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Resolvability of some special algebras with topologies

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Abstract. Let G be an infinite I_nP-n-groupoid. We construct a disjoint family $\{B_\mu : \mu \in M\}$ of non-empty subsets of G such that the sets $\{B_\mu\}$ are dense in all Choban's totally bounded topologies on G, $|M| = |G|$, $G = \cup\{B_\mu : \mu \in M\}$ and $\cup_{k=1}^{n} \Delta_\varphi \omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$ for all $\mu \in M$ and every finite subsets K of G. In particular, we continue the line of research from [6, 9].

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1 Introductory notions

A space X is called resolvable if in X there exist two disjoint dense subsets. In [6] M. Choban and L. Chiriac has proved the following assertion.

Theorem. *Let G be an infinite group of cardinality τ. Then there exists a disjoint family $\{B_\mu : \mu \in M\}$ of subsets of G such that:*
1. $|M| = |G|$.
2. $G = \cup\{B_\mu : \mu \in M\}$.
3. $(G \setminus B\mu) \cdot K \neq G$ *for all* $\mu \in M$ *and every finite subset* K *of* G.
4. *The sets* $\{B_\mu : \mu \in M\}$ *are dense in all totally bounded topologies on* G.

This fact is a generalization of one Protasov's result [9]. In this paper the assertions of Theorem are proved for the special algebras – I_nP_k-n-groupoids. We shall use the notation and terminology from [1–4, 7, 8]. In particular, $|X|$ is the cardinality of a set $|X|$, $N = 0, 1, 2, ...$, R is the space of reals. By ω_0 we denote the first infinite cardinal. If τ is an infinite cardinal, then τ^+ is the first cardinal larger than τ. If $\tau \geq 1$ is a cardinal, then the space X is called τ-resolvable if there exists a family of pairwise disjoint dense subsets $\{B_\alpha : \alpha \in A\}$ of X such that $|A| = \tau$. Every space is 1-resolvable. If the space X is 2-resolvable, then we say that X is resolvable.

Denote by a_1^m a sequence $a_1, a_2, ..., a_m$. If $a_1 = a_2 = ... = a_m$, then we denote this sequence by a^m. For every space X we put

$$m(X) = min\{|U| : U \neq \emptyset, U \subseteq X, U \in \tau\}.$$

A space X is maximal resolvable if it is $m(X)$-resolvable. It is clear that if X is τ-resolvable then $\tau \leq m(X)$. If $m(X) = |X| > 1$ and X is maximal resolvable, then we say that X is superresolvable.

For every mapping $f : X \to X$ we put $f' = f$ and $f^{n+1} = f \circ f^n$ for any $n \in N$. We can consider that $f^0 : X \to X$ is the identity mapping.

The problem of resolvability of totally bounded topological groups was solved by V.I. Malykhin, W.W. Comfort, S. Van Mill [5], I.V. Protasov [9] and M.M. Choban, L.L. Chiriac [6].

2 Groupoids with invertibility properties

Fix a sequence $\{E_n : n \in N\}$ of pairwise disjoint spaces. The discrete sum $E = \cup\{E_n : n \in N\}$ is called a signature or a set of fundamental operations. A universal algebra of signature E, or briefly, an E-algebra is a non-empty set G and a sequence of mappings $e_G = \{e_{nG} : E_n \times G^n \longrightarrow G : n \in N\}$. The set G is called a support of the E-algebra G and the mappings e_G are called the algebraical structure on G. Let G be an E-algebra. If $u \in E_0$, then the element $u_G = e_{0G}(\{u\} \times G^0)$ is called a constant of G and we put $u(x) = u_G$ for all $x \in G$. If $n \geq 1, u \in E_n$ and $x_1, ..., x_n \in G$, then we put $u(x_1, ..., x_n) = e_{nG}(u, x_1, ..., x_n)$. A pair (G, ω) is said to be a n-groupoid if G is a non-empty set and $\omega : G^n \to G$ is a mapping.

Definition 1. Let $k \leq n$. An n-groupoid (G, ω) is called:

1. an I_nP_k-n-groupoid if there exist the mappings $r_1, ..., r_{k-1}, r_{k+1}, ..., r_n : G \to G$ such that $\omega(r_1(x_1), ..., r_{k-1}(x_{k-1}), \omega(x_1, ..., x_{k-1}, y, x_{k+1}, ..., x_n), r_{k+1}(x_{k+1}), ..., r_n(x_n)) = y$ or $\omega(r_1^{k-1}(x_1^{k-1}), \omega(x_1^{k-1}, y, x_{k+1}^n), r_{k+1}^n(x_{k+1}^n)) = y$ for all $x_1, ..., x_{k-1}, x_{k+1}, ..., x_n, y \in G$. The mapping $r_i(x)$ is called k-involution, $i \in \{1, ..., k-1, k+1, ..., n\}$.

2. an I_nP-n-groupoid in the large sense if it is I_nP_k-n-groupoid for all $k = \overline{1, n}$. In this case the mapping $r_i(x)$ is called involution, $i \in \{1, ..., n\}$.

3. an I_nP-n-groupoid, or I_nP-n-groupoid in strong sense, if there exist the mappings $\{r_i : G \to G$: $i = \overline{1, n}\}$ such that $\{r_i : i \leqslant n, i \neq k\}$ is a family of k-involutions for any $k = \overline{1, n}$.

4. an I_0P_k-n-groupoid if there exist the mappings $r_1, ..., r_{k-1}, r_{k+1}, ..., r_n : G \to G$ such that $\omega(r_1(x), ..., r_{k-1}(x), \omega(x^{k-1}, y, x^{n-k}), r_{k-1}(x), ..., r_n(x)) = y$ for all $x, y \in G$.

5. an I_0P-n-groupoid if it is I_0P_k-n-groupoid for all $k = \overline{1, n}$.

Example 1. Let $(G, \cdot)$ be a topological non commutative group with the identity e. If we put $\omega(x, y, z, u) = y \cdot x \cdot u \cdot z$, then (G, ω) is an I_0P-4-quasigroup. Indeed:

1. (G, ω) is an I_0P_1-4-quasigroup for $r_2(y) = y^{-1}$, $r_3(z) = z^{-1}$, $r_4(u) = u^{-1}$. We have $\omega(\omega(x, t, t, t), r_2(t), r_3(t), r_4(t)) = r_2(t) \cdot t \cdot x \cdot t \cdot t \cdot r_4(t) \cdot r_3(t) = t^{-1} \cdot t \cdot x \cdot t \cdot t \cdot t^{-1} \cdot t^{-1} = e \cdot x \cdot t \cdot e \cdot t^{-1} = x \cdot t \cdot t^{-1} = x$.

2. (G, ω) is an I_0P_2-4-quasigroup for $r_1(x) = x^{-1}, r_3(z) = z^{-1}, r_4(u) = u^{-1}$. Really, $\omega(r_1(t), \omega(t, y, t, t), r_3(t), r_4(t)) = y \cdot t \cdot t \cdot t \cdot r_1(t) \cdot r_4(t) \cdot r_3(t) = y$.

3. (G, ω) is an I_0P_3-4-quasigroup for $r_1(x) = x^{-1}$, $r_2(y) = y^{-1}$, $r_4(u) = u^{-1}$. Really, $\omega(r_1(t), r_2(t), \omega(t, t, z, t), r_4(t)) = r_2(t) \cdot r_1(t) \cdot r_4(t) \cdot t \cdot t \cdot t \cdot z = z$.

