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Measure of stability for a finite cooperative game with a generalized concept of equilibrium *

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Abstract. We consider a finite cooperative game in the normal form with a parametric principle of optimality (the generalized concept of equilibrium). This principle is defined by the partition of the players into coalitions. In this situation, two extreme cases of this partition correspond to the lexicographically optimal situation and the Nash equilibrium situation, respectively. The analysis of stability for a set of generalized equilibrium situations under the perturbations of the coefficients of the linear payoff functions is performed. Upper and lower bounds of the stability radius in the l_∞-metric are obtained. We show that the lower bound of the stability radius is accessible.

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1 Introduction

Let us consider a finite game of several players in the normal form [1, 2], in which each player $i \in N_n = \{1, 2, \dots, n\}$, $n \geq 2$, has a finite number of options for the selection of a strategy $X_i \subset \mathbf{R}$, $2 \leq |X_i| < \infty$. The realization of the game and its result is uniquely determined by the choice of each player. Assume that, on the set of the situations $X = \prod\limits_{i \in N_n} X_i$ of the game, linear payoff functions of the players

$$f_i(x) = C_i x, \quad i \in N_n$$

are defined. Here C_i is the i-th row of the matrix $C = [c_{ij}]_{n \times n} \in \mathbf{R}^{n \times n}$, $x = (x_1, x_2, \dots, x_n)^T$, $x_j \in X_j$, $j \in N_n$. In the course of the game, which is called the game with matrix C, each player i receives the payoff $f_i(x)$, which he or she wants to maximize by using certain relationships of preference. For any game in normal form, the cooperative and noncooperative principles of optimality (equilibrium concepts) are used, which usually leads to different situations (results). In this paper a parametric principle of optimality is considered. Such principle leads to the set of generalized equilibrium situations. The parameter of this principle is the partition of players into coalitions, for which two extreme cases (one coalition of all players

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and the set of one-player coalitions) correspond to the lexicographically optimal situation and the Nash equilibrium situation, respectively. The analysis of stability for the set of situations, optimal for a given partition under the perturbations of coefficients of the linear payoff functions is performed. Lower and upper bounds of the stability radius for the problem of finding the set of generalized equilibrium situations are obtained. Note that back in [3–6], formulas of the stability radius of the optimal situation with various generalizations of the concept of equilibrium was obtained.

2 Basic definitions and properties

Now we introduce the binary relation of lexicographic order $\underset{L}{\prec}$ in the space $\mathbf{R}^d$ of any dimension $d \in \mathbf{N}$, assuming that, for any different vectors $y = (y_1, y_2, \ldots, y_d)$ and $y' = (y'_1, y'_2, \ldots, y'_d)$ of the space, the formula

$$y \underset{L}{\prec} y' \Leftrightarrow y_k < y'_k$$

holds, where $k = \min\{i \in N_d : y_i \neq y'_i\}$.

The following property is obvious.

Property 1. Let $y, y' \in \mathbf{R}^d$, $d \in \mathbf{N}$. If $y_1 < y'_1$, then $y \underset{L}{\prec} y'$.

We will call any nonempty subset $J \subseteq N_n$ of players a coalition. Here and below, x_J is the projection of the vector $x \in X$ onto the coordinate axes of the space $\mathbf{R}^n$ with the numbers of coalition J. For any coalition $J \subseteq N_n$ we introduce a binary relation $\Omega(C, J)$ on a set of situations X as follows:

$$x\ \Omega(C, J)\ x' \ \Leftrightarrow \ \begin{cases} C_J x \underset{L}{\prec} C_J x' \ \ \& \ \ x_{N_n \backslash J} = x'_{N_n \backslash J}, & \text{if } J \neq N_n, \\ Cx \underset{L}{\prec} Cx', & \text{if } J = N_n, \end{cases}$$

where C_J is the submatrix of C consisting of the rows with the numbers of the coalition J.

