat{R}, \widehat{\xi}\right)$ be an isomorphism. Then the following statements are equivalent:*

1. *The isomorphism $\varphi$ is a semi-isometric isomorphism on the left;*
2. *The inequalities $\dfrac{\xi\,(b \cdot a)}{\xi\,(a)} \le \widehat{\xi}\,(\varphi\,(b)) \le \xi\,(b)$ are fulfilled for any $a, b \in R \setminus \{0\}$;*
3. *There exists a pseudonormed ring $\left(\widetilde{R}, \widetilde{\xi}\right)$ such that the pseudonormed ring $(R, \xi)$ is a left ideal of the pseudonormed ring $\left(\widetilde{R}, \widetilde{\xi}\right)$ and the isomorphism $\varphi$ can be extended up to an isometric homomorphism $\widetilde{\varphi} : \left(\widetilde{R}, \widetilde{\xi}\right) \to \left(\widehat{R}, \widehat{\xi}\right)$, and $(\ker\widetilde{\varphi})^2 = \{0\}$.*

*Proof.* $1 \Rightarrow 2$. Let $\varphi : (R, \xi) \to \left(\widehat{R}, \widehat{\xi}\right)$ be a semi-isometric isomorphism on the left. Then there exists a pseudonormed ring $\left(\widetilde{R}, \widetilde{\xi}\right)$ such that the pseudonormed

[1] i.e. homomorphism $\varepsilon : R \to R/I$ such that $\varepsilon\,(r) = r + I$.

ring $(R, \xi)$ is a left ideal of the pseudonormed ring $\left(\widetilde{R}, \widetilde{\xi}\right)$ and the isomorphism $\varphi$ can be extended up to an isometric homomorphism $\widetilde{\varphi} : \left(\widetilde{R}, \widetilde{\xi}\right) \to \left(\widehat{R}, \widehat{\xi}\right)$.

Let $a, b \in R$ and $\varepsilon > 0$. Since $\widetilde{\varphi}$ is an extension of the isomorphism $\varphi$ then $R \cap \ker \widetilde{\varphi} = \ker \varphi = \{0\}$, and as $R$ is a left ideal of $\widetilde{R}$ and $\ker \widetilde{\varphi}$ is an ideal of $\widetilde{R}$ then $d \cdot a \in R \cap \ker \widetilde{\varphi} = \{0\}$ for any $d \in \ker \widetilde{\varphi}$, i.e. $(\ker \widetilde{\varphi}) \cdot a = 0$.

Since $\widetilde{\varphi} : \left(\widetilde{R}, \widetilde{\xi}\right) \to \left(\widehat{R}, \widehat{\xi}\right)$ is an isometric homomorphism then $\widehat{\xi}\left(\widetilde{\varphi}(b)\right) \leq \widetilde{\xi}(b) = \xi(b)$ and there exists an element $c \in \ker \widetilde{\varphi}$ such that $\widetilde{\xi}(b + c) < \widehat{\xi}\left(\widetilde{\varphi}(b)\right) + \varepsilon = \widehat{\xi}\left(\varphi(b)\right) + \varepsilon$. So as $c \cdot a \in \ker \widetilde{\varphi} \cdot a = 0$ then

$$\xi(b \cdot a) = \widetilde{\xi}(b \cdot a) = \widetilde{\xi}(b \cdot a + c \cdot a) = \widetilde{\xi}((b + c) \cdot a) \leq \widetilde{\xi}(b + c) \cdot \widetilde{\xi}(a) =$$

$$\widetilde{\xi}(b + c) \cdot \xi(a) < \left(\widehat{\xi}\left(\varphi(b)\right) + \varepsilon\right) \cdot \xi(a).$$

Since $\varepsilon > 0$ is any number then $\xi(b \cdot a) \leq \widehat{\xi}\left(\varphi(b)\right) \cdot \xi(a)$. It means that

$$\frac{\xi(b \cdot a)}{\xi(a)} \leq \widehat{\xi}\left(\varphi(b)\right) \leq \xi(b).$$

Hence $1 \Rightarrow 2$ is proved. □

*Proof.* **2 ⇒ 3.** Let $(R, \xi)$ and $\left(\widehat{R}, \widehat{\xi}\right)$ be pseudonormed rings and $\varphi : (R, \xi) \to \left(\widehat{R}, \widehat{\xi}\right)$ be an isomorphism such that the inequalities $\dfrac{\xi(b \cdot a)}{\xi(a)} \leq \widehat{\xi}\left(\varphi(b)\right) \leq \xi(b)$ are fulfilled for any $a, b \in R$.

