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A nonlinear hydrodynamic stability criterion derived by a generalized energy method

Cătălin Liviu Bichir, Adelina Georgescu, Lidia Palese

Abstract. By applying a new variant of the A. Georgescu – L. Palese – A. Redaelli (G-P-R) method [8], based on the symmetrization of a linear operator, we deduce a nonlinear stability criterion of a state of thermal conduction of a horizontal fluid layer subject to a vertical upwards uniform magnetic field and a vertical upwards constant temperature gradient. The Boussinesq approximation is used. The upper and lower surfaces of the layer are two rigid walls. It is assumed that the magnetic Prandtl number is strictly greater than unity.

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1 The perturbation problem

Consider an infinite horizontal layer of a homogeneous viscous electrically conducting fluid at rest ($\mathbf{V} = 0$) subject to the influence of a uniform vertical upwards magnetic field $\mathbf{H}$ and of an adverse constant vertical temperature gradient $\beta > 0$. Let $Oxyz$ be a Cartesian coordinate system, with $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ the unit vectors of the axes, where the vertical axis Oz has the direction opposite to the gravity. Suppose that the fluid is confined between the planes $z = 0$ and $z = 1$, on which the temperatures $T|_{z=0} = T_0$ and $T|_{z=1} = -\beta + T_0$ respectively are kept constant.

In the Oberbeck-Boussinesq approximation, the stability of the basic state m_0 ($\mathbf{V} = 0$, $\mathbf{H} = H\mathbf{k}$, $T = -\beta z + T_0, P$) is governed [1] by the following dimensionless equations for the perturbation fields $(\mathbf{u}, \mathbf{h}, \theta, p_1)$ of the state m_0

$$\begin{aligned}&\partial\mathbf{u}/\partial t + (\mathbf{u}\cdot\mathbf{grad})\mathbf{u} - P_m(\mathbf{h}\cdot\mathbf{grad})\mathbf{h} = \\ &= -\mathbf{grad}\, p_1 + R\theta\mathbf{k} + \triangle\mathbf{u} + Q\partial\mathbf{h}/\partial z,\end{aligned} \tag{1.1}$$

$$div\ \mathbf{u} = 0, \tag{1.2}$$

$$P_m(\partial\mathbf{h}/\partial t + (\mathbf{u}\cdot\mathbf{grad})\mathbf{h} - (\mathbf{h}\cdot\mathbf{grad})\mathbf{u}) = \triangle\mathbf{h} + Q\partial\mathbf{u}/\partial z, \tag{1.3}$$

$$div\ \mathbf{h} = 0, \tag{1.4}$$

$$P_r(\partial\theta/\partial t + (\mathbf{u}\cdot\mathbf{grad})\theta) = Rw + \triangle\theta, \tag{1.5}$$

where $(t, \mathbf{x}) \in (0, \infty) \times \mathbb{R}^2 \times (0, 1)$, $\mathbf{x} = (x, y, z)$, and by the conditions

$$\mathbf{u}(0,\mathbf{x}) = \mathbf{u}_0(\mathbf{x}), \quad \mathbf{h}(0,\mathbf{x}) = \mathbf{h}_0(\mathbf{x}), \quad \theta(0,\mathbf{x}) = \theta_0(\mathbf{x}), \quad \mathbf{x} \in \mathbb{R}^2 \times (0,1), \tag{1.6}$$

$$div\ \mathbf{u}_0 = div\ \mathbf{h}_0 = 0, \tag{1.7}$$

$$\mathbf{u}(t,\mathbf{x}) = \mathbf{h}(t,\mathbf{x}) = \mathbf{0}, \quad \theta(t,\mathbf{x}) = 0 \quad \text{at} \quad z=0,\ z=1,\ t \geq 0. \tag{1.8}$$