Let $s \in N_n$, $N_n = \bigcup\limits_{r \in N_s} J_r$ be the partition of the set N_n into s coalitions, i. e. $J_r \neq \emptyset$, $r \in N_s$; $p \neq q \Rightarrow J_p \cap J_q = \emptyset$. Under the game with matrix C we understand the problem $Z^n(C, J_1, J_2, \ldots, J_s)$ of finding the set of generalized equilibrium or, in other words of $(J_1, J_2, \ldots, J_s)$-optimal situations according to the formula

$$Q^n(C, J_1, J_2, \ldots, J_s) = \{x \in X : \forall r \in N_s \, \forall x' \in X \ (x \ \overline{\Omega(C, J_r)} \ x')\},$$

where $\overline{\Omega(C, J_r)}$ denotes the negation of relation $\Omega(C, J_r)$.

Thus, in each coalition the relationships of players are constructed on the basis of the lexicographic principle. Therefore, any N_n-optimal situation $x \in Q^n(C, N_n)$ (all players form one coalition) is lexicographically optimal in the space X of all situations. This means that all players are ordered (enumerated) by importance in such a way that each preceding one is more important than all the next. This situation corresponds to the generic setup of an optimization problem with several criteria (payoffs) applied consecutively [7, 8]. It is easy to see that the set $Q^n(C, N_n)$ of N_n-optimal situations is a lexicographic set

$$L^n(C) = \{x \in X : \quad \forall\, x' \in X \quad (Cx \underset{L}{\overline{\prec}}\, Cx')\},$$

which is a subset of the Pareto set.

Clearly, in another extreme case, where the game is noncooperative ($s = n$), any individually optimal situation $x \in Q^n(C, \{1\}, \{2\}, \dots, \{n\})$ is the Nash equilibrium situation (or equilibrium) [9] (see also [1, 2]). Indeed, by the definition, situation x is equilibrium if and only if the following formula

$$\nexists\, k \in N_n \;\; \nexists\, x' \in X \;\; (C_k x < C_k x' \;\;\&\;\; x_{N_n \setminus \{k\}} = x'_{N_n \setminus \{k\}})$$

holds. Therefore, the reasonability of equilibrium situation x means that any player does not benefit from a deviation from it (while all others stick to it). We denote by $NE^n(C)$ the set of all Nash equilibrium situations.

In this context, by the parametrization of the principle of optimality we mean introducing a characteristic of binary relation $\Omega(C, J)$ of preference of situations that allows us to relate the classical concepts of lexicographic optimality and Nash equilibrium.

Without loss of generality, below we will assume that the elements of the partition $N_n = \bigcup\limits_{r \in N_s} J_r$ have the form

$$J_r = \{t_{r-1} + 1, t_{r-1} + 2, \dots, t_r\},$$

$$r \in N_s,\; t_0 = 0,\; t_s = n.$$

By taking into account the separability of the linear payoff functions $C_i x,\; i \in N_n$, we derive the following formula from the definition of the set $(J_1, J_2, \dots, J_s)$-optimal situations

$$Q^n(C, J_1, J_2, \dots, J_s) = \prod_{r=1}^{s} L^{|J_r|}(C^r), \tag{1}$$

where each factor $L^{|J_r|}(C^r)$ is the set of lexicographically optimal solutions of a $|J_r|$-criteria vector problem

$$C^r z \to \operatorname{lex} \max_{z \in X_{J_r}},$$

i. e.

$$L^{|J_r|}(C^r) = \{z \in X_{J_r} \;:\; \forall z' \in X_{J_r} \; (C^r z \underset{L}{\overline{\prec}}\, C^r z')\}.$$

Here C^r is a square $|J_r| \times |J_r|$ matrix consisting of the entries of matrix C, standing at the intersection of the rows and columns with numbers from J_r; X_{J_r} is the projection of the set X onto J_r, i. e.

$$X_{J_r} = \prod_{j \in J_r} X_j \subset \mathbf{R}^{|J_r|}.$$

It is known [7, 8] that the set $L^{|J_r|}(C^r)$ is the result of solving the sequence of scalar problems

$$L_i^{|J_r|} = \mathrm{Arg}\max\{C_i^r z : \ z \in L_{i-1}^{|J_r|}\}, \ i \in N_{|J_r|}, \tag{2}$$

where $L_0^{|J_r|} = X_{J_r}$; C_i^r is the i-th row of matrix C^r. Thus, $L^{|J_r|}(C^r) = L_{|J_r|}^{|J_r|}$ for each index $r \in N_s$.