We shall lead the proof to some stages.

I. Construction of the ring $\widetilde{R}$ and checking some of its properties.

I.1. Let's consider a discrete ring $\widetilde{R}$ such that its additive group is the direct sum of the additive groups of the rings $R$ and $\widehat{R}$, and the multiplication is certain as follows: $(r_1, \widehat{r}_1) \cdot (r_2, \widehat{r}_2) = (r_1 \cdot r_2, \varphi(r_1) \cdot \widehat{r}_2)$.

I.2. It is easy to notice that $\widetilde{R}$ is a ring with respect to these operations of addition and multiplication, and the set $R' = \{(r, 0) | \, r \in R\}$ is a left ideal of $\widetilde{R}$.

I.3. Let's define the mapping $\alpha : R \to \widetilde{R}$ as follows $\alpha(r) = (r, 0)$ for any $r \in R$. It is easy to notice that $\alpha : R \to R' = \{(r, 0) | \, r \in R\}$ is a ring isomorphism. Hence, if we identify an element $r \in R$ with the element $(r, 0) \in R'$ then we can suppose that $R$ is a left ideal in the ring $\widetilde{R}$.

I.4. Let's define the mapping $\widetilde{\varphi} : \widetilde{R} \to \widehat{R}$ as follows $\widetilde{\varphi}(r, \widehat{r}) = \varphi(r)$. It's easy to notice that $\widetilde{\varphi} : \widetilde{R} \to \widehat{R}$ is a ring homomorphism, and (considering I.3.) $\widetilde{\varphi}(r) = \widetilde{\varphi}(r, 0) = \varphi(r)$ for any $r \in R$, i.e. $\widetilde{\varphi}|_R = \varphi$. Since $\ker \widetilde{\varphi} = \left\{(0, \widehat{r}) | \, \widehat{r} \in \widehat{R}\right\}$ then $(\ker \widetilde{\varphi})^2 = 0$.

II. Definition of a pseudonorm $\widetilde{\xi}$ and checking some of its properties.

II.1. Let's define the real function $\widetilde{\xi}$ on the ring $\widetilde{R}$ as follows: $\widetilde{\xi}(r,\widehat{r}) = \xi\left(r - \varphi^{-1}(\widehat{r})\right) + \widehat{\xi}(\widehat{r})$.

II.2. Let's verify that $\widetilde{\xi}$ is a pseudonorm.

It is easy to notice that $\widetilde{\xi}(-\widetilde{r}) = \widetilde{\xi}(\widetilde{r}) \geq 0$ for any $\widetilde{r} \in \widetilde{R}$ and $\widetilde{\xi}(\widetilde{r}) = 0$ if and only if $\widetilde{r} = 0$, i.e. the first and second conditions of the definition of the pseudonorm are valid. Let $\widetilde{r}_1 = (r_1, \widehat{r}_1), \widetilde{r}_2 = (r_2, \widehat{r}_2) \in \widetilde{R}$. Then

$$\widetilde{\xi}(\widetilde{r}_1 + \widetilde{r}_2) = \widetilde{\xi}((r_1, \widehat{r}_1) + (r_2, \widehat{r}_2)) = \widetilde{\xi}((r_1 + r_2, \widehat{r}_1 + \widehat{r}_2)) =$$

$$= \xi\left(r_1 + r_2 - \varphi^{-1}(\widehat{r}_1 + \widehat{r}_2)\right) + \widehat{\xi}(\widehat{r}_1 + \widehat{r}_2) =$$

$$= \xi\left(r_1 + r_2 - \varphi^{-1}(\widehat{r}_1) - \varphi^{-1}(\widehat{r}_2)\right) + \widehat{\xi}(\widehat{r}_1 + \widehat{r}_2) \leq$$

$$\leq \xi\left(r_1 - \varphi^{-1}(\widehat{r}_1)\right) + \xi\left(r_2 - \varphi^{-1}(\widehat{r}_2)\right) + \widehat{\xi}(\widehat{r}_1) + \widehat{\xi}(\widehat{r}_2) = \widetilde{\xi}(\widetilde{r}_1) + \widetilde{\xi}(\widetilde{r}_2).$$