Here $\mathbf{u} = (u,v,w) = (u_1,u_2,u_3)$, $w = \mathbf{u}\cdot\mathbf{k}$, $\mathbf{h} = (h_1,h_2,h_3)$, θ, p_1 are the perturbations of the velocity, magnetic, temperature and pressure (including the magnetic pressure) fields respectively. The dimensionless numbers are the Prandtl number $P_r = \nu/\kappa$, the Rayleigh number $R^2 = g\alpha\beta d^4/(\kappa\nu)$, the magnetic Prandtl number $P_m = \nu/\eta$, and the Chandrasekhar number $Q^2 = \mu H^2 d^2/(4\pi\rho\nu\eta)$, where ν is the coefficient of kinematic viscosity, κ is the coefficient of thermometric conductivity, $-g\mathbf{k}$ is the gravitational acceleration, α is the coefficient of volume expansion, ρ is the density, $\eta = 1/(4\pi\mu\sigma)$ is the resistivity, μ is the magnetic permeability, and σ is the coefficient of electrical conductivity. Assume that the perturbation fields are periodic functions of x and y, of periods $2\pi/a_x$ and $2\pi/a_y$ respectively, where $a_x, a_y > 0$. Denote by V the periodicity cell, $V = [0,2\pi/a_x]\times[0,2\pi/a_y]\times[0,1]$ and let ∂V_h be the horizontal boundary. We have $\partial V_h = \partial V_1 \cup \partial V_0$, where ∂V_1 and ∂V_0 are the upper and lower boundary respectively. In the sequel, the brackets $\langle\ \cdot\ \rangle$ stand for the integration over V, i.e. $\langle\ \cdot\ \rangle = \int\limits_V \cdot\ dV$. We impose the extra conditions

$$\langle u\rangle = \langle v\rangle = 0. \tag{1.9}$$

2 Energy relation

In order to obtain nonlinear stability criteria, let us apply the G-P-R method [8] to the perturbation problem (1.1) – (1.8). To this aim, first we write the system (1.1) – (1.5) as the equivalent system consisting of the equations (1.1), (1.3) and (1.5), in the space

$$\begin{aligned}\mathcal{N}_1 = \{(\theta,\mathbf{u},\mathbf{h}) \in H^2(V)^7 \mid div\ \mathbf{u} = div\ \mathbf{h} = 0;\\ \mathbf{u} = \mathbf{h} = \mathbf{0},\ \theta = 0 \text{ on } \partial V_h\}.\end{aligned}$$

In turn, this system is equivalent to the modified system in $\mathcal{N}_1$

$$\partial\theta/\partial t + (\mathbf{u}\cdot\mathbf{grad})\theta = P_r^{-1}\triangle\theta + P_r^{-1}R\mathbf{u}\cdot\mathbf{k}, \tag{2.1}$$

$$\begin{aligned}&a(\partial\mathbf{u}/\partial t + (\mathbf{u}\cdot\mathbf{grad})\mathbf{u}) + ag_3P_m(\partial\mathbf{h}/\partial t + (\mathbf{u}\cdot\mathbf{grad})\mathbf{h}) =\\ &= -a\ \mathbf{grad}\ p_1 + aR\theta\mathbf{k} + a\triangle\mathbf{u} + aQ\partial\mathbf{h}/\partial z + aP_m(\mathbf{h}\cdot\mathbf{grad})\mathbf{h} +\\ &+ ag_3Q\partial\mathbf{u}/\partial z + ag_3\triangle\mathbf{h} + ag_3P_m(\mathbf{h}\cdot\mathbf{grad})\mathbf{u},\end{aligned} \tag{2.2}$$

$$\begin{aligned}&bP_m(\partial\mathbf{h}/\partial t + (\mathbf{u}\cdot\mathbf{grad})\mathbf{h}) + bg_2(\partial\mathbf{u}/\partial t + (\mathbf{u}\cdot\mathbf{grad})\mathbf{u}) =\\ &= bQ\partial\mathbf{u}/\partial z + b\triangle\mathbf{h} + bP_m(\mathbf{h}\cdot\mathbf{grad})\mathbf{u} - bg_2\mathbf{grad}\ p_1 +\\ &+ bg_2R\theta\mathbf{k} + bg_2\triangle\mathbf{u} + bg_2Q\partial\mathbf{h}/\partial z + bg_2P_m(\mathbf{h}\cdot\mathbf{grad})\mathbf{h},\end{aligned} \tag{2.3}$$

obtained by the following algebraic operations: (2.1) = (1.5)P_r^{-1}, (2.2) = a(1.1) + ag_3 (1.3), (2.3) = b(1.3) + bg_2(1.1), where a, b, g_2 and g_3 are, so far, undetermined nonnull constants.

