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The solvability and properties of solutions of an integral convolutional equation

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Abstract. The work defines the conditions of solvability of one integral convolutional equation with degreely difference kernels. This type of integral convolutional equations was not studied earlier, and it turned out that all methods used for the investigation of such equations with the help of Riemann boundary problem at the real axis are not applied there. The investigation of such type equations is based on the investigation of the equivalent singular integral equation with the Cauchy type kernel at the real axis. It is determined that the equation is not a Noetherian one. Besides, there shown the number of the linear independent solutions of the homogeneous equation and the number of conditions of solvability for the heterogeneous equation. The general form of these conditions is also shown and there determined the spaces of solutions of that equation. Thus the convolutional equation that wasn't studied earlier is presented at that work and the theory of its solvability is built there. So some new and interesting theoretical results are got at that paper.

Mathematics subject classification: 45E05, 45E10.
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The present work is devoted to defining conditions of solvability and some properties of solutions of the next integral equation

$$P_m(x)\varphi(x)+\frac{1}{\sqrt{2\pi}}\int\limits_0^{+\infty} k(t,x-t)\varphi(t)\,dt+\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{0} n(t,x-t)\varphi(t)\,dt = h(x), x\in \mathbf{R}, \quad (1)$$

where $\mathbf{R}$ is the real axis;

$$k(t,x-t)=\sum_{j=0}^{n} k_j(x-t)t^j, n(t,x-t)=\sum_{\nu=0}^{s} n_\nu(x-t)t^\nu,$$

where

$$P_m(x)=\sum_{k=0}^{m} A_k x^k$$

is the known polynomial with degree m and $k_j(x), n_\nu(x)\in \mathbf{L}$, $j=\overline{1,n}$, $\nu=\overline{1,s}$, $h(x)\in \mathbf{L}_2$ are known functions.

Let $D^+ = \{z \in \mathbf{C} : \mathbf{Imz} > \mathbf{0}\}$ be an upper half plane and $D^- = \{z \in \mathbf{C} : \mathbf{Imz} < \mathbf{0}\}$ be a lower half plane of the complex plane $\mathbf{C}$. According to the properties of Fourier transformation [1, p. 77], [2, p. 16] the investigation of the equation (1) reduces to the investigation of the following differential boundary problem

$$\left[\sum_{k=0}^{m} A_k(-1)^k \Phi^{+(k)}(x) + \sum_{j=0}^{n} (-1)^j K_j(x)\Phi^{+(j)}(x)\right] -$$
$$- \left[\sum_{k=0}^{m} A_k(-1)^k \Phi^{-(k)}(x) + \sum_{\nu=0}^{s} (-1)^\nu N_\nu(x)\Phi^{-(\nu)}(x)\right] = H(x), x \in \mathbf{R}, \qquad (2)$$

where $K_j(x), N_\nu(x), H(x)$ are the Fourier transformations of functions $k_j(x), n_\nu(x)$, $h(x)$, $j = \overline{1,n}$, $\nu = \overline{1,s}$ accordingly. $\Phi^{+(p)}(x)$ and $\Phi^{-(q)}(x)$ are the boundary values at $\mathbf{R}$ of the functions $\Phi^{+(p)}(z)$ and $\Phi^{-(q)}(z)$ accordingly, where $\Phi^+(z)$, $\Phi^-(z)$ are unknown functions, which are analytical at the domains D^+ and D^- accordingly. As all the transformations of the differential boundary problem (2) and the equation (1) are identical, then the problem and the equation are equivalent in such a sense that they are solvable or unsolvable at the same time, and there is one and only one solution $\Phi^\pm(x)$ of the differential boundary problem (2) for every solution $\varphi(x)$ of the equation (1) and vice versa. The solutions of the equation (1) are expressed over solutions of the problem (2) according to the formula

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} \int\limits_{\mathbf{R}} \left[\Phi^+(t) - \Phi^-(t)\right] e^{-ixt}\, dt, x \in \mathbf{R}. \qquad (3)$$