4. (G, ω) is an I_0P_4-4-quasigroup for $r_1(x) = x^{-1}$, $r_2(y) = y^{-1}, r_3(z) = z^{-1}$. Really, $\omega(r_1(t), r_2(t), r_3(t), \omega(t, t, t, u)) = r_2(t) \cdot r_1(t) \cdot t \cdot t \cdot u \cdot t \cdot r_3(t) = u$.

In this case (G,ω) is an I_0P_i-4-quasigroup for every $i \in \{1,2,3,4\}$. Hence, (G,ω) is an I_0P-4-quasigroup.

Example 2. Let $(G,\cdot)$ be a topological group with the identity e. We put $\omega(x,y,z) = x \cdot y \cdot z$. In this case:

1. (G,ω) is a 3-groupoid;

2. (G,ω) is an I_0P_i-3-groupoid for every $i \in \{1,2,3\}$ and for $r_1(x) = r_2(x) = r_3(x) = x^{-1}$;

3. (G,ω) is an I_3P_2-3-groupoid for $r_1(x) = x^{-1}, r_3(x) = z^{-1}$. Indeed, $\omega(r_1(x), \omega(x,y,z), r_3(z)) = x^{-1} \cdot x \cdot y \cdot z \cdot z^{-1} = e \cdot y \cdot e = y$;

4. If the group G is non commutative, then (G,ω) is not an I_3P_i-3-groupoid for $i = \{1,3\}$.

Example 3. Let C be the field of the complex numbers, R be the field of the reals numbers. Let $A = C \setminus \{0\}$, $B = R \setminus \{0\}$ and $G = \{r \in R : r > 0\}$. Then $(A,\cdot)$, $(B,\cdot)$ and $(G,\cdot)$ are commutative multiplicative groups. We put $\omega(x,y,z) = x \cdot y^n \cdot z$, $n \geq 1$.

1. If $n = 1$, then (A,ω), (B,ω) and (G,ω) are I_3P-3 quasigroups.

2. If $n \geq 2$, then (A,ω), is a 3-groupoid with divisions. The equation $\omega(a,y,c) = d$ has n solutions.

3. If $n > 1$ and n is odd, then (B,ω) and (G,ω) are 3-quasigroups.

4. If $n \geq 2$ and n is even, then (B,ω) is not a 3-groupoid with divisions and (G,ω) is a 3-quasigroup.

5. (A,ω), (B,ω), (G,ω) are I_3P_1-3-groupoids and I_3P_3-3-groupoids. If $n \geq 2$, then (A,ω), (B,ω) and (G,ω) are not I_3P_2-3-groupoids.

Example 4. Let C be the field of the complex numbers and $A = C \setminus \{0\}$. We fix $k \in A$ and put $\omega_n(x_1, x_2, ..., x_n) = k \cdot x_1 \cdot x_2 \cdot ... \cdot x_n$, $(n \geq 2)$. In this case:

1. (A,ω_n) is a commutative quasigroup.

2. (A,ω_n) is an I_nP-n-groupoid in strong sense. Denote $r_i(x_i) = \sqrt[n-1]{\frac{1}{k^2}} \cdot x_i^{-1}$. Hence, $\omega_n(r_1(x_1), ..., r_{i-1}(x_{i-1}), \omega_n(x_1, ..., x_{i-1}, x_i, x_{i+1}, ..., x_n),$ $r_{i+1}(x_{i+1}), ..., r_n(x_n)) = k \cdot (\sqrt[n-1]{\frac{1}{k^2}})^{i-1} \cdot x_1^{-1} \cdot x_2^{-1} \cdot ... \cdot x_{i-1}^{-1} \cdot k \cdot x_1 \cdot x_2 \cdot ... \cdot x_{i-1} \cdot x_i \cdot$ $x_{i+1} \cdot ... \cdot x_n \cdot (\sqrt[n-1]{\frac{1}{k^2}})^{n-i} \cdot x_{i+1}^{-1} \cdot ... \cdot x_n^{-1} = k^2(\sqrt[n-1]{\frac{1}{k^2}})^{n-1} \cdot x_i = k^2 \cdot \frac{1}{k^2} \cdot x_i = x_i$. In strong sense there are $n-1$ complete involutions.

3. Let $n \geq 2$ and $m = 2+(n-1)$. There is $k \in A$ such that $k^m = 1$ and $k^i \neq 1$ for $i < m$. If $r_i(x_i) = k \cdot x_i^{-1}$ then $\{r_1, r_2, ..., r_n\}$ are involutions in strong sense. Hence, $\omega_n(r_1(x_1), ..., r_{i-1}(x_i), \omega_n(x_1, x_2, ..., x_n), r_{i+1}(x_{i+1}), ..., r_n(x_n)) = k^{n-1} \cdot k^2 \cdot x_1^{-1} \cdot ... \cdot$ $x_{i-1}^{-1} \cdot x_1 \cdot ... x_{i-1} \cdot x_i \cdot x_{i+1} \cdot ... \cdot x_n \cdot x_{i+1}^{-1} \cdot ... \cdot x_n^{-1} = k^{2+n-1} \cdot x_i = k^m \cdot x_i = x_i$.

4. Let $n = 2$, $m \geq 3$, $k^m = 1$ and $k^i \neq 1$ for $i < m$. We put $\omega(x,y) = k \cdot x \cdot y$, $r_1(x) = k^{m-2}x^{-1}$, $r_2(y) = k^{m-2}y^{-1}$. In this case $\{r_1(x), r_2(x)\}$ are unique involutions in strong sense and $r_i^2(x_i) = k^{m-2}(r_i(x_i))^{-1} = k^{m-2}((k^{m-2} \cdot x^{-1})^{-1}) = k^{m-2} \cdot \frac{1}{k^{m-2}} \cdot x_i = x_i$.

Example 5. Let $(G, \cdot)$ be a topological group with the identity. If we put $\omega(x, y) = x \cdot y$, then:

1. (G, ω) is a 2-groupoid or, briefly, groupoid;
2. (G, ω) is an RIP-groupoid for $r_2(x) = x^{-1}$. Indeed, $\omega(\omega(y, x), r_2(x)) = (y \cdot x) \cdot x^{-1} = y$;
3. (G, ω) is an LIP-groupoid for $r_1(x) = x^{-1}$. Indeed, $\omega(r_1(x_1), \omega(x, y)) = x^{-1}(x \cdot y) = y$;
4. (G, ω) is an IP-groupoid if it is both an RIP-groupoid and an LIP-groupoid.

The notions LIP, RIP in the class of groupoids were introduced by R. H. Bruck [4].

Proposition 1. *Let (G, ω) be an I_nP_1-n-groupoid and $r_2, r_3, ..., r_n : G \to G$ be 1-involutions. Then the following assertions are equivalent:*

1. $\omega(\omega(y, x_2, ..., x_n), r_2(x_2), ..., r_n(x_n)) = y$;

2. $\omega(\omega(y, r_2(x_2), ..., r_n(x_n)), x_2, ..., x_n) = y$ for all $x_2^n \in G$.