Owing to the fact that the set X_{J_r} is finite for any index $r \in N_s$, we conclude that the lexicographic set $L^{|J_r|}(C^r)$ is nonempty for any index $r \in N_s$. Therefore (in view of (1)) the set of $(J_1, J_2, \ldots, J_s)$-optimal situations $Q^n(C, J_1, J_2, \ldots, J_s)$ is nonempty for any matrix $C \in \mathbf{R}^{n\times n}$ and for any partition. In particular, the equilibrium situations exist for any matrix $C \in \mathbf{R}^{n\times n}$ (see Corollary 5).

Under the measure of stability in cooperative game with matrix C we understand the stability radius of the problem $Z^n(C, J_1, J_2, \ldots, J_s)$ of finding the set $Q^n(C, J_1, J_2, \ldots, J_s)$ which analogously to [6, 10, 11] is defined as follows:

$$\rho^n(C, J_1, J_2, \ldots, J_s) = \begin{cases} \sup \ \Phi & \text{if } \Phi \neq \emptyset, \\ 0 & \text{otherwise,} \end{cases}$$

where

$$\Phi = \{\varepsilon > 0 \ : \ \forall B \in \Xi(\varepsilon) \quad (Q^n(C+B, J_1, J_2, \ldots, J_s) \subseteq Q^n(C, J_1, J_2, \ldots, J_s))\},$$

$$\Xi(\varepsilon) = \{B \in \mathbf{R}^{n\times n} \ : \ ||B||_\infty < \varepsilon\},$$

$$||B||_\infty = \max\{|b_{ij}| \ : \ (i,j) \in N_n \times N_n\}, B = [b_{ij}]_{n\times n}.$$

In other words, the stability radius determines the limit level of perturbations of the parameters of payoff function in the l_∞-metric, for which new generalized optimal situations do not appear. Obviously, the problem $Z^n(C, J_1, J_2, \ldots, J_s)$ is stable and the stability radius is infinite if the equality $Q^n(C, J_1, J_2, \ldots, J_s) = X$ holds. If the set

$$\overline{Q^n}(C, J_1, J_2, \ldots, J_s) = X \setminus Q^n(C, J_1, J_2, \ldots, J_s)$$

is nonempty, then we say that the problem $Z^n(C, J_1, J_2, \ldots, J_s)$ is non-trivial.

Suppose

$$\overline{L^{|J_r|}}(C^r) = X_{J_r} \setminus L^{|J_r|}(C^r),$$

$$K(C) = \{r \in N_s \ : \ \overline{L^{|J_r|}}(C^r) \neq \emptyset\},$$

$$||a||_1 = \sum_{i=1}^{m} |a_i|, \ a = (a_1, a_2, \ldots, a_m) \in \mathbf{R}^m.$$

The following properties are obvious.

Property 2. The situation $x^0 \in \overline{Q^n}(C, J_1, J_2, \ldots, J_s)$ if and only if there exists an index $k \in K(C)$ such that $x^0_{J_k} \in \overline{L^{|J_k|}}(C^k)$.

Property 3. The problem $Z^n(C, J_1, J_2, \dots, J_s)$ is non-trivial if and only if the set $K(C)$ is nonempty.