Later on we will consider that the functions $K_j(x), N_\nu(x) \in \mathbf{H}_\alpha^{(r)}$, $r \geq 0, 0 < \alpha \leq 1$, $\mathbf{H}_\alpha^{(0)} = \mathbf{H}_\alpha$, $j = \overline{1,n}$, $\nu = \overline{1,s}$ and the function $H(x) \in \mathbf{L}_2^{(r)}$, $r \geq 0$, $\mathbf{L}_2^{(0)} = \mathbf{L}_2$. As the functions $k_j(x), n_\nu(x) \in \mathbf{L}$, $j = \overline{1,n}$, $\nu = \overline{1,s}$, then according to Riemann-Lebesgue theorem [1, p. 42] $\lim\limits_{x\to\infty} K_j(x) = 0$, $\lim\limits_{x\to\infty} N_\nu(x) = 0$, $j = \overline{1,n}$, $\nu = \overline{1,s}$. The equation (1) is a generalization of a convolutional type equation "with two kernels", and we will study it basing on the investigation of the differential boundary problem (2). The investigation of the solvability of the problem (2) we will do basing on the investigation of the singular integral equation at the real axis. The investigation of the differential boundary problem (2) reduces to the investigation of the singular integral equation with the help of integral representations for the functions and derivatives of them built in [4]. Let construct functions $\Phi^+(z)$ and $\Phi^-(z)$ such that they are analytical at the domains D^+, D^- accordingly and disappearing at infinity. Besides, the boundary values at $\mathbf{R}$ of functions $\Phi^{+(p)}(z)$ and $\Phi^{-(q)}(z)$ satisfy the following condition $\Phi^{+(p)}(x), \Phi^{-(q)}(x) \in \mathbf{L}_2^{(r)}$, $r \geq 0, p \geq 0, q \geq 0$. According to [4] these conditions satisfy such functions as:

$$\Phi^\pm(z) = (2\pi\imath)^{-1} \int\limits_{\mathbf{R}} P^\pm(x,z)\rho(x)\, dx, z \in D^\pm, \qquad (4)$$

where

$$P^{+}(x,z)=\frac{(-1)^{p}(x+\imath)^{-p}}{(p-1)!}\times$$
$$\times\left[(x-z)^{p-1}\ln\left(1-\frac{x+\imath}{z+\imath}\right)-\sum_{k=0}^{p-2}d_{p-k-2}(x+\imath)^{k+1}(z+\imath)^{p-k-2}\right],$$
$$x\in\mathbf{R},\mathbf{z}\in\mathbf{D}^{+};$$

$$P^{-}(x,z)=\frac{(-1)^{q}(x-\imath)^{-q}}{(q-1)!}\times$$
$$\times\left[(x-z)^{q-1}\ln\left(1-\frac{x-\imath}{z-\imath}\right)-\sum_{k=0}^{q-2}l_{q-k-2}(x-\imath)^{k+1}(z-\imath)^{q-k-2}\right],$$
$$x\in\mathbf{R},\mathbf{z}\in\mathbf{D}^{-};$$
$$d_{p-k-2}=(-1)^{k+1}\sum_{j=0}^{k}C_{p-1}^{p-1-j}(k-j+1)^{-1},$$
$$l_{q-k-2}=(-1)^{k+1}\sum_{j=0}^{k}C_{q-1}^{q-1-j}(k-j+1)^{-1},$$

where C_n^m are binomial coefficients and the function $\ln\left[1-\dfrac{x+\imath}{z+\imath}\right]$ is the main branch ($\ln 1=0$) of the logarithmic function in the complex plane with the cut connecting such points as $z=-\imath$ and $z=\infty$, following the negative direction of the axis of ordinate. The function $\ln\left[1-\dfrac{x-\imath}{z-\imath}\right]$ is the main branch ($\ln 1=0$) of the logarithmic function in the complex plane with the cut connecting such points as $z=\imath$ and $z=\infty$, following the positive direction of the axis of ordinate. It's easy to verify, that defined by (4) functions $\Phi^{+}(z)$ and $\Phi^{-}(z)$ are unique analytical functions in the domains D^{+}, D^{-} accordingly. According to the method from the work [4], it's easy to check that the function $\rho(x)\in\mathbf{L}_2$ or the density of the integral representations, is defined uniquely by the functions $\Phi^{+}(z)$ and $\Phi^{-}(z)$ and vice versa, so with the help of the given function $\rho(x)\in\mathbf{L}_2$ both functions $\Phi^{+}(z)$ and $\Phi^{-}(z)$ are constructing uniquely. According to the work [4] the following representations take place:

$$\Phi^{+(p)}(z)=(2\pi\imath)^{-1}\int_{\mathbf{R}}(z+\imath)^{-p}(x-z)^{-1}\rho(x)\,dx, z\in D^{+},$$
$$\Phi^{-q)}(z)=(2\pi\imath)^{-1}\int_{\mathbf{R}}(z-\imath)^{-q}(x-z)^{-1}\rho(x)\,dx, z\in D^{-}. \qquad (5)$$

