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The numerical analysis of the tense condition of a solid body with the asymmetrical tensor of strains

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Abstract. In this paper an approach permitting to make calculation of non-steady fields of elastic bodies with an asymmetrical stress tensor is proposed. On the basis of integral equations the explicit difference network, founded on S.K. Godunov method named "disintegrations of a gap" is constructed. The versions are considered, when the difference network approximates an initial set of equations with the first and second order of accuracy.

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The classical theory of elasticity is based on the model of a solid body in which interaction between particles is realized only by central forces.

However, it is impossible to explain satisfactorily the regularities of some phenomena, for instance, the spreading of the short acoustic waves in a crystalline solid body, polycrystalline metal and high polymer.

The theoretical results do not give satisfactory concordance with experimental data for a body with obviously expressed polycrystalline structure, in the complex tense condition with high gradient of the tension.

The model elaborated for the explanation of these phenomena differers from classical one by the fact that tense condition on elementary platform is characterized alongside with the vector of the power tension by the vector of moment tension, referred to the same unit platform as the vector of the usual tension.

The model of elastic moments medium was created in which infinitely small volume possesses six degrees of freedom, and the interaction between elements of the medium is realized by power and moments tensions [1–4].

Here we are concentrated on the consideration of the variant in which the motion of the medium point is completely described by the vector of the onward displacement , but the vector of the angular tumbling is equal to the local rotation of the medium in the sense of the usual theory of elasticity, i.e.

$$\overrightarrow{\omega} = \frac{1}{2} rot\, \overrightarrow{u}.$$

Such model is known under the name of continuum Kossera with straight rotation [5–7]. Each point of the medium has 3 degrees of freedom, and its motion is completely defined by the vector $\overrightarrow{u}$. In this variant of the moment theory of elasticity four independent elasticity constants: λ, μ, l, k are considered ; λ, μ are the Lame parameters, l is the constant with dimension of the length, k is a dimensionless constant of the type of Poison coefficient.

The waves of the expansion in the Kossera medium with straight rotation do not differ from the expansion waves in the classical theory of elasticity, however, the tension moments influence essentially on the distortion waves consideration. We shall present basic system of the equations for the case of the asymmetrical theory of elasticity we are interested in.

The equations of the motion for the plane case in a rectangular coordinate system will be of the form:

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}=\rho\frac{\partial^2 u}{\partial t^2},$$
$$\frac{\partial\sigma_{yx}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}=\rho\frac{\partial^2 v}{\partial t^2}, \tag{1}$$

where u, v are components of the displacement vector $\overrightarrow{u}$, and $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}, \sigma_{yx}$ are components of the tension tensor.

In this case $\sigma_{xy} \neq \sigma_{yx}$, and the relations between components of the tension and deformation taking in account inertness of the medium's particles rotation are of the form [8]

$$\sigma_{xx}=\lambda(\varepsilon_{xx}+\varepsilon_{yy})+2\mu\varepsilon_{xx}, \qquad \sigma_{yy}=\lambda(\varepsilon_{xx}+\varepsilon_{yy})+2\mu\varepsilon_{yy},$$
$$\sigma_{xy}=\mu\left[\varepsilon_{xy}-l^2\left(\frac{\partial^2\omega}{\partial x^2}+\frac{\partial^2\omega}{\partial y^2}-\frac{k^2\rho}{\mu}\frac{\partial^2\omega}{\partial t^2}\right)\right] \tag{2}$$
$$\sigma_{xy}=\mu\left[\varepsilon_{xy}+l^2\left(\frac{\partial^2\omega}{\partial x^2}+\frac{\partial^2\omega}{\partial y^2}-\frac{k^2\rho}{\mu}\frac{\partial^2\omega}{\partial t^2}\right)\right],$$

where ρ is the density, $\omega = \partial u/\partial y - \partial v/\partial x$.

As a consequence of the relations (2) the equation of the medium particles rotation is examined in the following form:

$$2k^2\rho l^2\frac{\partial^2\omega}{\partial t^2}=\frac{\partial M_{zx}}{\partial x}+\frac{\partial M_{zy}}{\partial y}+\left(\sigma_{xy}-\sigma_{yx}\right),$$

where $\sigma_{xy}+\sigma_{yx}=2\mu\varepsilon_{xy}$.

