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Computation of inertial manifolds in biological models. FitzHugh-Nagumo model

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Abstract. Inertial manifolds are related to the large time behaviour of dynamical systems. An algorithm, based on the Lyapunov-Perron method, is implemented here and used to construct a sequence of approximate inertial manifolds for a biological model. The hypotheses of the Jolly, Rosa, Temam's algorithm are verified for the FitzHugh-Nagumo model in the case of real eigenvalues. This algorithm is used for the construction of approximate inertial manifolds.

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1 Introduction

The purpose of this paper is to study the approximate inertial manifolds for FitzHugh-Nagumo model, in the case of real eigenvalues using an algorithm developed by Jolly, Rosa and Temam in [5, 6].

Let us consider the abstract evolution equation

$$\frac{du}{dt} + Au = f(u), \tag{1}$$

with the initial condition $u(0) = u_0$. Using the associated semigroup $\{S(t)\}_{t\geq 0}$, where $S(t) : u_0 \to u(t)$, $u(\cdot)$ is the solution of (1), with $u(0) = u_0$, the definition of inertial manifolds is given below.

Definition 1. [8]. An **inertial manifold** $\mathcal{M}$ is a finite-dimensional Lipschitz manifold, positively invariant (i.e. $S(t)\mathcal{M} \subset \mathcal{M}$, $t \geq 0$) and which exponentially attracts all orbits of (1).

Any inertial manifold contains the global attractor; and it is easier to describe then the attractor.

An *approximate inertial manifold* (a.i.m.) is a smooth finite dimensional manifold of the phase space which attracts all orbits to a thin neighborhood of it in a finite time uniformly with respect to the initial conditions from a given bounded set. This neighborhood contains the global attractor. The a.i.m.s are useful when an inertial manifold is not known to exist or its exact representation is not known, or when the dimension of the inertial manifold is too high and we want an approximation by a lower finite dimensional system. The algorithm we use in this paper keeps constant the dimension of the a.i.m.s.

2 The algorithm

In [5] and [6] was developed an algorithm for the computation of inertial manifolds. The assumptions presented below guarantee the existence of an inertial manifold and also the convergence of the algorithm.

Consider the equation (1), $u(0) = u_0$, where A is a linear operator, $u \in E$ and E is a Banach space.

A1. The nonlinear term f is globally Lipschitz continuous from E into another Banach space F, $E \subset F \subset \mathcal{E}$, the injections being continuous, each space dense in the following one, and $\mathcal{E}$ is a Banach space. It follows that

$$|f(u)|_F \leq M_0 + M_1 |u|_E,$$

for $M_0 \geq 0$.

A2. The linear operator $-A$ generates a strongly continuous semigroup $\{e^{-tA}\}_{t\geq 0}$ of bounded operators on $\mathcal{E}$ such that $e^{-tA}F \subset E$ for all $t > 0$.

A3. There exist two sequences of numbers $\{\lambda_n\}_{n=n_0}^{n_1}, \{\Lambda_n\}_{n=n_0}^{n_1}$, $n_0 \in \mathbb{N}$, $n_1 \in \mathbb{N} \cup \infty$ such that $0 < \lambda_n \leq \Lambda_n$, for all $n_0 \leq n \leq n_1$, and a sequence of finite-dimensional projectors $\{P_n\}_{n=n_0}^{n_1}$ such that $P_n\mathcal{E}$ is invariant under e^{-tA} for $t \geq 0$, and $\{e^{-tA}|P_n\mathcal{E}\}_{t\geq 0}$ can be extended to a strongly continuous semigroup $\{e^{-tA}P_n\}_{t\in\mathbb{R}}$ of bounded operators on $P_n\mathcal{E}$ with

$$\|e^{-tA}P_n\|_{\mathcal{L}(E)} \leq K_1 e^{-\lambda_n t},\ t \leq 0,$$

$$\|e^{-tA}P_n\|_{\mathcal{L}(F,E)} \leq K_1 \lambda_n^{\alpha} e^{-\lambda_n t},\ t \leq 0,$$

$Q_n\mathcal{E}$ is positively invariant under e^{-tA} for $t \geq 0$, with

$$\|e^{-tA}Q_n\|_{\mathcal{L}(E)} \leq K_2 e^{-\Lambda_n t},\ t \geq 0,$$

$$\|e^{-tA}Q_n\|_{\mathcal{L}(F,E)} \leq K_2 (t^{-\alpha} + \Lambda_n^{\alpha}) e^{-\Lambda_n t},\ t > 0,$$

where $K_1, K_2 \geq 1$ and $0 \leq \alpha < 1$.