We consider the case, when $m=n=s$. Using the properties [4] of partial derivatives of functions $P^{\pm}(x,z)$ with respect to z and Sohotski formulas for derivatives from

[7, p. 42], with the help of the representations (4), (5), we will transform the differential boundary problem (2) into the following singular integral equation and later on investigate it. The singular integral equation is

$$A(x)\rho(x) + B(x)(\pi\imath)^{-1}\int\limits_{\mathbf{R}}(t-x)^{-1}\rho(t)\,dt + (T\rho)(x) = H(x), x\in\mathbf{R}, \tag{6}$$

where

$$A(x) = 0,5(-1)^m\left\{[A_m + K_m(x)](x+\imath)^{-m} + [A_m + N_m(x)](x-\imath)^{-m}\right\},$$

$$B(x) = 0,5(-1)^m\left\{[A_m + K_m(x)](x+\imath)^{-m} - [A_m + N_m(x)](x-\imath)^{-m}\right\}, \tag{7}$$

$$(T\rho)(x) = \int\limits_{\mathbf{R}} K(x,t)\rho(t)\,dt, x\in\mathbf{R}, \tag{8}$$

$$K(x,t) = \frac{1}{2\pi\imath}\left[\sum_{j=0}^{m-1}(-1)^j\left[[A_j + K_j(x)]\frac{\partial^j P^+(t,x)}{\partial x^j} - [A_j + N_j(x)]\frac{\partial^j P^-(t,x)}{\partial x^j}\right]\right], \tag{9}$$

and $\dfrac{\partial^j P^{\pm}(t,x)}{\partial x^j}$ is a limiting value at $\mathbf{R}$ of the function $\dfrac{\partial^j P^{\pm}(t,z)}{\partial z^j}$, $j=\overline{0,m-1}$.

1 Lemma. *If the functions* $K_j(x), N_\nu(x) \in \mathbf{H}_\alpha^{(r)}$, $j=\overline{1,n}$, $\nu=\overline{1,s}$, *then the operator*

$$T:\mathbf{L_2^{(r)}}\to\mathbf{L_2^{(r)}},$$

$r\geq 0$, *defined by the formula (8) is a compact operator.*

The proof of lemma follows from Frechet-Kolmogorov-Riesz criterion of compactness of integral operators at the real axis in the space $\mathbf{L_p}, \mathbf{p}>\mathbf{1}$, the properties of functions $P^{\pm}(x,z)$ from [4] and the results of the work [8].

According to the work [9, p. 406], the problem (2) and the singular integral equation (6) are equivalent in such a sense that they are solvable or unsolvable at the same time, and for every solution $\rho(x)$ of the equation (6) there exists maybe unique solution $\Phi^{\pm}(x)$ of the problem (2) and vice versa. In order to make this accord unique it is necessary to set initial conditions for the problem (2). As its solutions $\Phi^{\pm}(x)$ are found in spaces of disappearing at infinity functions, then according to the properties of Cauchy type integral the solutions of the problem (2) are such that $\Phi^{\pm(j)}(\infty)=0$, $j=\overline{0,m-1}$, that is the initial conditions of (2) are trivial and set automatically. Thus it follows that the differential boundary problem (2) and the singular integral equation (6) are equivalent in such a sense that they are solvable or unsolvable at the same time, and there is one and only one solution $\rho(x)$ of the equation (6) for every solution $\Phi^{\pm}(x)$ of the problem (2) and vice versa. By the force of formula (4), the solutions of the problem (2) are expressed over solutions of the equation (6) according to the formula

$$\Phi^{\pm}(x) = \frac{1}{\sqrt{2\pi}}\int\limits_{\mathbf{R}} P^{\pm}(t,x)\rho(t)\,dt, x\in\mathbf{R}, \tag{10}$$

where $p = q = m$; $P^{\pm}(t,x)$ are the boundary values at $x \in \mathbf{R}$ of functions $P^{\pm}(t,z)$, and $\rho(x)$ is the solution of the equation (6). As the equation (1) and the problem (2) are equivalent, the problem (2) and the singular integral equation (6) are equivalent, too, it follows that the equation (1) and the equation (6) are equivalent in such a sense that they are solvable or unsolvable at the same time, and there is one and only one solution $\varphi(x)$ of the equation (1) for every solution $\rho(x)$ of the equation (6) and vice versa. Thus the solutions of the equation (1) are expressed over solutions of the equation (6) according to the formulas (10), (3). That is why the equation (1) we will call a Noetherian if the equation (6) is a Noetherian one.