The internal moments M_{zx} and M_{zy} are expressed by ω as follows

$$M_{zx}=2\mu l^2\frac{\partial\omega}{\partial x}, \quad M_{zy}=2\mu l^2\frac{\partial\omega}{\partial y}. \tag{3}$$

The numerical integration of the hyperbolic system (4) is realized in the presence of the initial and border conditions by the method, founded on approximations of the equations by finite differences taking into account relations along characteristic directions.

For the numerical solution of the moment theory of elasticity equations, S. K. Godunov [9] explicit difference scheme will be considered. It requires to pass to the system of equations in first-order partial derivatives. Taking in consideration the velocity of the displacement, and differentiating (2) by time, after uncomplicated transformations (4) will be obtained.

Here with the bar the corresponding functions differentiated by time are marked. From now on for convenience, the bar will be omitted. It is known that the method requires the determination of the characteristic equations and relations for them. In accordance with S.K. Godunov method for the determination of the characteristics in x direction free terms and derivatives by y are excluded from the system (4).

It is easy to note that in the obtained system of equations we have:

a) Two equations form up closed group giving the unknown functions σ_{xx}, u;

b) Two equations form up closed group giving the unknown functions σ_{xy}, v;

c) Two equations form up closed group giving the unknown functions M_{zx}, ω

$$\begin{gathered}
\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}=\rho\frac{\partial u}{\partial t},\qquad \frac{\partial\sigma_{yx}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}=\rho\frac{\partial v}{\partial t},\\
\frac{\partial\sigma_{xx}}{\partial t}=\lambda\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+2\mu\frac{\partial u}{\partial x},\\
\frac{\partial\sigma_{yy}}{\partial t}=\lambda\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+2\mu\frac{\partial v}{\partial y},\\
\frac{\partial\sigma_{xy}}{\partial t}+\frac{\partial\sigma_{yx}}{\partial t}=2\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right),\\
2k^2l^2\rho\frac{\partial\omega}{\partial t}=\frac{\partial M_{zx}}{\partial x}+\frac{\partial M_{zy}}{\partial y}+\sigma_{xy}-\sigma_{yx},\\
\frac{\partial M_{zx}}{\partial t}=2\mu l^2\frac{\partial\omega}{\partial x},\qquad \frac{\partial M_{zy}}{\partial t}=2\mu l^2\frac{\partial\omega}{\partial y}.
\end{gathered}\tag{4}$$

Father, following the known procedure, we shall multiply the first (the second) equation of the obtained first group of "one-dimensional" system on the α_1 (α_2) and add both equations.

As a result we get

$$\frac{\partial}{\partial t}\{\alpha_1\rho\, u+\alpha_2\sigma_{xx}\}=\frac{\partial}{\partial x}\{\alpha_2(\lambda+2\mu)\, u+\alpha_1\sigma_{xx}\}.\tag{5}$$

Our purpose consists in obtaining the characteristic equation for the Reman invariant F in the following form:

$$\frac{\partial F}{\partial t}=\eta\frac{\partial F}{\partial x}.$$

The linear combination of the velocity and tension (5) will be an invariant F if the following conditions are fulfilled

$$\alpha_1\rho = \alpha_2(\lambda + 2\mu)/\eta; \quad \alpha_1/\eta = \alpha_2.$$

In this homogeneous system one needs to determine such a value η for which its nonzero solution exists. Consequently,

$$(\rho - (\lambda + 2\mu)/\eta^2)\alpha_1 = 0; \quad (\rho - (\lambda + 2\mu)/\eta^2)\alpha_2 = 0. \tag{6}$$

Since for the existence of a nonzero solution either α_1, or α_2 must be different from zero, we find eigenvalues η_i:

$$\eta_{1,2} = \pm\sqrt{(\lambda + 2\mu)/\rho} = \pm c_1,$$

where c_1 is the velocity of the longitudinal wave spreading.

Proceeding similarly, for the second group of equations we shall get

$$\eta_{3,4} = \pm\sqrt{\mu/\rho} = \pm c_2,$$

where c_2 is the velocity of the transversal wave spreading.