Consider on $\mathcal{N}_1$ the scalar product $(\cdot,\cdot)$ of $\mathbf{L}^2(V)$ ($\equiv L^2(V)^7$). Introduce two linear operators $L_1 \in L(\mathcal{N}_1, \mathbf{L}^2(V))$, $L_2 \in L(\mathcal{N}_1,\mathcal{N}_1)$ and use the notation $\mathbf{U} =$

$(\theta, \mathbf{u}, \mathbf{h})^T \in \mathcal{N}_1,\ \mathbf{U}_1 = L_2\mathbf{U} = (\theta, a\mathbf{u} + ag_3P_m\mathbf{h}, bg_2\mathbf{u} + bP_m\mathbf{h})^T$, where L_1 and L_2 are defined by

$$L_1 = \begin{bmatrix} P_r^{-1}\triangle & P_r^{-1}R\mathbf{k} & 0 \\ aR\mathbf{k} & a\triangle + ag_3Q\partial/\partial z & ag_3\triangle + aQ\partial/\partial z \\ bg_2R\mathbf{k} & bg_2\triangle + bQ\partial/\partial z & b\triangle + bg_2Q\partial/\partial z \end{bmatrix},$$

$$L_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & a & ag_3P_m \\ 0 & bg_2 & bP_m \end{bmatrix}.$$

In addition, we define the nonlinear mapping

$$T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & ag_3P_m(\mathbf{h}\cdot\mathbf{grad}) & aP_m(\mathbf{h}\cdot\mathbf{grad}) \\ 0 & bP_m(\mathbf{h}\cdot\mathbf{grad}) & bg_2P_m(\mathbf{h}\cdot\mathbf{grad}) \end{bmatrix}.$$

It follows that the system (2.1) - (2.3) in $\mathbf{U} \in \mathcal{N}_1$ reads

$$(\partial/\partial t + \mathbf{u}\cdot\mathbf{grad})\mathbf{U}_1 = L_1\mathbf{U} + (0, -a\ \mathbf{grad}\ p_1, -bg_2\ \mathbf{grad}\ p_1)^T + T(\mathbf{U})$$

or, equivalently,

$$\partial\mathbf{U}_1/\partial t = L_1\mathbf{U} + N(\mathbf{U}) + T(\mathbf{U}), \tag{2.4}$$

where the mapping $N(\mathbf{U})$ corresponds to the advective and pressure terms, i.e.

$$N(\mathbf{U}) = -(\mathbf{u}\cdot\mathbf{grad})\mathbf{U}_1 + (0, -a\ \mathbf{grad}\ p_1, -bg_2\ \mathbf{grad}\ p_1)^T.$$

According to the Weyl decomposition lemma, a vector from $\mathbf{L}^2(V)$ is uniquely written as a sum of a solenoidal vector and a gradient of a scalar function. Then a projection of $\mathbf{L}^2(V)$ to $\mathcal{N}_1$ can be defined. If $(\cdot,\cdot)$ stands for the inner product in $\mathbf{L}^2(V)$, this projection of the system (2.1)-(2.3) to $\mathcal{N}_1$ is defined by the inner product of (2.4) by $\mathbf{U}$. As a result, from (2.4), we obtain the energy relation

$$(\partial\mathbf{U}_1/\partial t, \mathbf{U}) = (L_1\mathbf{U}, \mathbf{U}) + (N(\mathbf{U}), \mathbf{U}) + (T(\mathbf{U}), \mathbf{U}). \tag{2.5}$$

If $a = b$, then $(T(\mathbf{U}), \mathbf{U}) = 0$ because $\int_V (\mathbf{h}\cdot\mathbf{grad})\mathbf{u}\cdot\mathbf{u}\,dV = \int_V (\mathbf{h}\cdot\mathbf{grad})\mathbf{h}\cdot\mathbf{h}\,dV = 0$ and $\int_V (\mathbf{h}\cdot\mathbf{grad})\mathbf{h}\cdot\mathbf{u}\,dV = -\int_V (\mathbf{h}\cdot\mathbf{grad})\mathbf{u}\cdot\mathbf{h}\,dV$. Moreover, in order for the coefficients of $(\partial\mathbf{u}/\partial t)\cdot\mathbf{h}$ and $(\partial\mathbf{h}/\partial t)\cdot\mathbf{u}$ in the left - hand side of (2.5) be equal, we must have $g_3P_m = g_2$. Then (2.5) becomes

$$\frac{1}{2}\,d(\mathbf{U}_1, \mathbf{U})/dt = (L_1\mathbf{U}, \mathbf{U}) + (N(\mathbf{U}), \mathbf{U}). \tag{2.6}$$