A4. The equation (1) has a continuous semiflow $\{S(t)\}_{t\geq 0}$ in E.

A5. There exists $K_3 \geq 0$ independent of n such that $\|AP_n\|_{\mathcal{L}(E)} \leq K_3 \lambda_n$.

A6. A is invertible.

A7. The spectral gap condition

$$\Lambda_n - \lambda_n > 3M_1K_1K_2[\lambda_n^{\alpha} + (1+\gamma_\alpha)\Lambda_n^{\alpha}],$$

holds for some $n \in \mathbb{N}$, where $\gamma_\alpha = \begin{cases} \int\limits_0^\infty e^{-r} r^{-\alpha} dr, & \text{if } 0 < \alpha < 1, \\ 0, & \text{if } \alpha = 0. \end{cases}$

3 An alternative formulation of the FitzHugh-Nagumo Model

The FitzHugh-Nagumo system [1], modelling the electrical potential in the nodal system of the heart, reads

$$\begin{cases} \dot{x} = c(x + y - x^3/3), \\ \dot{y} = -(x - a + by)/c. \end{cases} \tag{2}$$

To its solution the initial condition $x(0) = x_0$, $y(0) = y_0$ is imposed, where x, y represent the electrical potential of the cell membrane and the excitability, respectively, a, b are real parameters depending on the number of channels of the cell membrane which are open for the ions of K^+ and Ca^{++} and $c > 0$ is the relaxation parameter.

In [2, 3] the global bifurcation diagram provides the qualitative responses of the model for all values of the parameters.

In order to apply to the FitzHugh-Nagumo model the numerical algorithm, this model must be reformulated in an appropriate way. This is done in the present section.

With the notation

$$A = \begin{pmatrix} -c & -c \\ 1/c & b/c \end{pmatrix}, \quad \mathbf{f}(x, y) = \begin{pmatrix} -cx^3/3 \\ a/c \end{pmatrix}. \tag{3}$$

system (2) can be written as

$$\dot{\mathbf{x}} + A\mathbf{x} = \mathbf{f}(\mathbf{x}), \tag{4}$$

where $\mathbf{x} = (x, y)$.

The eigenvalues of A are

$$\lambda_1 = \frac{b - c^2 - \sqrt{(c^2 + b)^2 - 4c^2}}{2c}, \quad \lambda_2 = \frac{b - c^2 + \sqrt{(c^2 + b)^2 - 4c^2}}{2c}$$

and the corresponding eigenvectors, read $v_1 = (1, -\dfrac{c + \lambda_1}{c})$, $v_2 = (1, -\dfrac{c + \lambda_2}{c})$.

We perform the following change of variables

$$\mathbf{x} = T\mathbf{u}, \tag{5}$$

where $\mathbf{u} = (u_1, u_2)$ and T contains the eigenvectors of A, i.e.

$$T = \begin{pmatrix} 1 & 1 \\ -\dfrac{c + \lambda_1}{c} & -\dfrac{c + \lambda_2}{c} \end{pmatrix}. \tag{6}$$

Then, equation (4) becomes

$$T\dot{\mathbf{u}} + AT\mathbf{u} = \mathbf{f}(\mathbf{Tu}).$$

Multiplying the last equation by T^{-1}, we obtain

$$\dot{\mathbf{u}} + T^{-1}AT\mathbf{u} = T^{-1}\mathbf{f}(T\mathbf{u}),$$

Denoting $B = T^{-1}AT$ and $\mathbf{g}(\mathbf{u}) = T^{-1}\mathbf{f}(T\mathbf{u})$, we obtain the modified FitzHugh-Nagumo system, which will be studied further in this paper, namely

$$\dot{\mathbf{u}} + B\mathbf{u} = \mathbf{g}(\mathbf{u}), \tag{7}$$

where B is the diagonal matrix

$$\begin{pmatrix} \zeta & 0 \\ 0 & \eta \end{pmatrix} \tag{8}$$

with

$$\zeta = -\frac{c^4 + 2bc^2 + c^2\sqrt{(c^2+b)^2 - 4c^2} - 4c^2 + b^2 - b\sqrt{(c^2+b)^2 - 4c^2}}{2c\sqrt{(c^2+b)^2 - 4c^2}},$$