Proof. Suppose that

$$\omega(\omega(y, x_2, ..., x_n), r_2(x_2), ..., r_n(x_n)) = y \tag{1}$$

for all $x_2^n, y \in G$. From (1) we have

$$\omega(\omega(\omega(y, x_2, ..., x_n), r_2(x_2), ..., r_n(x_n)), r_2^2(x_2), ..., r_n^2(x_n)) = \omega(y, x_2, ..., x_n) \tag{2}$$

and

$$\omega(\omega(\omega(y, x_2, ..., x_n), r_2(x_2), ..., r_n(x_n)), r_2^2(x_2), ..., r_n^2(x_n)) = \tag{3}$$

$$= \omega(y, r_2^2(x_2), ..., r_n^2(x_n)).$$

Using (2) and (3) we obtain

$$\omega(y, x_2, ..., x_n) = \omega(y, r_2^2(x_2), ..., r_n^2(x_n)). \tag{4}$$

Let in (4) $y = \omega(y, r_2(x_2), ..., r_n(x_n))$. Therefore from (4)

$$\omega(\omega(y, r_2(x_2), ..., r_n(x_n)), x_2, ..., x_n)) =$$

$$= \omega(\omega(y, r_2(x_2), ..., r_n(x_n)), r_2^2(x_2), ..., r_n^2(x_n)).$$

The implication $1 \to 2$ is proved. Suppose that

$$\omega(\omega(y, r_2(x_2), ..., r_n(x_n)), x_2, ..., x_n) = y. \tag{5}$$

From (5) it follows that

$$\omega(\omega[y, r_2^2(x_2), ..., r_n^2(x_n)], r_2(x_2), ..., r_n(x_n)) = y. \tag{6}$$

It is clear that

$$\omega(\omega[\omega[y, r_2^2(x_2), ..., r_n^2(x_n)], r_2(x_2), ..., r_n(x_n)], x_2, ..., x_n) = \tag{7}$$

$$= \omega(y, r_2^2(x_2), ..., r_n^2(x_n)).$$

From (6) we obtain

$$\omega(\omega[\omega[y, r_2^2(x_2), ..., r_n^2(x_n)], r_2(x_2), ..., r_n(x_n)], x_2, ..., x_n) = \omega(y, x_2, ..., x_n). \tag{8}$$

Using (7) and (8) we have

$$\omega(y, r_2^2(x_2), ..., r_n^2(x_n)) = \omega(y, x_2, ..., x_n). \tag{9}$$

Therefore

$$\omega(\omega(y, x_2, ..., x_n), r_2(x_2), ..., r_n(x_n)) =$$

$$= \omega(\omega[y, r_2^2(x_2), ..., r_n^2(x_n)], r_2(x_2), ..., r_n(x_n)) = y.$$

The implication $2 \to 1$ is proved. The proof is complete. □

Definition 2. An n-groupoid (G, ω) is called:

1. a k-cancellative n-groupoid if for every $a, b, x_1, ..., x_{k-1}, x_{k+1}, ..., x_n \in G$ we have $\omega(x_1, ..., x_{k-1}, a, x_{k+1}, ..., x_n) = \omega(x_1, ..., x_{k-1}, b, x_{k+1}, ..., x_n)$ if and only if $a = b$.
2. a cancellative n-groupoid if it is k-cancellative groupoid for all $k = \overline{1, n}$
3. an n-quasigroup if the equation $\omega(a_1^{i-1}, x, a_{i+1}^n) = b$ has unique solution for every a_i^n, b and each $i = \overline{1, n}$.

Definition 3. An element e from (G, ω) is called:

1. a k-identity of n-groupoid (G, ω) if $\omega(e^{k-1}, x, e^{n-k}) = x$ for every $x \in G$.
2. an identity of n-groupoid (G, ω) if $\omega(e^{i-1}, x, e^{n-i}) = x$ for every $x \in G$ and each $i = \overline{1, n}$.

If n-quasigroup (G, ω) contains at least one identity, then (G, ω) is called n-loop.

Proposition 2. *Let (G, ω) be an $I_n P_1$-n-groupoid and $r_2, r_3, ..., r_n : G \to G$ be 1-involutions. Then:*

1. $\omega(x_1, x_2, ..., x_n) = \omega(x_1, r_2^2(x_2), ..., r_n^2(x_n))$ *for all* $x_1^n \in G$.
2. $\omega(\omega(y, r_2(x_2), ..., r_n(x_n)), x_2, ..., x_n) = y$ *for all* $x_2^n, y \in G$.
3. (G, ω) *is* 1*-cancellative .*
4. *For every* $b, a_2^n \in G$, *the equation* $\omega(y, a_2, ..., a_n) = b$ *has a unique solution.*

Proof. The proof of the assertion 1 is contained in the proof of Proposition 1. The assertion 2 follows from Proposition 1. Let $a, b, x_2^n \in G$ and $\omega(a, x_2, ..., x_n) = \omega(b, x_2, ..., x_n)$. Then $a = \omega(\omega(a, x_2, ..., x_n), r_2(x_2), ..., r_n(x_n)) = \omega(\omega(b, x_2, ..., x_n), r_2(x_2), ..., r_n(x_n)) = b$. The assertion 3 is proved. We consider the equation $\omega(y, a_2, ..., a_n) = b$. Then from Proposition 1 we have $y = \omega(b, r_2(x_2), ..., r_n(x_n))$. Hence the equation $\omega(y, a_2, ..., a_n) = b$ has a unique solution. The proof is complete. □

Corollary 1. *Let (G, ω) be an I_nP-n-groupoid in the large sense and $r_i : G \to G, i = \overline{1, n}$, are the involutions on G. Then (G, ω) is cancellative.*

Proof. The assertion follows from Proposition 2. □

Academician M.M. Choban observed the following interesting fact.

Proposition 3. *Let (G, ω) be an I_nP-n-groupoid in the large sense and $r_i : G \to G, i = \overline{1, n}$, are the involutions on G. Then $x_i = r_i^{2(n-1)}(x_i)$, for every $i = \overline{1, n}$ and $n \geq 2$.*

Proof. It is sufficient to prove that $x_1 = r_1^{2(n-1)}(x_1)$ for any $x_1 \in G$. Fix $x_1, x_2, ..., x_n \in G$. From Proposition 2 we have $\omega(x_1, x_2, ..., x_n) = \omega(x_1, r_2^2(x_2), ..., r_n^2(x_n)) = \omega(r_1^2(x_1), r_2^2(x_2), r_3^4(x_3), ..., r_n^4(x_n)) = ... = \omega(r_1^{2i}(x_1), ..., r_{i+1}^{2i}(x_{i+1}), r_{i+2}^{2(i+1)}(x_{i+2}), ..., r_n^{2(i+1)}(x_n)) = ... = \omega(r_1^{2(n-1)}(x_1), r_2^{2(n-1)}(x_2), ..., r_n^{2(n-1)}(x_n))$, i.e. It is obvious that $\omega(x_1, x_2, ..., x_n) = \omega(x_1, r_2^{2m}(x_2), ..., r_n^{2m}(x_n))$ for any $m \geq 1$. Hence for $m = n - 1$, we have $\omega(x_1, r_2^{2(n-1}(x_2), ..., r_n^{2(n-1)}(x_n)) = \omega(r_1^{2(n-1)}(x_1), r_2^{2(n-1)}(x_2), ..., r_n^{2(n-1)}(x_n))$. Therefore $x_1 = r_1^{2(n-1)}(x_1)$ for any $x_1 \in G$ and $x_i = r_i^{2(n-1)}(x_i)$, for every $i = \overline{1, n}$ and $n \geq 2$. The proof is complete. □