From formula (1) and property 2 we derive

Property 4. If $r \in K(C)$ and there exists a perturbing matrix $\widehat{B} \in \mathbf{R}^{n\times n}$ such that the following formula holds

$$\forall\ z \in L^{|J_r|}(C^r) \quad \Big(z \in \overline{L^{|J_r|}}(C^r + \widehat{B}^r) \Big), \tag{3}$$

then we have

$$\forall\ x \in Q^n(C, J_1, J_2, \dots, J_s) \quad \Big(x \in \overline{Q^n}(C + \widehat{B}, J_1, J_2, \dots, J_s) \Big). \tag{4}$$

3 Bounds of the stability radius

Suppose

$$\varphi^n(C, J_1, J_2, \dots, J_s) = \min_{r\in K(C)} \ \min_{z\in \overline{L^{|J_r|}}(C^r)} \ \max_{z'\in L^{|J_r|}(C^r)} \frac{C_1^r(z'-z)}{||z'-z||_1}.$$

Theorem. *The stability radius $\rho^n(C, J_1, J_2, \dots, J_s)$ of the non-trivial problem $Z^n(C, J_1, J_2, \dots, J_s)$, $n \geq 2$, $s \geq 1$, has the following bounds*

$$\varphi^n(C, J_1, J_2, \dots, J_s) \leq \ \rho^n(C, J_1, J_2, \dots, J_s) \ \leq \ \min\{||C_1^r||_\infty \ : \ r \in K(C)\}.$$

Proof. Note that in view of property 3 the non-triviality of the problem $Z^n(C, J_1, J_2, \dots, J_s)$ implies the non-emptiness of the set $K(C)$.

Let us introduce the notations

$$\varphi := \varphi^n(C, J_1, J_2, \dots, J_s), \quad \rho := \rho^n(C, J_1, J_2, \dots, J_s).$$

It is easy to see that $\varphi \geq 0$.

At first we prove the inequality $\rho \geq \varphi$. If $\varphi = 0$, then this inequality is obvious.

Let $\varphi > 0$, $B \in \Xi(\varphi)$, $x^0 \in \overline{Q^n}(C, J_1, J_2, \dots, J_s)$. Let us show that $x^0 \in$ $\in \overline{Q^n}(C + B, J_1, J_2, \dots, J_s)$.

It follows directly from the definition of φ that

$$\forall r \in K(C) \quad \forall z \in \overline{L^{|J_r|}}(C^r) \quad \left(\max_{z'\in L^{|J_r|}(C^r)} \frac{C_1^r(z'-z)}{||z'-z||_1} \geq \varphi \right). \tag{5}$$

According to the property 2 there exists an index $k \in K(C)$ such that $x^0_{J_k} \in \in \overline{L^{|J_k|}}(C^k)$. Therefore the formula (5) implies the existence of a vector $z' \in L^{|J_k|}(C^k)$, such that the following inequalities

$$\frac{C^k_1(z' - x^0_{J_k})}{||z' - x^0_{J_k}||_1} \ \geq \ \varphi \ > \ ||B||_\infty \ \geq \ ||B^k||_\infty$$

hold. Due to the obvious inequality

$$|uv| \leq ||u||_1 ||v||_\infty,$$

which is valid for all $u = (u_1, u_2, \ldots, u_n) \in \mathbf{R}^n$ and $v = (v_1, v_2, \ldots, v_n)^T \in \mathbf{R}^n$, we obtain

$$(C^k_1 + B^k_1)(z' - x^0_{J_k}) = C^k_1(z' - x^0_{J_k}) + B^k_1(z' - x^0_{J_k}) \geq$$
$$\geq C^k_1(z' - x^0_{J_k}) - ||B^k||_\infty ||z' - x^0_{J_k}||_1 > 0.$$

Thus, according to property 1 we have $(C^k + B^k)x^0_{J_k} \underset{L}{\prec} (C^k + B^k)z'$, i. e. $x^0_{J_k} \in \overline{L^{|J_k|}}(C^k + B^k)$. In view of property 2, we conclude that $x^0 \in \overline{Q^n}(C + +B, J_1, J_2, \ldots, J_s)$.

So, the following formula is true

$$\forall B \in \Xi(\varphi) \quad \big(Q^n(C + B, J_1, J_2, \ldots, J_s) \subseteq Q^n(C, J_1, J_2, \ldots, J_s)\big),$$

which means that $\rho \geq \varphi$.

To prove the upper bound we need to show that for any index $r \in K(C)$ the following formula $\rho \leq ||C^r_1||_\infty$ is valid.