The third group of equations gives

$$\frac{\partial}{\partial t}\{\alpha_1 M_{zx} + 2\alpha_2 k^2 \rho l^2 \omega\} = \frac{\partial}{\partial x}\{\alpha_2 M_{zx} + 2\alpha_1 \mu l^2 \omega\}. \tag{7}$$

The linear combination (7) will be an invariant F if the following conditions are fulfilled:

$$\alpha_1 = \alpha_2/\eta, \qquad \alpha_1\mu/\eta = \alpha_2 k^2 \rho. \tag{8}$$

In homogeneous system (8) it is necessary to determine such value of η, for which a nonzero solution for this system exists. Consequently,

$$(k^2\rho - \mu/\eta^2)\alpha_1 = 0, \qquad (k^2\rho - \mu/\eta^2)\alpha_2 = 0. \tag{9}$$

Since the existence of a nonzero solution necessitates that either α_1, or α_2 must be different from zero, we find eigenvalues η_i:

$$\eta_{5,6} = \pm\sqrt{\mu/k^2/\rho} = \pm c_3.$$

Because in the medium only one transversal wave must be, we require $k = 1$ and $c_3 = c_2$. Really, if the volume deformation $\delta = \partial u/\partial x + \partial v/\partial y$ will be introduced, then it is possible to reduce the system of equations (4) to the following form:

$$\Delta\delta - \frac{1}{c_1^2}\delta_{tt} = 0; \quad \Delta\omega - \frac{1}{c_2^2}\omega_{tt} - l^2\left(\Delta\omega - \frac{k^2}{c_2^2}\omega_{tt}\right) = 0. \tag{10}$$

As can be seen from (10), this equation is a wave one relative to σ, as well as in the classical linear theory of elasticity while shift deformation ω satisfies the fourth order equation. By the study of the flat waves spreading in the elastic medium three-dimensional and shift deformation are expressed through four arbitrary functions depending on the flat waves [10]

$$\mathrm{Z}_{1,2} = t - \theta x \mp \sqrt{c_1^2 - \theta^2}y, \qquad \mathrm{Z}_{3,4} = t - \theta x \mp \sqrt{c_2^2 - \theta^2}y,$$

where θ is an arbitrary parameter.

The equation (10) relative to δ allows arbitrary solutions for flat waves through $\mathrm{Z}_{1,2}$.

We research the equation (10) for ω. Let $\omega = f(\gamma t - \alpha y - \beta x)$. Substituting in equation (10), we shall get

$$\left(\alpha^2 + \beta^2 - \gamma^2/c_2^2\right) f'' - l^2 \left(\alpha^2 + \beta^2\right) \left(\alpha^2 + \beta^2 - k^2\gamma^2/c_2^2\right) f^{IV} = 0 \tag{11}$$

From the given equation it follows that flat waves of the type $\omega = f(\gamma t - \alpha y - \beta x)$ for arbitrary type of the functions f are possible only by the condition $k = 1$, $\alpha^2 + \beta^2 - \gamma^2/c_2^2 = 0$.

So, further we shall everywhere consider $k = 1$.

We shall consider further the case of $\eta_{1,2} = \pm c_1$. Then from (6) it follows that α_1 is an arbitrary constant value, which we take equal to ± 1, and the factor $\alpha_2 = \pm 1/c_1$.

Then the invariant $F_1 = \rho u \pm \sigma_{xx}/c_1$.

Similarly it is possible to obtain $F_2 = \rho v \pm \sigma_{xy}/c_2$; $F_3 = M_{zx} \pm 2\alpha_2 \rho l^2/c_2\omega$.

This means that along the characteristics $\frac{\partial x}{\partial t} = \pm\eta_k$ the relations $F_k = const$ are fulfilled, because

$$dF_k = \frac{\partial F_k}{\partial t}dt + \frac{\partial F_k}{\partial x}dx = \left(\frac{\partial F_k}{\partial t} \pm \eta_k \frac{\partial F_k}{\partial x}\right) dt = 0.$$

By analogy with "one-dimensional" system relative to the variable x, "one-dimensional" system relative to the variable y can be considered, obtained from system (4) when excluding derivatives by y.

The direct comparison of these systems shows that they become completely identical after establishing the correspondence:

$$x \leftrightarrow y, \quad u \leftrightarrow v, \quad \sigma_{xx} \leftrightarrow \sigma_{yy}, \quad \sigma_{xy} \leftrightarrow \sigma_{yx}, \quad M_{zx} \leftrightarrow M_{zy}, \quad F_k \leftrightarrow \Phi_k.$$

The characteristics and relations for them are automatically obtained with account of this correspondence.

The invariants F_k and $\Phi_k (k = 1, 2, 3)$ possess important features that along straight line $\pm c_k t + x = const$ they keep constant values.