Proposition 4. *Let (G, ω) be an I_nP-n-groupoid in the large sense and $r_i : G \to G, i = \overline{2, n}$, are the involutions on G. If $e_1, e_2, ..., e_n \in G$, $e_i = r_i^{2m}(e_i)$, for all $i = \overline{2, n}$, then $x_i = r_i^{2m}(x_i)$, for every $x_i \in G$ and $n \geq 2$.*

Proof. From Proposition 2 it follows that $\omega(x_1, x_2, ..., x_n) = \omega(x_1, r_2^{2m}(x_2), ..., r_n^{2m}(x_n))$. Fix $i = \overline{2, n}$. Then $\omega(e_1, e_2, ..., e_{i-1}, x_i, e_{i+1}, ..., e_n) = \omega(e_1, e_2, ..., e_{i-1}, r_i^{2m}(x_i), e_{i+1}, ..., e_n)$. Hence, $x_i = r_i^{2m}(x_i)$, for every $x_i \in G$, $i = \overline{2, n}$ and $n \geq 2$. The proof is complete. □

3 Topologies on algebras

We consider arbitrary topologies on universal algebras. There are a lot of types of bounded topology. We fix $n \geq 2$ and $k \leq n$. Consider a mapping $\varphi : \{1, 2, ..., n\} \to \{1, 2, ..., n\}$. We will use Choban's bounded topology.

Definition 4. Let (G,ω) be an n-groupoid and $L_1, L_2, ..., L_n$ be a family of subsets of G. Then:

1. The sets $L_1, L_2, ..., L_n$ are k-α-associated with the mapping φ and denote $(L_1, L_2, ..., L_n)\alpha(k)\varphi$ if $L_i = L_j$ provided $\varphi(i) = \varphi(j)$ and $i \neq k$, $j \neq k$.

2. If $x_1, x_2, ..., x_n \in G$ and $(\{x_1\}, \{x_2\}, ..., \{x_n\})\alpha(k)\varphi$, then we put $(x_1, x_2, ..., x_n)\alpha(k)\varphi$.

3. We put $\Delta_{\varphi(k)}\omega(L_1, L_2, ..., L_n) = \{\omega(x_1, x_2, ..., x_n) : x_1 \in L_1, x_2 \in L_2, ..., x_n \in L_n$ and $(x_1, x_2, ..., x_n)\alpha(k)\varphi\}$.

Remark 1. Let $L_1, L_2, ..., L_n$ be subsets of G, and $L_k^{'}{=}L_k$ and $L_i^{'}{=}\bigcap\{L_j : j \leq n, \varphi(j) = \varphi(i)\}$ for any $i \neq k$. Then $(L_1^{'}, L_2^{'}, ..., L_n^{'})\alpha(k)\varphi$ and $\Delta_{\varphi(k)}\omega(L_1^{'}, L_2^{'}, ..., L_n^{'}){=}\Delta_{\varphi(k)}\omega(L_1, L_2, ..., L_n)$.

Definition 5. Let $k \leq n$. An n-groupoid (G,ω) is called an $I_\varphi P_k$-n-groupoid if there exist the mappings $r_i : G \to G$, $i \in \{1, ..., k-1, k+1, ..., n\}$ such that $\omega(r_1(x_1), ..., r_{k-1}(x_{k-1}), \omega(x_1, ..., x_{k-1}, y, x_{k+1}, ..., x_n), r_{k+1}(x_{k+1}), ..., r_n(x_n)) = y$ provided $(x_1, ..., x_{k-1}, x_{k+1}, ..., x_n)$ $\alpha(k)\varphi$ for all $x_1, ..., x_{k-1}, x_{k+1}, ..., x_n, y \in G$.

We say that the mapping $r_i : G \to G$, $i \in \{1, ..., k-1, k+1, ..., n\}$ is called k-φ-involution.

If $\varphi(i) = \varphi(j)$ for all $i, j \leq n$, then $I_\varphi P_k$-n-groupoid is an $I_0 P_k$-n-groupoid.

Definition 6. Let (G,ω) be an n-groupoid and λ be an infinite cardinal. A topology $\mathcal{T}$ on G is called:

– a λ-k-φ-bounded topology if for every non-empty open set $U \in \mathcal{T}$ there exists a subset $K \subseteq G$ such that $|K| < \lambda$ and $\Delta_{\varphi(k)}\omega(K^{k-1}, U, K^{n-k}) = G$.

– a λ-φ-bounded topology if it is λ-k-φ-bounded topology for every $k = \overline{1, n}$. An ω_0-k-φ-bounded topology is called a k-φ-totally bounded topology. The topology is said to be φ-totally bounded if it is a k-φ-totally bounded topology for every $k = \overline{1, n}$.

Remark 2. If in Definition 6 the mapping φ is one-to-one, then a topology $\mathcal{T}$ on G is called respectively: a λ-k-bounded topology, a λ-bounded topology, a ω_0-k-bounded topology, a k-totally bounded topology and totally bounded topology, for every $k = \overline{1, n}$.

Proposition 5. *Let $\varphi : \{1, 2, ..., n\} \to \{1, 2, ..., n\}$ be a mapping, (G,ω) be an n-groupoid with the properties:*

1. The equation $\omega(a^{k-1}, x, a^{n-k}) = b$ is solvable for every $a, b \in G$.

2. For every $a, b \in G$ there exist $a_1, a_2, ..., a_n \in G$ such that $a_k = a$, $(a_1, a_2, ..., a_n)\alpha(k)\varphi$ and $\omega(a_1, a_2, ..., a_n) = b$.

Then the minimal compact T_1-topology $\mathcal{T} = \{\varnothing\} \cup \{G \setminus F : F$ is a finite subset of $G\}$ is a k-φ-totally bounded topology on G.

Proof. Let $U \in \mathcal{T}$ and $U \neq \varnothing$. Then the set $F = G \setminus U$ is finite. Fix $a \in U$. Then $h_a : G \to G$, where $h_a(x) = \omega(a^{k-1}, x, a^{n-k})$ for any $x \in G$ is a mapping of G onto G. Thus $F^{'} = G \setminus h_a(U) \subseteq h_a(F)$ is a finite set. For any $x \in G$ there exist

$y_1(x), y_2(x), ..., y_n(x) \in G$ such that $y_k(x) = a$, $(y_1(x), y_2(x), ..., y_n(x))\alpha(k)\varphi$ and $\omega(y_1(x), y_2(x), ..., y_n(x)) = x$. We put $\Phi = \{a\} \cup \{\{y_1(x), y_2(x), ..., y_n(x)\} : x \in F^{'}\}$. The set Φ is finite. By construction, $\Delta_{\varphi(k)}\omega(\Phi^{k-1}, U, \Phi^{n-k}) = G$. The proof is complete. □

Proposition 6. *Let $\varphi : \{1, 2, ..., n\} \to \{1, 2, ..., n\}$ be a mapping, (G, ω) be an n-groupoid with the properties:*

1. For every $a, b \in G$ there exist $a_1, a_2, ..., a_n \in G$ such that $a_k = a$, $(a_1, a_2, ..., a_n)\alpha(k)\varphi$ and $\omega(a_1, a_2, ..., a_n) = b$.