$$\eta = \frac{c^4 + 2bc^2 - c^2\sqrt{(c^2+b)^2 - 4c^2} - 4c^2 + b^2 + b\sqrt{(c^2+b)^2 - 4c^2}}{2c\sqrt{(c^2+b)^2 - 4c^2}},$$

and

$$\mathbf{g}(\mathbf{u}) = \begin{pmatrix} -\dfrac{(c^2 + b + \sqrt{(c^2+b)^2 - 4c^2})(u_1+u_2)^3 c}{2\sqrt{(c^2+b)^2 - 4c^2}} + \dfrac{ca}{\sqrt{(c^2+b)^2 - 4c^2}} \\ \\ \dfrac{(c^2 + b - \sqrt{(c^2+b)^2 - 4c^2})(u_1+u_2)^3 c}{2\sqrt{(c^2+b)^2 - 4c^2}} - \dfrac{ca}{\sqrt{(c^2+b)^2 - 4c^2}} \end{pmatrix}. \tag{9}$$

4 Checking the hypotheses of the algorithm for the modified FitzHugh-Nagumo model

We deal only with the case of real eigenvalues, i.e. $b \in (-\infty, -c^2 - 2c] \cup [-c^2 + 2c, +\infty)$, because for complex eigenvalues we cannot choose λ_n and Λ_n to satisfy the conditions A3 and A7 of the numerical algorithm.

We consider $E = F = \mathcal{E} = \mathbb{R}^2$.

Assumption A1. The first assumption is that the nonlinear term $\mathbf{g}$ is globally Lipschitz. In order to have this condition fulfilled, we shall further use the prepared equation, as in like [6]. First we verify the Lipschitz condition for $\mathbf{g}$ restricted to the disk of radius r and then we construct the prepared equation, inside the ball of radius ρ the flow of the initial one, being the same with that of the prepared one.

First we compute the Lipschitz constant for each component of $\mathbf{g} = (g_1, g_2)$, and then for $\mathbf{g}$. Let $\mathbf{u} = (u_1, u_2), \mathbf{v} = (v_1, v_2)$ be in the disk of radius r, i.e. $u_1^2 + u_2^2 \le r^2$ and $v_1^2 + v_2^2 \le r^2$. We use the norm $\|\mathbf{u}\| = \max\{|u_1|, |u_2|\}$.

$$|g_1(u_1,u_2) - g_1(v_1,v_2)| = \left| \frac{c(c^2 + b + \sqrt{(c^2+b)^2 - 4c^2})[-(u_1+u_2)^3 + (v_1+v_2)^3]}{2\sqrt{(c^2+b)^2 - 4c^2}} \right| =$$

$$= c\left|\frac{c^2+b+\sqrt{(c^2+b)^2-4c^2}}{2\sqrt{(c^2+b)^2-4c^2}}\right| \cdot |v_1+v_2-u_1-u_2| \cdot |v_1^2+v_2^2-v_1v_2-u_1^2-u_2^2+u_1u_2|.$$

Using $|v_1+v_2-u_1-u_2| \le |v_1-u_1|+|v_2-u_2| \le 2\max\{|v_1-u_1|,|v_2-u_2|\} = \|(u_1,u_2)-(v_1,v_2)\|$ and $|u_1|,|u_2|,|v_1|,|v_2| \le r$, we obtain

$$|g_1(u_1,u_2)-g_1(v_1,v_2)| \le c\left|\frac{c^2+b+\sqrt{(c^2+b)^2-4c^2}}{2\sqrt{(c^2+b)^2-4c^2}}\right| \cdot 2\|(u_1,u_2)-(v_1,v_2)\| \cdot 6r^2.$$

Hence,

$$|g_1(u_1,u_2)-g_1(v_1,v_2)| \le c\left|\frac{c^2+b+\sqrt{(c^2+b)^2-4c^2}}{\sqrt{(c^2+b)^2-4c^2}}\right| \cdot 6r^2\|(u_1,u_2)-(v_1,v_2)\|$$

and

$$|g_2(u_1,u_2)-g_2(v_1,v_2)| \le c\left|\frac{c^2+b-\sqrt{(c^2+b)^2-4c^2}}{\sqrt{(c^2+b)^2-4c^2}}\right| \cdot 6r^2\|(u_1,u_2)-(v_1,v_2)\|.$$