These features are put in the base of the finite difference scheme construction.

In the domain of the arguments x and y variation we shall introduce uniform differences schemes as follows. The area will be limited by the contour, given by the restrictions $0 \le x \le a$, $0 \le y \le b$. We shall cover this square-wave area with the net as follows: for x

$$h_x = a/I, \quad x_i = i \cdot h_x, i = 1, 2, \cdots, I,$$

and for y

$$h_y = a/J, \quad y_j = j \cdot h_y, j = 1, 2, \cdots, J.$$

The nodes of the deference net, in which we shall define the unknown functions, choose in the cell center, formed by orthogonal net. All unknown functions are related to the center of accounting cell and are considered constant within a separate cell. Since the problem is dynamic then in the difference scheme the function values are present on two temporary stratums t_n and t_{n+1}, with step on time $\tau_n = t_{n+1} - t_n$.

We shall mark the functions defined on upper temporary layer t_{n+1}, by $\varphi^{i-1/2,j-1/2}$ or $\overline{\varphi}$, and on the under-stratums by $\varphi_{i-1/2,j-1/2}$ or φ. The approximation of the system of equations (4) is built on the base of integral identity, equivalent to the system (4).

The solution will be defined in the three-dimensional space with coordinate x, y, t.

We shall consider the elementary volume V, formed by the coordinate planes $x = x_i, x = x_{i-1}, y = y_j, y = y_{j-1}, t = t_n, t = t_{n-1}$. Let us take integral from the first equation of the system (4) on volume V:

$$\rho \int_V \frac{\partial u}{\partial t} dV = \int_V \left[\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} \right] dV$$

Considering that $dV = dxdydt$ and applying integrations formulas by parts, we get

$$\rho \int_{S_1} u|_{t_n}^{t_{n+1}} dxdy = \int_{S_2} \sigma_{xx}|_{x_i}^{x_{i+1}} dtdy + \int_{S_3} \sigma_{xy}|_{y_j}^{y_{j+1}} dxdt.$$

Here through S_1, S_2, S_3 we shall mark the areas sides of the elementary volume. Approximating the obtained integrals on the areas S_1, S_2, S_3 by square formula of the central rectangle, get the following difference equation

$$\rho \left(\overline{u} - u \right) h_x h_y = \left(\Sigma_{xxi,j-1/2} - \Sigma_{xxi-1,j-1/2} \right) h_y \tau +$$

$$+ \left(\Sigma_{xyi-1/2,j} - \Sigma_{xyi-1/2,j-1} \right) h_x \tau.$$

The functions $u, \sigma_{xx}, \sigma_{xy}$ are determined on the lower temporary layer t_n, and $\overline{u}, \overline{\sigma}_{xx}, \overline{\sigma}_{xy}$ are determined on the temporary layer t_{n+1}. In the center of the lateral sides S_1, S_2 "greater" values are defined, their expressions through "small" values are given below.

The obtained equation will be divided by $h_x \cdot h_y \cdot \tau$

$$\rho \frac{\overline{u} - u}{\tau} = \frac{\Sigma_{xxi,j-1/2} - \Sigma_{xxi-1,j-1/2}}{h_x} + \frac{\Sigma_{xyi-1/2,j} - \Sigma_{xyi-1/2,j-1}}{h_y}.$$

The rest of the equations are approximated similarly, and that gives

$$\rho \frac{\overline{v} - v}{\tau} = \frac{\Sigma_{yxi,j-1/2} - \Sigma_{yxi-1,j-1/2}}{h_x} + \frac{\Sigma_{yyi-1/2,j} - \Sigma_{yyi-1/2,j-1}}{h_y},$$

$$\frac{\overline{\sigma}_{xx}-\sigma_{xx}}{\tau}=(\lambda+2\mu)\,\frac{U_{i,j-1/2}-U_{i-1,j-1/2}}{h_x}+\lambda\frac{V_{i-1/2,j}-V_{i-1/2,j-1}}{h_y},$$

$$\frac{\overline{\sigma}_{yy}-\sigma_{yy}}{\tau}=\lambda\frac{U_{i,j-1/2}-U_{i-1,j-1/2}}{h_x}+(\lambda+2\mu)\,\frac{V_{i-1/2,j}-V_{i-1/2,j-1}}{h_y},$$