2 Theorem. *The equation (1) is not a Noetherian one.*

Proof. According to the work [7, p. 208–212] the equation (6) is a Noetherian one if and only if when $A(x) + B(x) \neq 0$, $A(x) - B(x) \neq 0$ at $x \in \mathbf{R}$. From the formula (7) it follows that

$$A(x) + B(x) = (-1)^m [A_m + K_m(x)](x + \imath)^{-m},$$

$$A(x) - B(x) = (-1)^m [A_m + N_m(x)](x - \imath)^{-m}.$$

So we have got that the functions $A(x)+B(x)$, $A(x)-B(x)$ have a null at least with order m in infinity. It means that the equation (6) is not a Noetherian one. Then as the equations (1) and (6) are equivalent, the equation (1) is not a Noetherian one, too. The theorem is proved. □

Firstly we consider the case when $A_m + K_m(x) \neq 0$, $A_m + N_m(x) \neq 0$ at $\mathbf{R}$. Let determine $\varkappa = ind \dfrac{A_m + N_m(x)}{A_m + K_m(x)}$.

3 Theorem. *Let the functions $k_j(x), n_\nu(x) \in \mathbf{L}$, $j, \nu = \overline{1,m}$, $h(x) \in \mathbf{L}_2$; the functions $K_j(x), N_\nu(x) \in \mathbf{H}_\alpha^{(r)}$, $j, \nu = \overline{1,m}$, $H(x) \in \mathbf{L}_2^{(r)}$, $r \geq m$; $A_m + K_m(x) \neq 0$, $A_m + N_m(x) \neq 0$ at $\mathbf{R}$. If $\varkappa - m \geq 0$, then the homogeneous equation (1) has not less than $\varkappa - m$ linearly independent solutions; the heterogeneous equation (1) is an unconditionally solvable one and its general solution depends upon not less than $\varkappa - m$ arbitrary constants. If $\varkappa - m < 0$, then generally speaking the heterogeneous equation (1) is an unsolvable one. It will be a solvable one when not less than $m - \varkappa$ conditions of solvability*

$$\int_{\mathbf{R}} H(x)\psi_j(x)\, dt = 0, \tag{11}$$

will be executed. Here $H(x)$ is a right part of the equation (6), and $\psi_j(x)$ are linearly independent solutions of the homogeneous equation

$$A(x)\psi(x) - (\pi\imath)^{-1} \int_{\mathbf{R}} (t-x)^{-1} B(t)\psi(t)\, dt + \int_{\mathbf{R}} K(t,x)\psi(t)\, dt = 0,$$

allied to the equation (6), where the functions $A(x), B(x), K(x,t)$ are defined by the formulas (7), (9) accordingly.

Proof. According to the work [7, p. 208–212] if $\varkappa - m \geq 0$, then the homogeneous equation (6) has not less than $\varkappa - m$ linearly independent solutions; the heterogeneous equation (6) is an unconditionally solvable one and its general solution depends upon not less than $\varkappa - m$ arbitrary constants. If $\varkappa - m < 0$, then generally speaking the heterogeneous equation (6) is an unsolvable one. It will be a solvable one when not less than $m - \varkappa$ conditions of solvability (11) will be executed. As the equations (6) and (1) are equivalent, then theorem is proved. □

According to the work [2, p. 262], let define by $\mathbf{L}_2[-\mu;0]$ the space of functions $\varphi(x) \in \mathbf{L}_2$ which satisfy such condition as $(x+\imath)^{\mu}\varphi(x) \in \mathbf{L}_2$.

4 Theorem. *Let the functions $k_j(x), n_\nu(x) \in \mathbf{L}$, $j,\nu = \overline{1,m}$, $h(x) \in \mathbf{L}_2$; the functions $K_j(x), N_\nu(x) \in \mathbf{H}_\alpha^{(r)}$, $j,\nu = \overline{1,m}$, $H(x) \in \mathbf{L}_2^{(r)}$, $r \geq m$; $A_m + K_m(x) \neq 0$, $A_m + N_m(x) \neq 0$ at $\mathbf{R}$ and the equation (1) is a solvable one. Then its solutions belong the space $\mathbf{L}_2[-r;0]$, $r \geq m$.*