Using Green identities, the relation $g_3P_m = g_2$ and the fact that $\mathbf{grad}\ p_1$ is orthogonal to the solenoidal vectors $\mathbf{u}$ and $\mathbf{h}$, it follows that $(N(\mathbf{U}), \mathbf{U}) = 0$. Consequently, the energy relation (2.6) becomes

$$\frac{1}{2}\,d(\mathbf{U}_1, \mathbf{U})/dt = (L_1\mathbf{U}, \mathbf{U}). \tag{2.7}$$

The symmetric part of L_1 reads

$$L_{1s} = \begin{bmatrix} P_r^{-1}\triangle & \delta_1\mathbf{k} & \delta_2\mathbf{k} \\ \delta_1\mathbf{k} & a\triangle & \delta_3\triangle \\ \delta_2\mathbf{k} & \delta_3\triangle & b\triangle \end{bmatrix},$$

where $\delta_1 = 0.5(a + P_r^{-1})R$, $\delta_2 = 0.5ag_2R$, $\delta_3 = 0.5a(g_3 + g_2)$. Since $(L_1\mathbf{U},\mathbf{U}) = (L_{1s}\mathbf{U},\mathbf{U})$, (2.7) becomes

$$\begin{aligned}\frac{1}{2}\cdot d(\mathbf{U}_1,\mathbf{U})/dt = P_r^{-1}[-\langle|\mathbf{grad}\ \theta|^2\rangle - P_r a\langle|\mathbf{grad}\ \mathbf{u}|^2\rangle -\\ - P_r a\langle|\mathbf{grad}\ \mathbf{h}|^2\rangle - P_r a(g_3+g_2)\langle\mathbf{grad}\ \mathbf{u}\cdot\mathbf{grad}\ \mathbf{h}\rangle + \\ + P_rR(a+P_r^{-1})\langle\theta w\rangle + P_rRag_2\langle\theta h_3\rangle].\end{aligned} \tag{2.8}$$

3 The algebraic associated system

Introduce the functions

$$\phi_1 = a_1\mathbf{u} + a_2\mathbf{h}, \quad \phi_2 = b_1\mathbf{u} + b_2\mathbf{h},$$

where the constants $a_1, a_2, b_1, b_2 \in \mathbb{R}$ are to be determined and $\phi_1 = \phi_1(t,\mathbf{x})$, $\phi_2 = \phi_2(t,\mathbf{x})$. Remark that this choice represents an extension of the G-P-R method, because here ϕ_1 and ϕ_2 are vector functions. Thus, the expression

$$(\mathbf{U}_1,\mathbf{U}) = \langle\ |\theta|^2\rangle + a\langle|\mathbf{u}|^2 + (g_3P_m + g_2)\ \mathbf{u}\cdot\mathbf{h} + P_m|\mathbf{h}|^2\rangle$$

must read, equivalently,

$$(\mathbf{U}_1,\mathbf{U}) = \langle\ |\theta|^2\rangle + d_1\langle\ |\phi_1|^2\rangle + d_2\langle\ |\phi_2|^2\rangle,$$

where $d_1, d_2 \in \mathbb{R}^+$, implying

$$d_1a_1^2 + d_2b_1^2 = a, \quad d_1a_1a_2 + d_2b_1b_2 = ag_2, \quad d_1a_2^2 + d_2b_2^2 = aP_m, \tag{3.1}$$

where d_1, d_2, b_1, b_2 are determined up to some factor. Eliminating d_1 and d_2 between these equalities, we obtain the relationship between b_1 and b_2

$$a_2b_2 + P_ma_1b_1 - g_2(a_2b_1 + a_1b_2) = 0, \quad a_2^2b_1^2 - a_1^2b_2^2 \neq 0, \tag{3.2}$$

defining ϕ_2 up to a factor.