We conclude that

$$\|\mathbf{g}(\mathbf{u})-\mathbf{g}(\mathbf{v})\| \le M_r\|\mathbf{u}-\mathbf{v}\|, \tag{10}$$

where

$$M_r = \frac{6cr^2}{\sqrt{(c^2+b)^2-4c^2}}\max\{|c^2+b+\sqrt{(c^2+b)^2-4c^2}|, |c^2+b-\sqrt{(c^2+b)^2-4c^2}|\}. \tag{11}$$

Now we determine M such that $\|\mathbf{g}(\mathbf{u})\| \le M$, for $\mathbf{u}$ inside the disk of radius r. We have

$$\|\mathbf{g}(\mathbf{u})\| = \max\left\{\left| -\frac{(c^2+b+\sqrt{(c^2+b)^2-4c^2})(u_1+u_2)^3c}{2\sqrt{(c^2+b)^2-4c^2}} + \frac{ca}{\sqrt{(c^2+b)^2-4c^2}}\right|,\right.$$

$$\left.\left|\frac{(c^2+b-\sqrt{(c^2+b)^2-4c^2})(u_1+u_2)^3c}{2\sqrt{(c^2+b)^2-4c^2}} - \frac{ca}{\sqrt{(c^2+b)^2-4c^2}}\right|\right\} \le$$

$$\le \frac{\max\{|c^2+b+\sqrt{(c^2+b)^2-4c^2}|, |c^2+b-\sqrt{(c^2+b)^2-4c^2}|\}}{2\sqrt{(c^2+b)^2-4c^2}}c|u_1+u_2|^3 +$$

$$+\frac{c|a|}{\sqrt{(c^2+b)^2-4c^2}}.$$

Since $|u_1|,|u_2| \le r$, we have $|u_1+u_2| \le 2r$ and $|u_1+u_2|^3 \le 8r^3$. Thus,

$$\|\mathbf{g}(\mathbf{u})\| \le \frac{4cr^3\max\{|c^2+b+\sqrt{(c^2+b)^2-4c^2}|, |c^2+b-\sqrt{(c^2+b)^2-4c^2}|\} + c|a|}{\sqrt{(c^2+b)^2-4c^2}}. \tag{12}$$

The prepared equation is

$$\frac{d\mathbf{u}}{dt} + B\mathbf{u} = \mathbf{g}_\rho(\mathbf{u}), \tag{13}$$

where $\mathbf{g}_\rho(\mathbf{u}) = \chi_\rho(r)\mathbf{g}(\mathbf{u})$, $\chi_\rho(r) = \chi\left(\frac{r^2}{\rho^2}\right)$, $\chi \in \mathcal{C}^1(\mathbb{R}_+)$, $\chi/_{[0,1]} = 1$, $\chi/_{[2,\infty)} = 0$, $0 \le \chi(s) \le 1, \forall s \in [1,2]$. Thus, the nonlinear term, $\mathbf{g}_\rho(\mathbf{u})$ is zero outside the ball of radius $\rho\sqrt{2}$. For $\chi(s) = 2(s-1)^3 - 3(s-1)^2 + 1$, $s \in [1,2]$, $\chi'(s) = 6(s^2-3s+2)$, hence $\chi'(s) \in \left[-\frac{3}{2}, 0\right]$, i.e. $\chi'(s) \le \frac{3}{2}$. For $s \in \mathbb{R}\backslash[1,2], \chi'(s) = 0 \le \frac{3}{2}$.

Let us compute the Lipschitz constant for $\mathbf{g}_\rho$. For $u_1^2+u_2^2 \le r_1^2$ and $v_1^2+v_2^2 \le r_2^2$, we have

$$\|\mathbf{g}_\rho(\mathbf{u}) - \mathbf{g}_\rho(\mathbf{v})\| = \|\chi_\rho(r_1)\mathbf{g}(\mathbf{u}) - \chi_\rho(r_2)\mathbf{g}(\mathbf{v})\| = \|\chi\left(\frac{r_1^2}{\rho^2}\right)\mathbf{g}(\mathbf{u}) - \chi\left(\frac{r_2^2}{\rho^2}\right)\mathbf{g}(\mathbf{v})\| =$$