$$\frac{\overline{\sigma}_{xy}-\sigma_{xy}}{\tau}+\frac{\overline{\sigma}_{yx}-\sigma_{yx}}{\tau}=2\mu\left(\frac{U_{i-1/2,j}-U_{i-1/2,j-1}}{h_y}+\frac{V_{i,j-1/2}-V_{i-1,j-1/2}}{h_x}\right),\quad(12)$$

$$2\rho l^2\frac{\overline{\omega}-\omega}{\tau}=\frac{M_{zxi,j-1/2}-M_{zxi-1,j-1/2}}{h_x}+\frac{M_{zyi-1/2,j}-M_{zyi-1/2,j-1}}{h_y}+\sigma_{xy}-\sigma_{yx},$$

$$\frac{\overline{M}_{zx}-M_{zx}}{\tau}=2\mu l^2\frac{\Omega_{i,j-1/2}-\Omega_{i-1,j-1/2}}{h_x},$$
$$\frac{\overline{M}_{zy}-M_{zy}}{\tau}=2\mu l^2\frac{\Omega_{i-1/2,j}-\Omega_{i-1/2,j-1}}{h_y}.$$

The built system of equations in finite differences (12) approximates the system of the differential equations (4). For finishing of the equations in finite differences building is required to indicate the way of the "greater" values calculation through "small". "Greater" values are defined in the center of the lateral sides of the volume V. Their expression through "small" values φ in internal nodes of the net are found from characteristic correlations, considered above.

"Greater" values Φ in the x direction are calculated as follows . We shall consider two nearby cells on temporary layer t_n, the centers of which are marked by $i-1/2$, $j-1/2$ and $i+1/2$, $j-1/2$. We shall conduct from these point characteristics with slopping $\pm c_1$, but from point of their intersection we shall lower characteristics with slopping $\pm c_2$. Since along these straight linear combinations of the unknown function maintain constant values, it is possible to write following correlations

$$\rho U_{i,j-1/2}+\Sigma_{xxi,j-1/2}/c_1=\rho u_{i+1/2,j-1/2}+\sigma_{xxi+1/2,j-1/2}/c_1,$$
$$\rho U_{i,j-1/2}-\Sigma_{xxi,j-1/2}/c_1=\rho u_{i-1/2,j-1/2}-\sigma_{xxi-1/2,j-1/2}/c_1.\quad(13)$$

Solving system (13) relative to "greater" values, we get

$$U_{i,j-1/2}=(u_{i+1/2,j-1/2}+u_{i-1/2,j-1/2})/2+$$
$$+(\sigma_{xxi+1/2,j-1/2}-\sigma_{xxi-1/2,j-1/2})/2/\rho/c_1,$$

$$\Sigma_{xxi,j-1/2}=\rho c_1(u_{i+1/2,j-1/2}-u_{i-1/2,j-1/2})/2+$$

$$+(\sigma_{xxi+1/2,j-1/2} + \sigma_{xxi-1/2,j-1/2})/2.$$

Doing similar discourses and transformations, we shall define the rest "greater" values, in direction y similarly as in direction x.

"Greater" values, on the border of the area are defined from three relations for the characteristics and three border conditions.

The initial conditions for the examined scheme are written by change continuous function by "small" values, determined on the net.

By decompositions of the discrete function, entering into difference scheme, in Taylor row in the neighborhood of the point (x_i, y_j, t_n) it is possible to show that built accounting scheme has a first approximation order. Stability condition can be received in the base of the Churant-Fridrix-Levi criterion, which confirms that velocity in differences of the wave spreading on each directions, must not be less, than velocity of the wave for differential approach.

Then we get

$$\tau \le \left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)^{-1},$$

where $\tau_1 \le \frac{h_x}{c_1}$, $\tau_1 \le \frac{h_y}{c_1}$.

When execution of this condition is ensured, the solution of the built difference problem will be converging with the first order accuracy to exact solution. In order to study possibilities of the using at calculation of the second order accuracy schemes we shall consider certain modification of the scheme built above. Increasing of the scheme accuracy can be reached by account of centering differences function on temporary interval and taking into account of the change tangent to the verge of the sought function [11].

In obtained above grid-characteristic scheme, auxiliary "greater" values on the cell border were calculated at condition of constancy values of the vector solution components within each elementary cell. In this case, for determination of the values in cross point of the characteristics with lower time plane, in essence, is used zero order interpolation that provides the first order accuracy and monotonicity of the difference schemes. Increasing approximation till the second oder can be reached by considering more exact interpolation formulas.