Let $r \in K(C)$, $\varepsilon > ||C^r_1||_\infty$, $\psi_i = ||C^r_i||_\infty$, $i \in N_{|J_r|}$. We build a perturbing matrix $\widehat{B} = \left[\widehat{b}_{ij}\right]_{n\times n}$, assuming

$$\widehat{b}_{ij} = \begin{cases} -c_{ij} - \delta c_{i+p-1,j} & \text{if } i = t_{r-1} + 1, \ j \in J_r, \\ 0 & \text{otherwise,} \end{cases}$$

where $0 < \delta < \frac{\varepsilon - \psi_1}{\psi_p}$. Here

$$p = \min\{i \in N_{|J_r|} : \ X_{J_r} \neq L^{|J_r|}_i\},$$

and $L^{|J_r|}_i$ is defined by the formula (2). It is easy to see that $\psi_p \neq 0$. After a simple calculation we obtain $||\widehat{B}||_\infty < \varepsilon$, i. e. $\widehat{B} \in \Xi(\varepsilon)$.

Let $z^* \in X \backslash L^{|J_r|}_p$. Then for any vector $z \in L^{|J_r|}(C^r)$ we have

$$C^r_p(z^* - z) < 0.$$

Using this and taking into account the construction of the row $\widehat{B}^r_1$, we derive

$$(C^r_1 + \widehat{B}^r_1)(z - z^*) = \delta C^r_p(z^* - z) < 0.$$

This inequality in view of property 1 is equivalent to the following relation

$$(C^r + \widehat{B}^r)z \underset{L}{\prec} (C^r + \widehat{B}^r)z^*.$$

From this we obtain the formula (3), and therefore by virtue of property 4 we have (4). Hence

$$Q^n(C + \widehat{B}, J_1, J_2, \ldots, J_s) \nsubseteq Q^n(C, J_1, J_2, \ldots, J_s). \tag{6}$$

Resuming all the said above, we conclude that for any index $r \in K(C)$ and for any number $\varepsilon > ||C_1^r||_\infty$ there exists a matrix $\widehat{B} \in \Xi(\varepsilon)$ such that the formula (6) is true. This means that the stability radius $\rho \le ||C_1^r||_\infty$ for any index $r \in K(C)$. That complete the proof.

4 Some of special cases

The theorem allows us to formulate the following corollaries.

Corollary 1. *If $|X_j| = 2,\ j \in N_n$, then for the stability radius $\rho^n(C, J_1, J_2, \ldots, J_s)$ of non-trivial problem $Z^n(C, J_1, J_2, \ldots, J_s)$ the formula*

$$\rho^n(C, J_1, J_2, \ldots, J_s) = \varphi^n(C, J_1, J_2, \ldots, J_s) \tag{7}$$

holds.

Proof. Taking into account the proved inequality $\rho \ge \varphi$ (see theorem) for deriving the formula (7) it remains to show that $\rho \le \varphi$. Let us introduce the notations:

$$X_j = \{x_j^-, x_j^+\}, \quad x_j^-, x_j^+ \in \mathbf{R}, \quad x_j^- < x_j^+,\ j \in N_n.$$

By the definition of number φ, there exist an index $r \in K(C)$ and a vector $z^* \in \overline{L^{|J_r|}}(C^r)$ such that for any vector $z \in L^{|J_r|}(C^r)$ we have

$$C_1^r(z - z^*) \ \le\ \varphi||z - z^*||_1.$$

Then, assuming $\varepsilon > \varphi,\ \widehat{B} = \left[\widehat{b}_{ij}\right]_{n\times n} \in \Xi(\varepsilon)$, where

$$\widehat{b}_{ij} = \begin{cases} -\alpha & \text{if } z^*_{j-t_{r-1}} = x_j^-,\ i = t_{r-1}+1,\ j \in J_r, \\ \alpha & \text{if } z^*_{j-t_{r-1}} = x_j^+,\ i = t_{r-1}+1,\ j \in J_r, \\ 0 & \text{otherwise,} \end{cases}$$

$$\varepsilon > \alpha > \varphi,$$

we derive

$$(C_1^r + \widehat{B}_1^r)(z - z^*) = C_1^r(z - z^*) + \widehat{B}_1^r(z - z^*) =$$

$$= C_1^r(z - z^*) - \alpha||z - z^*||_1 \le \varphi||z - z^*||_1 - \alpha||z - z^*||_1 < 0,$$

i. e. $z \in \overline{L^{|J_r|}}(C^r + \widehat{B}^r)$. Therefore we have $(C^r + \widehat{B}^r)z \underset{L}{\prec} (C^r + \widehat{B}^r)z^*$.