Besides that, because the inequalities $\xi(b \cdot a) \leq \widehat{\xi}(\varphi(b)) \cdot \xi(a)$ and $\widehat{\xi}(\varphi(a)) \leq \xi(a)$ for any $a, b \in R$ are true (see the statement 2 of formulation of the theorem) we have:

$$\widetilde{\xi}(\widetilde{r}_1 \cdot \widetilde{r}_2) = \widetilde{\xi}((r_1, \widehat{r}_1) \cdot (r_2, \widehat{r}_2)) = \widetilde{\xi}((r_1 \cdot r_2, \varphi(r_1) \cdot \widehat{r}_2)) =$$

$$= \xi\left(r_1 \cdot r_2 - \varphi^{-1}(\varphi(r_1) \cdot \widehat{r}_2)\right) + \widehat{\xi}(\varphi(r_1) \cdot \widehat{r}_2) =$$

$$= \xi\left(r_1 \cdot r_2 - r_1 \cdot \varphi^{-1}(\widehat{r}_2)\right) + \widehat{\xi}(\varphi(r_1) \cdot \widehat{r}_2) =$$

$$\xi\left(r_1 \cdot \left(r_2 - \varphi^{-1}(\widehat{r}_2)\right)\right) + \widehat{\xi}(\varphi(r_1) \cdot \widehat{r}_2) \leq$$

$$\leq \widehat{\xi}(\varphi(r_1)) \cdot \xi\left(r_2 - \varphi^{-1}(\widehat{r}_2)\right) + \widehat{\xi}(\varphi(r_1) \cdot \widehat{r}_2) =$$

$$= \widehat{\xi}(\varphi(r_1)) \cdot \xi\left(r_2 - \varphi^{-1}(\widehat{r}_2)\right) + \widehat{\xi}((\varphi(r_1) - \widehat{r}_1 + \widehat{r}_1) \cdot \widehat{r}_2) \leq$$

$$\leq \widehat{\xi}(\varphi(r_1)) \cdot \xi\left(r_2 - \varphi^{-1}(\widehat{r}_2)\right) + \widehat{\xi}((\varphi(r_1) - \widehat{r}_1) \cdot \widehat{r}_2) + \widehat{\xi}(\widehat{r}_1 \cdot \widehat{r}_2) \leq$$

$$\leq \widehat{\xi}(\varphi(r_1)) \cdot \xi\left(r_2 - \varphi^{-1}(\widehat{r}_2)\right) + \widehat{\xi}(\varphi(r_1) - \widehat{r}_1) \cdot \widehat{\xi}(\widehat{r}_2) + \widehat{\xi}(\widehat{r}_1) \cdot \widehat{\xi}(\widehat{r}_2) \leq$$

$$\leq \widehat{\xi}(\varphi(r_1)) \cdot \xi\left(r_2 - \varphi^{-1}(\widehat{r}_2)\right) + \xi\left(r_1 - \varphi^{-1}(\widehat{r}_1)\right) \cdot \widehat{\xi}(\widehat{r}_2) + \widehat{\xi}(\widehat{r}_1) \cdot \widehat{\xi}(\widehat{r}_2) =$$

$$= \widehat{\xi}(\varphi(r_1) - \widehat{r}_1 + \widehat{r}_1) \cdot \xi\left(r_2 - \varphi^{-1}(\widehat{r}_2)\right) +$$

$$\xi\left(r_1 - \varphi^{-1}(\widehat{r}_1)\right) \cdot \widehat{\xi}(\widehat{r}_2) + \widehat{\xi}(\widehat{r}_1) \cdot \widehat{\xi}(\widehat{r}_2) \leq$$

$$\leq \widehat{\xi}(\varphi(r_1) - \widehat{r}_1) \cdot \xi\left(r_2 - \varphi^{-1}(\widehat{r}_2)\right) + \widehat{\xi}(\widehat{r}_1) \cdot \xi\left(r_2 - \varphi^{-1}(\widehat{r}_2)\right) +$$