2. There exists $e \in G$ such that $G \setminus \omega(e^{k-1}, G, e^{n-k})$ is a finite set (in particular, $\omega(e^{k-1}, x, e^{n-k}) = x$ for every $x \in G$).

Then the minimal compact T_1-topology $\mathcal{T} = \{\varnothing\} \cup \{G \setminus F : F$ is a finite subset of $G\}$ is a k-φ-totally bounded topology on G.

Proof. Let $U \in \mathcal{T}$ and $U \neq \varnothing$. Then the set $F = G \setminus U$ is finite. Fix $a \in U$. Consider the mapping $h_e : G \to G$, where $h_e(x) = \omega(e^{k-1}, x, e^{n-k})$ for any $x \in G$. The set $G \setminus h_e(G)$ is finite. Thus the set $F^{'} = G \setminus h_e(U) \subseteq (G \setminus h_e(G)) \bigcup h_e(F)$ is a finite set. For any $x \in F^{'}$ fix $\{y_1(x), y_2(x), ..., y_n(x)\} \subseteq G$ such that $y_k(x) = a$, $(y_1(x), y_2(x), ..., y_n(x))\alpha(k)\varphi$ and $\omega(y_1(x), y_2(x), ..., y_n(x)) = x$. Let $\Phi = \{e\} \cup \cup\{\{y_1(x), y_2(x), ..., y_n(x)\} : x \in F^{'}\}$. The set Φ is finite. By construction, $\Delta_{\varphi(k)}\omega(\Phi^{k-1}, U, \Phi^{n-k}) = G$. The proof is complete. □

Proposition 7. *Let $\varphi : \{1, 2, ..., n\} \to \{1, 2, ..., n\}$ be a mapping, (G, ω) be an infinite I_nP_k-n-groupoid, $B \subseteq G$, m be an infinite cardinal and $\Delta_{\varphi(k)}\omega(K^{k-1}, G \setminus B, K^{n-k}) \neq G$ for every subset K of cardinality $|K| < m$. Then the set B is dense in every m-k-φ-bounded topology $\mathcal{T}$ on G.*

Proof. Suppose that $\mathcal{T}$ is an m-k-φ-bounded topology on G and $U = G \setminus cl_G B \neq \varnothing$. Then $U \in \mathcal{T}$ and $U \subseteq G \setminus B$. By assumption there exists a subset K of G such that $\Delta_{\varphi(k)}\omega(K^{k-1}, U, K^{n-k}) = G$ and $|K| < m$. Since $U \subseteq G \setminus B$, we have $G \supseteq \Delta_{\varphi(k)}\omega(K^{k-1}, G \setminus B, K^{n-k}) \supseteq \Delta_{\varphi(k)}\omega(K^{k-1}, U, K^{n-k}) = G$, a contradiction. The proof is complete. □

4 Decomposition of I_nP_k-n-groupoids

We fix $n \geq 2$ and $k \leq n$. Consider a mapping $\varphi : \{1, 2, ..., n\} \to \{1, 2, ...n\}$.

Lemma 1. *Let G be an infinite I_nP_k-n-groupoid, $r_1, ..., r_{k-1}, r_{k+1}, ..., r_n : G \to G$ be k-involutions, L and M be subsets of G and $|L \cup M| < |G|$. Then there exists an element $a \in G$ such that $\omega(L^{k-1}, a, L^{n-k}) \cap M = \varnothing$ and $\Delta_{\varphi(k)}\omega(L^{k-1}, a, L^{n-k}) \cap M = \varnothing$.*

Proof. Let $H = \{\omega(r_1(y_1), ..., r_{k-1}(y_{k-1}), x, r_{k+1}(y_{k+1}), ..., r_n(y_n)) : x \in M, y_1, ..., y_{k-1}, y_{k+1}, ..., y_n \in L\}$. Thus $|H| < |G|$ and there exists an element $a \in G \setminus H$.

Suppose that $\omega(L^{k-1}, a, L^{n-k}) \cap M \neq \varnothing$. Fix $\omega(L^{k-1}, a, L^{n-k}) \cap M$. Then $x = \omega(y_1, ..., y_{k-1}, a, y_{k+1}, ..., y_n)$ for some $y_1, ..., y_{k-1}, y_{k+1}, ..., y_n \in L$. Hence

$$a = \omega(r_1(y_1), ..., r_{k-1}(y_{k-1}), \omega(y_1^{k-1}, a, y_{k+1}^n), r_{k+1}(y_{k+1}), ..., r_n(y_n)) =$$

$$= \omega(r_1(y_1), ..., r_{k-1}(y_{k-1}), x, r_{k+1}(y_{k+1}), ..., r_n(y_n)) \in$$

$$\in \omega(r_1(y_1), ..., r_{k-1}(y_{k-1}), M, r_{k+1}(y_{k+1}), ..., r_n(y_n)) \subseteq H,$$

a contradiction. By construction, $\Delta_{\varphi(k)}\omega(L^{k-1}, M, L^{n-k}) \subseteq \omega(L^{k-1}, M, L^{n-k})$. Hence, $\Delta_{\varphi(k)}\omega(L^{k-1}, a, L^{n-k}) \cap M = \varnothing$. The proof is complete. □

Theorem 1. *Let G be an infinite I_nP_k-n-groupoid, $\mathcal{L}$ be a a non-empty family of non-empty subsets of G, $|\mathcal{L}| \leq |G|$ and for every set A and mapping $\Psi : A \to \mathcal{L}$ we have $| \cup \{\Psi(\alpha) : \alpha \in A\} |<| G |$ provided $| A |<| G |$. Then there exists a family $\{B_\mu : \mu \in M\}$ of non-empty subsets of G such that:*

1. *$| M |=| G |$.*
2. *$B_\mu \cap B_\eta = \varnothing$ for all $\alpha, \beta \in M$ and $\alpha \neq \beta$.*
3. *$G = \cup\{B_\mu : \mu \in M\}$.*
4. *$\omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$ for all $\mu \in M$ and $K \in \mathcal{L}$.*
5. *$\Delta_{\varphi(k)}\omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$ for all $\mu \in M$ and $K \in \mathcal{L}$.*