Using this we obtain the formula (3), and therefore by virtue of property 4 we have (4). Hence the formula (6) is true.

Resuming all the information given above, we conclude that for any number $\varepsilon > \varphi$ there exists a matrix $\widehat{B} \in \Xi(\varepsilon)$ such that the formula (6) is true. This means that $\rho \leq \varphi$. That completes the proof of Corollary 1.

Note, that Corollary 1 shows the accessibility of the lower bound of the stability radius ρ.

Corollary 2. *If* $Q^n(C, J_1, J_2, \ldots, J_s) = \{x^0\}$, *then*

$$\rho^n(C, J_1, J_2, \ldots, J_s) = \min_{r \in N_s} \min_{z \in X_{J_r} \setminus \{x^0_{J_r}\}} \frac{C^r_1(x^0_{J_r} - z)}{||x^0_{J_r} - z||_1}. \qquad (8)$$

Proof. Denote the right side of the formula (8) by ζ. It is easy to see that the problem $Z^n(C, J_1, J_2, \ldots, J_s)$ is nontrivial and the number ζ is $\varphi = \varphi^n(C, J_1, J_2, \ldots, J_s)$. Therefore, in view of inequality $\rho \geq \varphi$ (see the theorem) it remains to show that $\rho \leq \zeta$.

By the definition of number ζ, we have that there exist an index $r \in N_s$ and a vector $z^* \in X_{J_r} \setminus \{x^0_{J_r}\}$ such that

$$C^r_1(x^0_{J_r} - z^*) = \zeta||x^0_{J_r} - z^*||_1, \quad \{x^0_{J_r}\} = L^{|J_r|}(C^r).$$

Therefore, assuming $\varepsilon > \zeta$ and building a perturbing matrix $\widehat{B} = \left[\widehat{b}_{ij}\right]_{n \times n} \in \Xi(\varepsilon)$ with elements

$$\widehat{b}_{ij} = \begin{cases} -\alpha & \text{if } x^0_j \geq z^*_{j-t_{r-1}},\ i = t_{r-1}+1,\ j \in J_r, \\ \alpha & \text{if } x^0_j < z^*_{j-t_{r-1}},\ i = t_{r-1}+1,\ j \in J_r, \\ 0 & \text{otherwise,} \end{cases}$$

where $\varepsilon > \alpha > \zeta$, we obtain

$$(C^r_1 + \widehat{B}^r_1)(x^0_{J_r} - z^*) = C^r_1(x^0_{J_r} - z^*) - \alpha||x^0_{J_r} - z^*||_1 = \zeta||x^0_{J_r} - z^*||_1 - \alpha||x^0_{J_r} - z^*||_1 < 0.$$

Hence, taking into account property 1 we have

$$(C^r + \widehat{B}^r)x^0_{J_r} \underset{L}{\prec} (C^r + \widehat{B}^r)z^*.$$

From here we find the formula (3), and therefore according to the property 4 we have (4), i. e. the formula (6) is true.

Resuming the said above we conclude that for any number $\varepsilon > \varphi$ there exists a perturbing matrix $\widehat{B} \in \Xi(\varepsilon)$ such that the formula (6) is true. This means that $\rho \leq \varphi$. The proof of Corollary 2 is completed.

The theorem implies

Corollary 3. *The stability radius $\rho^n(C, N_n)$ of a non-trivial problem $Z^n(C, N_n)$ of finding the lexicographic set $L^n(C)$ has the following bounds:*

$$\min_{x \in \overline{L^n}(C)} \max_{x' \in L^n(C)} \frac{C_1(x'-x)}{||x'-x||_1} \leq \rho^n(C, N_n) \leq ||C_1||_\infty .$$

Here $\overline{L}^n(C) = X \backslash L^n(C) = X \backslash Q^n(C, N_n)$.