$$\xi\left(r_1 - \varphi^{-1}(\widehat{r}_1)\right) \cdot \widehat{\xi}(\widehat{r}_2) + \widehat{\xi}(\widehat{r}_1) \cdot \widehat{\xi}(\widehat{r}_2) \leq$$

$$\xi\left(r_1 - \varphi^{-1}(\widehat{r}_1)\right) \cdot \xi\left(r_2 - \varphi^{-1}(\widehat{r}_2)\right) + \widehat{\xi}(\widehat{r}_1) \cdot \xi\left(r_2 - \varphi^{-1}(\widehat{r}_2)\right) +$$

$$\xi\left(r_1 - \varphi^{-1}(\widehat{r}_1)\right) \cdot \widehat{\xi}(\widehat{r}_2) + \widehat{\xi}(\widehat{r}_1) \cdot \widehat{\xi}(\widehat{r}_2) =$$

$$= \left(\xi\left(r_1 - \varphi^{-1}(\widehat{r}_1)\right) + \widehat{\xi}(\widehat{r}_1)\right) \cdot \left(\xi\left(r_2 - \varphi^{-1}(\widehat{r}_2)\right) + \widehat{\xi}(\widehat{r}_2)\right) = \widetilde{\xi}(\widetilde{r}_1) \cdot \widetilde{\xi}(\widetilde{r}_2).$$

Hence the function $\widetilde{\xi}$ satisfies also the last condition of definition of pseudonorm. It means that $\widetilde{\xi}$ is a pseudonorm on the ring $\widetilde{R}$.

II.3. Since $\widetilde{\xi}(r) = \widetilde{\xi}(r,0) = \xi(r) + \widehat{\xi}(0) = \xi(r)$ for any $r \in R$ then $\widetilde{\xi}\Big|_R = \xi$.

II.4. Let's verify that $\widetilde{\varphi} : \left(\widetilde{R}, \widetilde{\xi}\right) \to \left(\widehat{R}, \widehat{\xi}\right)$ is an isometric homomorphism, i.e. $\widehat{\xi}(\widetilde{\varphi}(\widetilde{r})) = \inf\left\{\widetilde{\xi}(\widetilde{r} + \widetilde{a}) \Big| \widetilde{a} \in \ker \widetilde{\varphi}\right\}$ for any $\widetilde{r} \in \widetilde{R}$.

Let $\widetilde{r} = (r, \widehat{r}) \in \widetilde{R}$. Then $\widetilde{r}_1 = (0, \varphi(r) - \widehat{r}) \in \ker \widetilde{\varphi}$, and

$$\inf\left\{\widetilde{\xi}(\widetilde{r} + \widetilde{a}) \Big| \widetilde{a} \in \ker \widetilde{\varphi}\right\} \leq \widetilde{\xi}(\widetilde{r} + \widetilde{r}_1) = \widetilde{\xi}((r, \widehat{r}) + (0, \varphi(r) - \widehat{r})) =$$

$$= \widetilde{\xi}(r, \widehat{r} + \varphi(r) - \widehat{r}) = \widetilde{\xi}(r, \varphi(r)) =$$
$$\xi\left(r - \varphi^{-1}(\varphi(r))\right) + \widehat{\xi}(\varphi(r)) = \widehat{\xi}(\varphi(r)) = \widehat{\xi}(\widetilde{\varphi}(\widetilde{r})).$$

On the other hand, since $\widehat{\xi}(\varphi(b)) \leq \xi(b)$ for any element $b \in R$, then for any element $\widetilde{a} = (0, \widehat{a}) \in \ker \widetilde{\varphi}$ we have

$$\widetilde{\xi}(\widetilde{r} + \widetilde{a}) = \widetilde{\xi}((r + 0, \widehat{r} + \widehat{a})) = \widetilde{\xi}(r, \widehat{r} + \widehat{a}) = \xi\left(r - \varphi^{-1}(\widehat{r} + \widehat{a})\right) + \widehat{\xi}(\widehat{r} + \widehat{a}) \geq$$

$$\geq \widehat{\xi}\left(\varphi\left(r - \varphi^{-1}(\widehat{r} + \widehat{a})\right)\right) + \widehat{\xi}(\widehat{r} + \widehat{a}) \geq \widehat{\xi}\left(\varphi\left(r - \varphi^{-1}(\widehat{r} + \widehat{a})\right) + \widehat{r} + \widehat{a}\right) =$$
$$= \widehat{\xi}(\varphi(r) - \widehat{r} - \widehat{a} + \widehat{r} + \widehat{a}) = \widehat{\xi}(\varphi(r)) = \widehat{\xi}(\widetilde{\varphi}(\widetilde{r})).$$