Proof. Consider on G some k-involutions, $r_1, ..., r_{k-1}, r_{k+1}, ..., r_n : G \to G$. Let $\tau = |G|$. Denote by $|\alpha|$ the cardinality of the ordinal number α. We put $\Omega_\tau = \{\alpha : 1 \leq |\alpha| < \tau\}$. If $K \subseteq G$, then $K_i^{-1} = \{r_i(x_i) : x_i \in K\}, i = 1, ..., k-1, k+1, ..., n$, and $K^{-1} = \cup\{K_i^{-1} : i = 1, 2, ..., k-1, k+1, ...n\}$. Let $\mathcal{L}_\infty = \{K^{-1} : K \in \mathcal{L}\} \cup \mathcal{L}$. It is clear that $|\mathcal{L}_1| \leq \tau$. Moreover, if A is a set, $|A| < \tau$ and $\Psi : A \to \mathcal{L}_1$ is a mapping, then $| \cup \{\Psi(\alpha) : \alpha \in A\}| < \tau$. Fix a set M of the cardinality τ. Since $|\Omega_\tau| = |M \times \mathcal{L}_1| = \tau$ then there exists a bijection $h : \Omega_\tau \to M \times \mathcal{L}_1$. If $\alpha \in \Omega_\tau$, then we consider that $h(\alpha) = (\mu_\alpha, K_\alpha) \in M \times \mathcal{L}_1$. If $\mu \in M$, then we put $A_\mu = h^{-1}(\{\mu\} \times \mathcal{L}_1)$. It is obvious that $A_\mu = \{\alpha \in \Omega_\tau : \mu_\alpha = \mu\}$ and $\{K_\alpha : \alpha \in A_\mu\} = \mathcal{L}_1$. Now we affirm that there exists a transfinite sequence $\{a_\alpha : \alpha \in \Omega_\tau\} \subseteq G$ such that $\omega(K_\alpha^{k-1}, a_\alpha, K_\alpha^{n-k}) \cap \omega(K_\beta^{k-1}, a_\beta, K_\beta^{n-k}) = \varnothing$ for all $\alpha, \beta \in \Omega_\tau$ and $\alpha \neq \beta$. We fix $a_1 \in G$. Let $1 < \beta, \beta \in \Omega_\tau$ and the elements $\{a_\alpha : \alpha < \beta\}$ are constructed. We put now $H_\beta = \cup\{\omega(K_\alpha^{k-1}, a_\alpha, K_\alpha^{n-k}) : \alpha < \beta\}$. Since $|\alpha \in \Omega_\tau : \alpha < \beta| \leq |\beta| < |G|$, then $|H_\beta| < |G|$. From Lemma 1 it follows that there exists $a_\beta \in G$ such that $\omega(K_\beta^{k-1}, a_\beta, K_\beta^{n-k}) \cap H_\beta = \varnothing$. By the transfinite induction if follows that the set $\{a_\alpha : \alpha \in \Omega_\tau\}$ is constructed. We put $P_\mu = \cup\{\omega(K_\alpha^{k-1}, a_\alpha, K_\alpha^{n-k}) : \alpha \in A_\mu\}$ for every $\mu \in H$. Fix $\mu, \eta \in M$ and $\mu \neq \eta$. Then $A_\mu \cap A_\eta = \varnothing$. Since $\omega(K_\alpha^{k-1}, a_\alpha, K_\alpha^{n-k}) \cap \omega(K_\beta^{k-1}, a_\beta, K_\beta^{n-k}) = \varnothing$ for all $\alpha \in A_\alpha$ and $\beta \in A_\eta$, then $P_\mu \cap P_\eta = \varnothing$. Fix $\mu \in M$ and $K \in \mathcal{L}$. Then $K^{-1} \in \mathcal{L}_1$ and $(\mu, K^{-1}) = (\mu_\alpha, K_\alpha)$ for some $\alpha \in A_\mu$. Suppose that $\omega(K^{k-1}, G \setminus P_\mu, K^{n-k}) = G$. Then $a_\alpha \in \omega(K^{k-1}, G \setminus P_\mu, K^{n-k})$, i.e. $a_\alpha = \omega(y_1^{k-1}, x, y_{k+1}^n)$ for some $x \in G \setminus P_\mu$ and $y_1, ..., y_{k-1}, y_{k+1}, ..., y_n \in K$. By construction, we have $r_1(y_1), ..., r_{k-1}(y_{k-1}), r_{k+1}(y_{k+1}), ..., r_n(x_n) \in K_\alpha$ and

$\omega(r_1(y_1), ..., r_{k-1}(y_{k-1}), a_\alpha, r_{k+1}(y_{k+1}), ..., r_n(x_n)) \in \omega(K_\alpha^{k-1}, a_\alpha, K_\alpha^{n-k}) \subseteq P_\mu$. By assumption, we have that

$$\omega(r_1(y_1), ..., r_{k-1}(y_{k-1}), a_\alpha, r_{k+1}(y_{k+1}), ..., r_n(x_n)) =$$

$$= \omega(r_1(y_1), ..., r_{k-1}(y_{k-1}), \omega(y_1^{k-1}, x, y_{k+1}^{n-k}), r_{k+1}(y_{k+1}), .., r_n(y_n)) = x \in G \setminus P_\mu,$$

a contradiction. Hence $\omega(K^{k-1}, G \setminus P_\mu, K^{n-k}) \neq G$ for all $\mu \in M$ and $K \in \mathcal{L}$. Now we fix $\mu_0 \in M$. We put $B_\mu = P_\mu$ for all $\mu \in M \setminus \{\mu_0\}$ and $B_{\mu_0} = G \setminus \cup\{P_\mu : \mu \in M\setminus\{\mu_0\}\}$. By construction, we have $P_\mu \subseteq B_\mu$ for all $\mu \in M$ and $G = \cup\{B_\mu : \mu \in H\}$. If $\mu \in M$, then $G \setminus B_\mu \subseteq G \setminus P\mu$ and $\omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$ for all $K \in \mathcal{L}$. The proof is complete. □

Theorem 2. *Let (G, ω) be an infinite I_nP_k-n-groupoid, $\tau = |G|$, m be an infinite cardinal, $\tau = \sum\{\tau^q : q < m\}$ and either $m < \tau$, or τ be a regular cardinal. If $\mathcal{L}_m = \{K \subseteq G : |K| < m\}$, then there exists a family $\{B_\mu : \mu \in M\}$ of non-empty subsets of G such that:*

1. *$|M| = \tau$.*
2. *$B_\mu \cap B_\eta = \varnothing$ for all $\mu, \eta \in M$ and $\mu \neq \eta$.*
3. *$G = \cup\{B_\mu : \mu \in M\}$.*
4. *$\omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$ for all $\mu \in M$ and $K \in \mathcal{L}_m$.*
5. *$\Delta_{\varphi(k)}\omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$ for all $\mu \in M$ and $K \in \mathcal{L}_m$*
6. *The sets B_μ are dense in every m-k-φ-bounded topology on G.*
7. *Relative to every m-k-φ-bounded topology G is super-resolvable.*
8. *The sets B_μ are dense in every m-k-bounded topology on G.*
9. *Relative to every m-k-bounded topology G is super-resolvable.*

Proof. Since $\tau = \sum\{\tau^q : q < m\}$, we have $m \leq \tau$. Let A be a set, $|A| < \tau, \Psi : A \to L_m$ be a mapping and $H = \cup\{\Psi(\alpha) : \alpha \in A\}$. If $m < \tau$, then $|H| \leq \omega(m, ...m, |A|, m, ..., m) = \omega(m^{k-1}, |A|, m^{n-k}) < \tau$. If $m = \tau$ and $|H| = \tau$, then $cf(\tau) \leq |A| < \tau$ and the cardinal τ is not regular. Hence $|H| < \tau$. Theorem 1 and Proposition 7 complete the proof. □

Corollary 2. *Let G be an infinite I_nP_k-n-groupoid. Then there exists a family $\{B_\mu : \mu \in M\}$ of non-empty subsets of G such that:*

1. *$|M| = |G|$.*
2. *$B_\mu \cap B_\eta = \varnothing$ for all $\mu, \eta \in M$ and $\mu \neq \eta$.*
3. *$G = \cup\{B_\mu : \mu \in M\}$.*
4. *$\omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$ for all $\mu \in M$ and every finte subset K of G.*
5. *$\Delta_{\varphi(k)}\omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$ for all $\mu \in M$ and every finte subset K of G.*
6. *The sets $\{B_\mu : \mu \in M\}$ are dense in every k-φ-totally bounded topology on G.*
7. *Relative to every k-φ-totally bounded topology G is super-resolvable.*
8. *The sets $\{B_\mu : \mu \in M\}$ are dense in every k-totally bounded topology on G.*
9. *Relative to every k-totally bounded topology G is super-resolvable.*