Obviously, in case we have a noncooperative game ($s = n$), for any index $r \in N_s$ the inequality $\overline{L^1}(C^r) \neq \emptyset$, where $C^r = c_{rr}$, is equivalent to the inequality $c_{rr} \neq 0$. Therefore, the theorem implies

Corollary 4 [12]. *For the stability radius $\rho^n(C, \{1\}, \{2\}, \ldots, \{n\})$, $n \geq 2$, of the problem $Z^n(C, \{1\}, \{2\}, \ldots, \{n\})$ of finding the set of Nash equilibrium situations $NE^n(C)$ the formula*

$$\rho^n(C, \{1\}, \{2\}, \ldots, \{n\}) = \begin{cases} \min\{|c_{kk}| \ : \ k \in K(C)\} & \textit{if } K(C) \neq \emptyset, \\ \infty & \textit{if } K(C) = \emptyset \end{cases}$$

holds.

Taking into account the formula (1) we obtain

Corollary 5 [3]. *The situation $x^0 = (x_1^0, x_2^0, \ldots, x_n^0) \in X$ of a noncooperative game with matrix $C \in \mathbf{R}^{n \times n}$ is a Nash equilibrium situation if and only if the strategy of each player $i \in N_n$ has the form*

$$x_i^0 = \begin{cases} \max\{\, x_i \ : \ x_i \in X_i\} & \textit{if } c_{ii} > 0, \\ \min\{\, x_i \ : \ x_i \in X_i\} & \textit{if } c_{ii} < 0, \\ x_i \in X_i & \textit{if } c_{ii} = 0. \end{cases}$$

References

[1] Moulen H. *Théorie des jeux pour l'économie et la politique.* Harmann Paris, 1979.

[2] Petrosyan L.A., Zenkevich N.A., Semina E.A. *Games theory.* Moscow, Vysshaya shkola, 1998 (in Russian).

[3] Emelichev V.A., Bukhtoyarov S.E. *Stability of generally efficient situation in finite cooperative games with parametric optimality principle ("from Pareto to Nash").* Computer Science Journal of Moldova, 2003, **11**, N 3, p. 316–323.

[4] Bukhtoyarov S.E., Emelichev V.A., Stepanishina Yu.V. *Stability of discrete vector problems with the parametric principle of optimality.* Cybernetics and Systems Analysis, 2003, **39**, p. 604–614.

[5] Emelichev V.A., Kuzmin K.G. *Finite cooperative games with a parametric concept of equilibrium under uncertainty conditions.* J. of Computer and Systems International, 2006, **45**, N 2, p. 276–281.

[6] Bukhtoyarov S.E., Emelichev V.A. *Measure of stability for a finite cooperative game with a parametric optimality principle (from Pareto to Nash).* Comput. Math. and Mathem. Physics, 2006, **46**, N 7, p. 1193–1199.

[7] Podinovskiy V.V., Gavrilov V.M. *Optimization on sequential applied criteria.* Moskow, Sovetskoe radio, 1975 (in Russian).

[8] Chervak Yu. *Unimprovable choice.* Uzhgorod, Uzhgorod National University, 2002 (in Ukrainian)

[9] Nash J.F. *Non-cooperative games.* Ann. Math., 1951, **54**, N 2, p. 286–295.

[10] Sotskov Yu., Leontev V., Gordeev E. *Some concepts of stability analysis in combinatorial optimization.* Discrete Appl. Math., 1995, **58,** N 2, p. 169–190.

[11] Emelichev V., Girlich E., Nikulin Yu., Podkopaev D. *Stability and regularization of vector problems of integer linear programming.* Optimization, 2002, **51**, N 4, p. 645–676.

[12] Bukhtoyarov S.E., Kuzmin K.G. *On stability in game problems of finding Nash set.* Computer Science Journal of Moldova, 2004, **12**, N 3, p. 381–388.

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