It means that $\inf\left\{\widetilde{\xi}(\widetilde{r} + \widetilde{a}) \Big| \widetilde{a} \in \ker \widetilde{\varphi}\right\} \geq \widehat{\xi}(\widetilde{\varphi}(\widetilde{r}))$.

Thus, $\inf\left\{\widetilde{\xi}(\widetilde{r} + \widetilde{a}) \Big| \widetilde{a} \in \ker \widetilde{\varphi}\right\} = \widehat{\xi}(\widetilde{\varphi}(\widetilde{r}))$, and $\widetilde{\varphi} : \left(\widetilde{R}, \widetilde{\xi}\right) \to \left(\widehat{R}, \widehat{\xi}\right)$ is an isometric homomorphism.

Hence $2 \Rightarrow 3$ is proved. □

For completion of the proof of the theorem it is necessary to verify that $3 \Rightarrow 1$. But this is obvious because the pseudonormed ring $\left(\widetilde{R}, \widetilde{\xi}\right)$ which is specified in the statement 3 satisfies all conditions from Definition 2.

Passing to antiisomorphic rings [2] from Theorem 1 easily follows:

**Theorem 2.** *If $(R, \xi)$ and $\left(\widehat{R}, \widehat{\xi}\right)$ are pseudonormed rings and $\varphi : (R, \xi) \to \left(\widehat{R}, \widehat{\xi}\right)$ is an isomorphism then the following statements are equivalent:*

1. *The isomorphism $\varphi$ is a semi-isometric isomorphism on the right;*
2. *The inequalities $\dfrac{\xi(b \cdot a)}{\xi(b)} \leq \widehat{\xi}(\varphi(a)) \leq \xi(a)$ are fulfilled for any $a, b \in R \setminus \{0\}$;*

[2] If $R$ and $R'$ are rings then a mapping $\varsigma : R \to R'$ is called an antiisomorphism when it is an isomorphism of the additive groups of these rings and $\varsigma(a \cdot b) = \varsigma(b) \cdot \varsigma(a)$ for any $a, b \in R$.

3. *There exists a pseudonormed ring* $\left(\widetilde{R}, \widetilde{\xi}\right)$ *such that the pseudonormed ring* $(R, \xi)$ *is a right ideal of the pseudonormed ring* $\left(\widetilde{R}, \widetilde{\xi}\right)$ *and the isomorphism* $\varphi$ *can be extended up to an isometric homomorphism* $\widetilde{\varphi} : \left(\widetilde{R}, \widetilde{\xi}\right) \to \left(\widehat{R}, \widehat{\xi}\right)$, *and* $(\ker \widetilde{\varphi})^2 = \{0\}$.

From Theorems 1 and 2 of the present article follows

**Corollary 1.** *If* $(R, \xi)$ *and* $\left(\widehat{R}, \widehat{\xi}\right)$ *are pseudonormed rings and an isomorphism* $\varphi : (R, \xi) \to \left(\widehat{R}, \widehat{\xi}\right)$ *is a semi-isometric isomorphism on the left and a semi-isometric isomorphism on the right then it is semi-isometric.*

*Remark* 3. The ring $\widetilde{R}$ which is constructed by the proof $2 \Rightarrow 3$ (see the proof of Theorem 1) is associative when the rings $R$ and $\widehat{R}$ are associative. Therefore Theorems 1 and 2 also are true for associative rings.

## References

[1] Aleschenko S. A., Arnautov V. I. *Quotient rings of pseudonormed rings.* Buletinul Academiei de Ştiinte a Republicii Moldova, Matematica, 2006, No. 2(51), 3–16.

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