Proof. According to the work [3, p. 139] the solutions of the equation (6) $\rho(x) \in \mathbf{L}_2^{(r-m)}$, $r \geq m$ in conditions of the theorem. Then in virtue of the representations (5) and the properties of Cauchy type integral the limiting values $\Phi^{\pm(m)}(x)$ at $\mathbf{R}$ of functions $\Phi^{\pm(m)}(z)$ belong the space $\mathbf{L}_2^{(r-m)}$, $r \geq m$. From the properties of Fourier transformation [2, p. 262] we have that the solutions of the equation (1) given by the formula (3) belong the space $\mathbf{L}_2[-r;0]$, $r \geq m$. The theorem is proved. □

Let the conditions $A_m + K_m(x) \neq 0$, $A_m + N_m(x) \neq 0$ at $\mathbf{R}$ are not executed. Then we suppose that the functions $A_m + K_m(x)$, $A_m + N_m(x)$ go to zero at the real axis in such points as $a_1, a_2, \ldots, a_u$ and $b_1, b_2, \ldots, b_\omega$ with accordingly integer orders $\gamma_1, \gamma_2, \ldots, \gamma_u$, $\mu_1, \mu_2, \ldots, \mu_\omega$. Then in virtue of the work [3, p. 199] the following representations take place

$$A(x) + B(x) = (x+\imath)^{-m} M(x)\rho_+(x), A(x) - B(x) = (x-\imath)^{-m} N(x)\rho_-(x), \quad (12)$$

where the functions $M(x) \neq 0, N(x) \neq 0$ at $\mathbf{R}$, $M(x), N(x) \in \mathbf{H}_\alpha^{(r)}$ and the functions $\rho_+(x), \rho_-(x)$ look as

$$\rho_+(x) = \prod_{k=1}^{u} \left(\frac{x-a_k}{x+\imath}\right)^{\gamma_k}, \rho_-(x) = \prod_{k=1}^{\omega} \left(\frac{x-b_k}{x-\imath}\right)^{\mu_k}. \quad (13)$$

Let

$$r_0 = \max\{\gamma_1, \gamma_2, \ldots, \gamma_u, \mu_1, \mu_2, \ldots, \mu_\omega, m\}, \quad (14)$$

$$\gamma = \sum_{k=1}^{u} \gamma_k, \mu = \sum_{j=1}^{\omega} \mu_k, \varkappa = ind\frac{N(x)}{M(x)}. \quad (15)$$

5 Theorem. *Let the functions $k_j(x), n_\nu(x) \in \mathbf{L}$, $j,\nu = \overline{1,m}$, $h(x) \in \mathbf{L}_2$; the functions $K_j(x), N_\nu(x) \in \mathbf{H}_\alpha^{(r)}$, $j,\nu = \overline{1,m}$, $H(x) \in \mathbf{L}_2^{(r)}$, $r \geq r_0$, where the number r_0 is defined by the formula (14) and the representations (12) take place. If*

$\varkappa-m-\gamma-\mu \geq 0$, where the numbers $\varkappa, \gamma, \mu$ are defined by the formulas (15), then the homogeneous equation (1) has not less than $\varkappa-m-\gamma-\mu$ linearly independent solutions; the heterogeneous equation (1) is an unconditionally solvable one and its general solution depends upon not less than $\varkappa-m-\gamma-\mu$ arbitrary constants. If $\varkappa-m-\gamma-\mu<0$, then generally speaking the heterogeneous equation (1) is an unsolvable one. It will be a solvable one when not less than $m+\gamma+\mu-\varkappa$ conditions of solvability (11) will be executed.

Proof. According to the work [3, p. 248–278] if $\varkappa-m-\gamma-\mu \geq 0$, then the homogeneous equation (6) has not less than $\varkappa-m-\gamma-\mu$ linearly independent solutions; the heterogeneous equation (6) is an unconditionally solvable one and its general solution depends upon not less than $\varkappa-m-\gamma-\mu$ arbitrary constants. If $\varkappa-m-\gamma-\mu<0$, then generally speaking the heterogeneous equation (6) is an unsolvable one. It will be a solvable one when not less than $m+\gamma+\mu-\varkappa$ conditions of solvability (11) will be executed. As the equations (6) and (1) are equivalent, then theorem is proved. □

6 Theorem. *Let the functions $k_j(x), n_\nu(x) \in \mathbf{L}$, $j,\nu=\overline{1,m}$, $h(x) \in \mathbf{L}_2$; the functions $K_j(x), N_\nu(x) \in \mathbf{H}_\alpha^{(r)}$, $j,\nu=\overline{1,m}$, $H(x) \in \mathbf{L}_2^{(r)}$, $r \geq r_0$, where the number r_0 is defined by the formula (14). Besides the representations (12) take place and the equation (1) is a solvable one. Then its solutions belong the space $\mathbf{L}_2[-r-m+r_0;0]$, $r \geq r_0$.*

The proof follows from the work [3, p. 248–298] because the equation's (6) solutions belong the space $\mathbf{L}_2[r-r_0;0]$ at that case.