$$= \|\chi\left(\frac{r_1^2}{\rho^2}\right)\mathbf{g}(\mathbf{u}) - \chi\left(\frac{r_2^2}{\rho^2}\right)\mathbf{g}(\mathbf{u}) + \chi\left(\frac{r_2^2}{\rho^2}\right)\mathbf{g}(\mathbf{u}) - \chi\left(\frac{r_2^2}{\rho^2}\right)\mathbf{g}(\mathbf{v})\| \le$$

$$\le |\chi\left(\frac{r_1^2}{\rho^2}\right) - \chi\left(\frac{r_2^2}{\rho^2}\right)| \cdot \|\mathbf{g}(\mathbf{u})\| + |\chi\left(\frac{r_2^2}{\rho^2}\right)| \cdot \|\mathbf{g}(\mathbf{u}) - \mathbf{g}(\mathbf{v})\| \le$$

$$\le |\chi'(\xi)| \cdot \left|\frac{r_1^2 - r_2^2}{\rho^2}\right| \cdot \|\mathbf{g}(\mathbf{u})\| + \|\mathbf{g}(\mathbf{u}) - \mathbf{g}(\mathbf{v})\|.$$

We have used the Lagrange Theorem, with ξ between $\frac{r_1^2}{\rho^2}$ and $\frac{r_2^2}{\rho^2}$, and $\left|\chi\left(\frac{r_2^2}{\rho^2}\right)\right| \le 1$. Since $|\chi'(\xi)| \le \frac{3}{2}$, using (10) we obtain

$$\|\mathbf{g}_\rho(\mathbf{u}) - \mathbf{g}_\rho(\mathbf{v})\| \le \frac{3}{2\rho^2}|r_1 + r_2| \cdot |r_1 - r_2| \cdot \|\mathbf{g}(\mathbf{u})\| + M_r\|\mathbf{u} - \mathbf{v}\|,$$

with M_r defined in (11).

If $r_{1,2}^2 > 2\rho^2$, then $\|\mathbf{g}_\rho(\mathbf{u}) - \mathbf{g}_\rho(\mathbf{v})\| = 0$. If $r_{1,2}^2 \le 2\rho^2$, then $|r_1 - r_2| \le \sqrt{2}\|\mathbf{u} - \mathbf{v}\|$ and thus, $\|\mathbf{g}_\rho(\mathbf{u}) - \mathbf{g}_\rho(\mathbf{v})\| \le \frac{3}{2\rho^2}2\rho\sqrt{2} \cdot \sqrt{2}\|\mathbf{u} - \mathbf{v}\| \cdot \|\mathbf{g}(\mathbf{u})\| + M_r\|\mathbf{u} - \mathbf{v}\|$. Using $r_{1,2}^2 \le 2\rho^2$ in (12) and (11), we obtain

$$\|\mathbf{g}_\rho(\mathbf{u}) - \mathbf{g}_\rho(\mathbf{v})\| \le M_\rho\|\mathbf{u} - \mathbf{v}\| \tag{14}$$

where

$$M_\rho = \frac{\max\{|c^2 + b + \sqrt{(c^2+b)^2 - 4c^2}|, |c^2 + b - \sqrt{(c^2+b)^2 - 4c^2}|\}}{\sqrt{(c^2+b)^2 - 4c^2}} \times$$

$$\times(48\sqrt{2} + 12)c\rho^2 + \frac{6c|a|}{\rho\sqrt{(c^2+b)^2 - 4c^2}} \tag{15}$$

Assumption A3. We choose the following projectors

$$P = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

We have $\|e^{-tB}P\| = e^{-\lambda_1 t}$ and $\|e^{-tB}Q\| = e^{-\lambda_2 t}$. We have to choose $0 < \lambda_n \leq \Lambda_n$ to satisfy the conditions A3.

I. The case $0 < \lambda_1 \leq \lambda_2$. We have $\|e^{-tB}P\| = e^{-\lambda_1 t} \leq 1e^{-\lambda_1 t}, \quad \forall t \leq 0$, $\|e^{-tB}Q\| = e^{-\lambda_2 t} \leq 1e^{-\lambda_2 t}, \quad \forall t \geq 0$. So, we can choose $\lambda_n = \lambda_1$, $\Lambda_n = \lambda_2$, $K_1 = 1$, $K_2 = 1$ and $\alpha = 0$.