For calculation of intermediate values of the sought function on previous temporary layer $n\tau$ we use modified formula for square interpolations.

$$u_\pm = u_{k\pm1/2} + g^*\left(u_{k\mp1/2} - u_{k\pm1/2}\right) +$$

$$+Ag^*\left(1-g^*\right)\left(u_{k\mp1/2} - 2u_{k\pm1/2} + u_{k\pm3/2}\right)/2.$$

For the bounded nodes formula with unilateral finite differences is truthful

$$u_\pm = u_{k\pm1/2} + g^*\left(u_{k\mp1/2} - u_{k\pm3/2}\right) +$$

$$+Ag^*\left(1-g^*\right)\left(u_{k\pm1/2} - 2u_{k\pm3/2} + u_{k\pm5/2}\right)/2.$$

Here signs "+", "−" correspond to positive and negative slopping of the characteristics $g^* = (1 - \tau c_1/h)/2$; A is the parameter, which value is defined from condition $\|u_n - [u]_h\| \to \min$, where u_n is obtained discrete solution; $[u]_h$ is projection of the sample problem exact solution on the grid; $\|\cdot\|$ is the norm in the space of grid function; h is the spatial step.

It is known that schemes of the raised accuracy order to do not possess monotonicity feature, but choice of the best value of the parameter A allows to minimize the dispersion of the finite differences solutions comparatively exact and vastly reduce drid viscosity, which show the first-order accuracy scheme.

So, the second order accuracy difference scheme differs from first-order accuracy scheme by way of expressing of the "greater" values through "small".

For nodes of the grid, located in internal area, we write following caracteristics correlations in x direction

$$\rho U_{i,j-1/2} + \Sigma_{xxi,j-1/2}/c_1 = s_{k+1/2} + g^* \left(s_{k-1/2} - s_{k+1/2}\right) +$$
$$+ A g^* \left(1 - g^*\right) \left(s_{k-1/2} - 2s_{k+1/2} + s_{k+3/2}\right)/2,$$

$$\rho U_{i,j-1/2} - \Sigma_{xxi,j-1/2}/c_1 = s_{k-1/2} + g^* \left(s_{k+1/2} - s_{k-1/2}\right) +$$
$$+ A g^* \left(1 - g^*\right) \left(s_{k+1/2} - 2s_{k-1/2} + s_{k-3/2}\right)/2,$$

were $s_{\pm} = \rho u_{i\pm1/2,j-1/2} + \sigma_{xxi\pm1/2,j-1/2}/c_1$.

We shall mark the right parts of the last correlations through $S_{i+1/2}$ and $S_{i-1/2}$. Then "greater" values are expressed through "small" by formula

$$U_{i,j-1/2} = 0.5\left(S_{i+1/2} + S_{i+1/2}\right)/\rho,$$

$$\Sigma_{xxi,j-1/2} = 0.5c_1\left(S_{i+1/2} - S_{i+1/2}\right).$$

Similarly all rest "greater" values are got.

Study of the stability by Furie method gives condition

$$\tau \le (a_+/b_-)\tau_+\tau_-/\sqrt{\tau_-^2 + (a_+/b_-)^2\tau_+^2},$$

were $\tau_+ = \max(\tau_x, \tau_y), \tau_- = \min(\tau_x, \tau_y), a_+ = \max(\lambda_i, \mu_i), b_- = \min(\lambda_i, \mu_i)$, $(i = 1, 2)$, $\tau_x = h_x/a_+$, $\tau_y = h_y/a_+$ are the limiting steps, defined from condition of stability corresponding to one dimentional scheme.

For square greed $\tau_+ = \tau_- = \tau_0 = h/a_+$ and $\tau \le (a_+/b_-)\tau_0[1 + (a_+/b_-)^2]^{-1/2}$.

A more severe condition have in the case of the first-order accuracy scheme

$$\tau \le (a_+/b_-)\tau_+\tau_-[\tau_- + (a_+/b_-)\tau_+],$$

from which for square grid follows $\tau \le (a_+/b_-)\tau_0/[1 + (a_+/b_-)]$.

In order to obtain more high accuracy order results brings to expediency of the hybrid difference schemes using with flows correction, taking as support, described above scheme variants.

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