Let consider the other cases of numbers' m,n,s correlation.

If $m>n, m>s$, then it will be $p=q=m$ in formulas (4), (5). So we have $A(x)+B(x)=(-1)^m A_m(x+\imath)^{-m}$, $A(x)-B(x)=(-1)^m A_m(x-\imath)^{-m}$ and theorems like 3, 4 take place.

If $m<n=s$, then it will be $p=q=n$ in formulas (4), (5). The representations

$$A(x)+B(x)=(x+\imath)^{-n}M(x)\rho_+(x), A(x)-B(x)=(x-\imath)^{-n}N(x)\rho_-(x), \quad (16)$$

where

$$\rho_+(x)=(x+\imath)^{-\gamma_0}\prod_{k=1}^{u}\left(\frac{x-a_k}{x+\imath}\right)^{\gamma_k}, \rho_-(x)=(x-\imath)^{-\mu_0}\prod_{k=1}^{\omega}\left(\frac{x-b_k}{x-\imath}\right)^{\mu_k} \quad (17)$$

take place. Besides,

$$r_0=\max\{\gamma_1,\gamma_2,\ldots,\gamma_u,\gamma_0+n,\mu_1,\mu_2,\ldots,\mu_\omega,\mu_0+n\}, \gamma=\sum_{k=0}^{u}\gamma_k, \mu=\sum_{j=0}^{\omega}\mu_k. \quad (18)$$

The numbers γ_0,μ_0 are integer orders of nulls in infinity of functions $K_n(x)$ and $N_n(x)$ accordingly.

Thus theorems like 5, 6 take place.

If $m < n < s$, then it will be $p = n, q = s$ in formulas (4), (5). The representations (16) take place where $\rho_+(x), \rho_-(x)$ are like in (17). The numbers γ, μ are defined by (18) and the number r_0 is

$$r_0 = \max\{\gamma_1, \gamma_2, \dots, \gamma_u, \gamma_0 + n, \mu_1, \mu_2, \dots, \mu_\omega, \mu_0 + s\}. \tag{19}$$

Theorems like 5, 6 take place there.

If $m = n < s$, then it will be $p = n, q = s$ in formulas (4), (5). The representations (16) take place where $\rho_+(x)$ is like in (13) and $\rho_-(x)$ is like in (17). The number r_0 is:

$$r_0 = \max\{\gamma_1, \gamma_2, \dots, \gamma_u, \mu_1, \mu_2, \dots, \mu_\omega, \mu_0 + s\}. \tag{20}$$

The number μ we define by the formula (18), and the number γ – by the formula (15). Theorems like 5, 6 take place there.

If $m < s < n$, then it will be $p = s, q = n$ in formulas (4), (5). The representations

$$A(x) + B(x) = (x + \imath)^{-s} M(x) \rho_+(x), A(x) - B(x) = (x - \imath)^{-n} N(x) \rho_-(x) \tag{21}$$

take place, where $\rho_+(x), \rho_-(x)$ are like in (17). The numbers γ, μ are defined by (18) and the number r_0 - by the formula

$$r_0 = \max\{\gamma_1, \gamma_2, \dots, \gamma_u, \gamma_0 + s, \mu_1, \mu_2, \dots, \mu_\omega, \mu_0 + n\}. \tag{22}$$

Theorems like 5, 6 take place there.

If $m = s < n$, then it will be $p = n, q = s$ in formulas (4), (5). The representations (16) take place where $\rho_+(x)$ 1s like in (17), and $\rho_-(x)$ is like in (13).The number r_0 is defined by the formula

$$r_0 = \max\{\gamma_1, \gamma_2, \dots, \gamma_u, \gamma_0 + n, \mu_1, \mu_2, \dots, \mu_\omega\}. \tag{23}$$

The number μ is defined by (15) and the number γ – by the formula (18). Theorems like 5, 6 take place there.

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