Corollary 3. *Let G be an infinite I_nP_k-n-groupoid, $\tau = |G|$, m be an infinite cardinal and $\tau^m = \tau$. Then there exists a family $\{B_\mu : \mu \in M\}$ of non-empty subsets of G such that:*

1. $|M| = |G|$.
2. $B_\mu \cap B_\eta = \varnothing$ *for all* $\mu, \eta \in M$ *and* $\mu \neq \eta$.
3. $G = \cup\{B_\mu : \mu \in M\}$.
4. *If* $\mu \in M$, $K \subseteq G$ *and* $|K| < m$ *then* $\omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$.
5. *If* $\mu \in M$, $K \subseteq G$ *and* $|K| < m$ *then* $\Delta_{\varphi(k)}\omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$.
6. *The sets* $\{B_\mu : \mu \in M\}$ *are dense in every* m^+-k-φ-*bounded topology on* G.
7. *Relative to every* m^+-k-φ-*bounded topology* G *is super-resolvable.*
8. *The sets* $\{B_\mu : \mu \in M\}$ *are dense in every* m^+-k-*bounded topology on* G.
9. *Relative to every* m^+-k-*bounded topology* G *is super-resolvable.*

5 Decomposition of I_nP-n-groupoids

We fix $n \geq 2$ and $k \leq n$. Consider a mapping $\varphi : \{1, 2, ..., n\} \to \{1, 2, ...n\}$.

Lemma 2. *Let G be an infinite I_nP-gruopoid, $r_1, ..., r_n : G \to G$ be involutions, L and M be subsets of G and $|L \cup M| < |G|$. Then there exists an element $a \in G$ such that:*

1. $\bigcup_{k=1}^n \omega(L^{k-1}, a, L^{n-k}) \cap M = \varnothing$, *where* $\bigcup_{k=1}^n \omega(L^{k-1}, a, L^{n-k}) = \omega(a, L^{n-1}) \cup \omega(L^1, a, L^{n-2}) \cup ... \cup \omega(L^{n-1}, a)$.

2. $\bigcup_{k=1}^n \Delta_{\varphi(k)}\omega(L^{k-1}, a, L^{n-k}) \cap M = \varnothing$, *where* $\bigcup_{k=1}^n \Delta_{\varphi(k)}\omega(L^{k-1}, a, L^{n-k}) = \Delta_{\varphi(k)}\omega(a, L^{n-1}) \cup \Delta_{\varphi(k)}\omega(L^1, a, L^{n-2}) \cup ... \cup \Delta_{\varphi(k)}\omega(L^{n-1}, a)$.

Proof. Let $H = \{\omega(x, r_2(y_2), ..., r_n(y_n)) : x \in M, y_2, ...y_n \in L\} \cup \{\omega(r_1(y_1), x, r_3(y_3), ..., r_n(y_n)) : x \in M, y_1, y_3, ..., y_n \in L\} \cup ... \cup \{\omega(r_1(y_1), ..., r_{n-1}(y_{n-1}), x) : x \in M, y_1, ...y_{n-1} \in L\}$. Since $|H| < |G|$, then there exists an element $a \in G \setminus H$. Let $\omega(a, L, ...L) \cap M \neq \varnothing$. Fix $x \in \omega(a, L, ...L) \cap M$. Then $x = \omega(a, y_2, ..., y_n)$ for some $y_2, ..., y_n \in L$. Hence $a = \omega(\omega(a, y_2, ..., y_n), r_2(y_2), ..., r_n(y_n)) = \omega(x, r_2(y_2), ..., r_n(y_n)) \in \omega(M, r_2(y_2), ..., r_n(y_n)) \leq H$, a contradiction. In similar way we prove that $\omega(L^{k-1}, a, L^{n-k}) \cap M$ for all $k = \overline{1, n}$. Hence $\bigcup_{k=1}^n \omega(L^{k-1}, a, L^{n-k}) \cap M = \varnothing$. By construction, $\Delta_{\varphi(k)}\omega(L^{k-1}, a, L^{n-k}) \subseteq \omega(L^{k-1}, a, L^{n-k})$. Hence, $\bigcup_{k=1}^n \Delta_{\varphi(k)}\omega(L^{k-1}, a, L^{n-k}) \cap M = \varnothing$. The proof is complete. □

Theorem 3. *Let G be an infinite I_nP-n-groupoid, $\mathcal{L}$ be a non-empty family of non-empty subsets of G, $|\mathcal{L}| \leq |G|$ and for every set A and mapping $\Psi : A \to \mathcal{L}$ we have $| \cup\{\Psi(\alpha) : \alpha \in A\} |<| G |$ provided $| A |<| G |$. Then there exists a family $\{B_\mu : \mu \in M\}$ of non-empty subsets of G such that:*

1. $| M |=| G |$.
2. $B_\mu \cap B_\eta = \varnothing$ *for all* $\alpha, \beta \in M$ *and* $\alpha \neq \beta$.
3. $G = \cup\{B_\mu : \mu \in M\}$.
4. $\bigcup_{k=1}^n \omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$ *for all* $\mu \in M$ *and* $K \in \mathcal{L}$
5. $\bigcup_{k=1}^n \Delta_{\varphi(k)}\omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$ *for all* $\mu \in M$ *and* $K \in \mathcal{L}$.