II. The case $\lambda_1 \leq 0 < \lambda_2$. $\|e^{-tB}P\| = e^{-\lambda_1 t} \leq e^0 < 1e^{-10^{-1}t}, \quad \forall t \leq 0$, $\|e^{-tB}Q\| = e^{-\lambda_2 t} \leq 1e^{-\lambda_2 t}, \quad \forall t \geq 0$. Consequently, for $\lambda_n = 10^{-1}$, $\Lambda_n = \lambda_2$, $K_1 = 1$, $K_2 = 1$ and $\alpha = 0$, we have A3 satisfied if $\lambda_2 \geq 10^{-1}$.

III. The case $\lambda_1 < \lambda_2 \leq 0$. In this case we can not have the conditions A3 satisfied. This would imply that $e^{-\lambda_2 t} \leq K_2 e^{-\Lambda_n t}$ for all $t \geq 0$, i.e. $\Lambda_n \leq \lambda_2 < 0$, which is impossible. Thus, in this case, we can not apply this algorithm.

Assumption A5. $\|BP\| = |\lambda_1|$. In the first case, $\lambda_1 > 0$, hence $\|BP\| = \lambda_1$, $\lambda_n = \lambda_1$, and $K_3 = 1$. In the second case $\lambda_1 < 0$ and we must have $\|BP\| = -\lambda_1 \leq K_3\lambda_n$, where $\lambda_n = \dfrac{1}{10}$.

In conclusion, there exists $K_3 \geq 0$ independent of n such that $\|BP\| \leq K_3\lambda_n$, for λ_n defined as above.

Assumption A7 (Spectral Gap Condition). We must have $\Lambda_n - \lambda_n > 3M_\rho K_1 K_2[\lambda_n^\alpha + (1+\gamma_\alpha)\Lambda_n^\alpha]$. For $\alpha = 0$, we have $\gamma_\alpha = 0$, the condition reads then

$$\Lambda_n - \lambda_n > 6M_\rho, \tag{16}$$

with M_ρ defined in (15).

5 The approximate inertial manifolds for the prepared equation

Using the Jolly, Rosa, Temam's algorithm (see [5],[6]), we have implemented a program, using Scilab software (see [10]), for the construction of approximate inertial manifolds.

The approximate inertial manifolds are the collections of trajectories given by $\mathcal{M}_j = \text{graph}\Phi_j$, where $\Phi_j : P\mathbb{R}^2 \to Q\mathbb{R}^2$, $\Phi_j(p_0) = Q\varphi^j(p_0)(0)$.

For the following choice of parameters, we have all conditions satisfied: $a = 0.01, b = 5, c = 1$; we also choose $\rho = 1/20$. The eigenvalues are $\lambda_1 = 2 - 2\sqrt{2}$ and $\lambda_2 = 2 + 2\sqrt{2}$, i.e. the second case. We take $\lambda_n = 10^{-1}$, $\Lambda_n = \lambda_2$ and then, the spectral gap condition becomes $\frac{19}{10} + 2\sqrt{2} > 2.68$, which is satisfied.

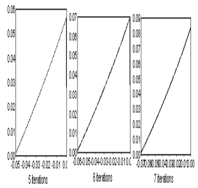

Fig. 1

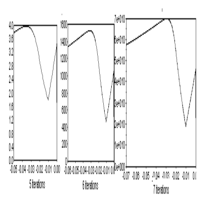

Fig. 2

The graphical representations of $Q\varphi^j$ vs time, for different numbers of iterations, for the initial conditions $u_0 = 1, v_0 = 1$ are shown in Fig. 1. For the same choice of parameters, but for $u_0 = 5, v_0 = 3$ we have the graphics in Fig. 2.

For $a = 0.01, b = 0.9, c = 0.1$, we are situated in the first case, real positive eigenvalues, $\lambda_n = \lambda_1 = 0.011, \Lambda_n = \lambda_2 = 8.89$. Choosing $\rho = 1/10$, the spectral gap condition becomes $8.88 > 1.038$, which is satisfied. For $u_0 = 5, v_0 = 3$ we have Fig. 3.

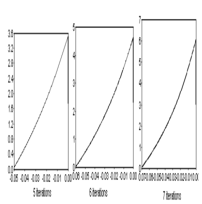

Fig. 3

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