Proof. Consider on G involutions, $r_1, ..., r_n : G \to G$. Let $\tau = |G|$. Denote by $|\alpha|$ the cardinality of the ordinal number α. We put $\Omega_\tau = \{\alpha : 1 \leq |\alpha| < \tau\}$. If $K \subseteq G$, then $K_i^{-1} = \{r_i(x_i) : i = \overline{1,n}, x_i \in K\}$. We put $K^{-1} = \cup K_i^{-1}$ and $\mathcal{L}_1 = \{K^{-1} : K \in \mathcal{L}\} \cup \mathcal{L}$. It is clear that $|\mathcal{L}_1| \leq \tau$. Moreover, if A is a set, $|A| < \tau$ and $\Psi : A \to \mathcal{L}_1$ is a mapping, then $|\cup\{\Psi(\alpha) : \alpha \in A\}| < \tau$. Fix a set M of the cardinality τ. Since $|\Omega_\tau| = |M \times \mathcal{L}_1| = \tau$, then there exists a bijection $h : \Omega_\tau \to M \times \mathcal{L}_1$. Let $A_\mu = h^{-1}(\{\mu\} \times \mathcal{L}_1) = \alpha \in \Omega_\tau : \mu_\alpha = \mu\}$. If $\alpha \in \Omega_\tau$, then we consider that $h(\alpha) = (\mu_\alpha, K_\alpha) \in M \times \mathcal{L}_1$. It is obvious that $A_\mu = \{\alpha \in \Omega_\tau : \mu_\alpha = \mu\}$ and $\{K_\alpha : \alpha \in A_\mu\} = \mathcal{L}_1$. As in the proof of Theorem 1 from Lemma 2 it follows that there exists a transfinite sequence $\{a_\alpha \in G : \alpha \in \Omega_\tau\}$ such that $(\bigcup_{k=1}^n \omega(K_\alpha^{k-1}, a_\alpha, K_\alpha^{n-k})) \cap (\bigcup_{k=1}^n \omega(K_\beta^{k-1}, a_\beta, K_\beta^{n-k})) = \varnothing$ for all $\alpha, \beta \in \Omega_\tau$ and $\alpha \neq \beta$. Now we put $P_\mu = \cup\{\bigcup_{k=1}^n \omega(K_\alpha^{k-1}, a_\alpha, K_\alpha^{n-k}) : \alpha \in A_\mu\}$ for every $\mu \in M$. If $P_\mu^k = \bigcup_{k=1}^n \omega\{(K_\alpha^{k-1}, a_\alpha, K_\alpha^{n-k}) : \alpha \in A_\mu\}$ for all $k = \overline{1,n}$, then $P_\mu = \bigcup_{k=1}^n P_\mu^k$ and $\omega(K^{k-1}, G \setminus P_\mu^k, K^{n-k}) \neq G$ for every $K \in \mathcal{L}$. Suppose that $K \in \mathcal{L}$, $\mu \in M$ and $G = \bigcup_{k=1}^n \omega(K^{k-1}, G \setminus P_\mu^k, K^{n-k})$. For some $\alpha \in A_\mu$ we have $K_\alpha = \bigcup_{i=1}^n K_i^{-1} = K^{-1}$. Then $\bigcup_{k=1}^n \omega(K_\alpha^{k-1}, a_\alpha, K_\alpha^{n-k}) \subseteq P_\mu$ and $a_\alpha \in G$. Suppose that $a_\alpha \in \omega(K^{k-1}, G \setminus P_\mu^k, K^{n-k})$. Then $a_\alpha = \omega(y_1, ..., y_{k-1}, x, y_{k+1}, ..., y_n)$ for some $y_1, ..., y_{k-1}, y_{k+1}, ..., y_n \in K$ and $x \in G \setminus P_\mu$. Therefore $\omega(r_1(y_1), ..., r_{k-1}(y_{k-1}), a_\alpha, r_{k+1}(y_{k+1}), ..., r_n(y_n)) = \omega(r_1(y_1), ..., r_{k-1}(y_{k-1}), \omega(y_1^{k-1}, x, y_{k+1}^{n-k}), r_{k+1}(y_{k+1}), .., r_n(y_n)) = x \in G \setminus P_\mu$. Since $r_i(y_i \in K_\alpha)$, $i = \overline{1,n}$, we have $x = \omega(r_1(y_1), ..., r_{k-1}(y_{k-1}), a_\alpha, r_{k+1}(y_{k+1}), ..., r_n(y_n)) \in \omega(K_\alpha^{k-1}, a_\alpha, K_\alpha^{n-k}) \subseteq P_\mu$, a contradiction. Hence $\bigcup_{k=1}^n \omega(K^{k-1}, G \setminus P_\mu, K^{n-k}) \neq G$ for all $\mu \in M$ and $K \in \mathcal{L}$. Now we fix $\mu_0 \in M$. We put $B_\mu = P_\mu$ for all $\mu \in M \setminus \{\mu_0\}$ and $B_{\mu_0} = G \setminus \cup\{P_\mu : \mu \in M \setminus \{\mu_0\}\}$. By construction, we have $P_\mu \subseteq B_\mu$ for all $\mu \in M$ and $G = \cup\{B_\mu : \mu \in H\}$. If $\mu \in M$, then $G \setminus B_\mu \subseteq G \setminus P\mu$ and $\bigcup_{k-1}^n \omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$ for all $K \in \mathcal{L}$. The proof is complete. □

Theorem 4. *Let (G) be an infinite I_nP-n-groupoid, $\tau = |G|$, m be an infinite cardinal, $\tau = \sum\{\tau^q : q < m\}$ and either $m < \tau$, or τ be a regular cardinal. If $\mathcal{L}_m = \{K \subseteq G : |K| < m\}$, then there exists a family $\{B_\mu : \mu \in M\}$ of non-empty subsets of G such that:*

1. *$|M| = \tau$.*
2. *$B_\mu \cap B_\eta = \varnothing$ for all $\mu, \eta \in M$ and $\mu \neq \eta$.*
3. *$G = \cup\{B_\mu : \mu \in M\}$.*
4. *$\bigcup_{k=1}^n \omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$ for all $\mu \in M$ and $K \in \mathcal{L}_m$.*
5. *$\bigcup_{k=1}^n \Delta_{\varphi(k)}\omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$ for all $\mu \in M$ and $K \in \mathcal{L}_m$.*
6. *The sets B_μ are dense in every m-φ-bounded topology on G.*
7. *Relative to every m-φ-bounded topology T on G the space (G, T) is super-resolvable.*
8. *The sets B_μ are dense in every m-bounded topology on G.*
9. *Relative to every m-bounded topology T on G the space (G, T) is super-resolvable.*

Proof. Is similar to the proof of Theorem 2. □

Corollary 4. *Let G be an infinite I_nP-n-groupoid. Then there exists a family $\{B_\mu : \mu \in M\}$ of non-empty subsets of G such that:*

1. $|M| = |G|$.
2. $B_\mu \cap B_\eta = \varnothing$ *for all* $\mu, \eta \in M$ *and* $\mu \neq \eta$.
3. $G = \cup\{B_\mu : \mu \in M\}$.
4. $\bigcup_{k=1}^{n} \omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$ *for all* $\mu \in M$ *and every finte subset* K *of* G.
5. $\bigcup_{k=1}^{n} \Delta_{\varphi(k)}\omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$ *for all* $\mu \in M$ *and every finte subset* K *of* G.
6. *The sets* $\{B_\mu : \mu \in M\}$ *are dense in every* φ*-totally bounded topology on* G.
7. *Relative to every* φ*-totally bounded topology* G *is super-resolvable.*
8. *The sets* $\{B_\mu : \mu \in M\}$ *are dense in every totally bounded topology on* G.
9. *Relative to every totally bounded topology* G *is super-resolvable.*

Corollary 5. *Let G be an infinite I_nP-n-groupoid, $\tau = |G|$, m be an infinite cardinal and $\tau^m = \tau$. Then there exists a family $\{B_\mu : \mu \in M\}$ of non-empty subsets of G such that:*

1. $|M| = |G|$.
2. $B_\mu \cap B_\eta = \varnothing$ *for all* $\mu, \eta \in M$ *and* $\mu \neq \eta$.
3. $G = \cup\{B_\mu : \mu \in M\}$.
4. *If* $\mu \in M$, $K \subseteq G$ *and* $|K| \leq m$ *then* $\bigcup_{k=1}^{n} \omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$.
5. *If* $\mu \in M$, $K \subseteq G$ *and* $|K| \leq m$ *then* $\bigcup_{k=1}^{n} \Delta_{\varphi(k)}\omega(K^{k-1}, G \setminus B_\mu, K^{n-k}) \neq G$.
6. *The sets* $\{B_\mu : \mu \in M\}$ *are dense in every* m^+*-*k*-*φ*-bounded topology on* G.
7. *Relative to every* m^+*-*k*-*φ*-bounded topology* G *is super-resolvable.*
8. *The sets* $\{B_\mu : \mu \in M\}$ *are dense in every* m^+*-*k*-bounded topology on* G.
9. *Relative to every* m^+*-*k*-bounded topology* G *is super-resolvable.*

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