Let us find a_1 and a_2 such that (2.8) has the simple form

$$\frac{1}{2}\cdot d(\mathbf{U}_1,\mathbf{U})/dt = P_r^{-1}[-\langle|\mathbf{grad}\ \theta|^2 + |\mathbf{grad}\ \phi_1|^2\rangle + P_rRk'\langle\theta\phi_1\cdot\mathbf{k}\rangle], \tag{3.3}$$

where k' is an undetermined factor. By identifying (2.8) and (3.3), it follows

$$\begin{aligned}-P_ra\langle|\ \mathbf{grad}\ \mathbf{u}|^2\rangle - P_ra\langle|\ \mathbf{grad}\ \mathbf{h}|^2\rangle -\\ -P_ra(g_3+g_2)\langle\mathbf{grad}\ \mathbf{u}\cdot\mathbf{grad}\ \mathbf{h}\rangle = -\langle|\mathbf{grad}\ \phi_1|^2\rangle,\end{aligned}$$

$$P_rR(a+P_r^{-1})\langle\theta w\rangle + P_rRag_2\langle\theta h_3\rangle = P_rRk'\langle\theta\phi_1\cdot\mathbf{k}\rangle.$$

If $P_m > 1$, we obtain

$$a=(P_m+1)[P_r(P_m-1)]^{-1},\quad a_1=\pm a_2,$$
$$g_2=\pm 2P_m(P_m+1)^{-1},\quad g_3=P_m^{-1}g_2,$$

where the signs $+$ and $-$ correspond, and

$$a_1=\pm\sqrt{(P_m+1)(P_m-1)^{-1}},\quad k'=\pm 2P_m(P_r\sqrt{P_m^2-1})^{-1},$$

where the signs $+$ and $-$ correspond.

From $(3.1_{1,3})$ it follows that

$$d_1=(b_2^2-P_mb_1^2)/[P_r(b_2^2-b_1^2)],\quad d_2=a(P_m-1)/(b_2^2-b_1^2).$$

Then, for $a_1=a_2$, (3.2) implies $b_2/b_1=P_m$, while, for $a_1=-a_2$, (3.2) implies $b_2/b_1=-P_m$. In both these cases, we have

$$d_1=P_m/[P_r(P_m+1)],\quad d_2=aP_m^2/[b_2^2(P_m+1)].$$

Therefore all these four solutions a, b, g_2, g_3, a_1, a_2, b_2/b_1, k' are convenient. In the next Section, we show that they lead to the same stability criterion.

4 The stability criterion

Introduce the functions

$$E(t)=\langle\ |\theta|^2+d_1\ |\phi_1|^2\rangle/2,\quad \Psi(t)=d_2\langle\ |\phi_2|^2\rangle/2$$

and the notation

$$\xi^2=\min_{\theta,\phi_1}\frac{2\langle\ |\mathbf{grad}\ \theta|^2+|\mathbf{grad}\ \phi_1|^2\rangle}{\langle\ |\theta|^2+\ |\phi_1|^2\rangle}\ ,$$
$$\frac{1}{\sqrt{R_a^*}}=\max_{\theta,\phi_1}\frac{2\langle\theta\phi_1\cdot\mathbf{k}\rangle}{\langle\ |\mathbf{grad}\ \theta|^2+|\mathbf{grad}\ \phi_1|^2\rangle}\ . \tag{4.1}$$

Then, due to the fact that $\phi_1=0$ on ∂V_h, for $k'>0$, the energy relation (2.7) becomes successively

$$\begin{aligned}\frac{dE}{dt}+\frac{d\Psi}{dt}&=P_r^{-1}[-\langle|\mathbf{grad}\ \theta|^2+|\mathbf{grad}\ \phi_1|^2\rangle+P_rRk'\langle\theta\phi_1\cdot\mathbf{k}\rangle]=\\&=-P_r^{-1}\langle|\mathbf{grad}\ \theta|^2+|\mathbf{grad}\ \phi_1|^2\rangle\cdot[1-\frac{P_rRk'\langle\theta\phi_1\cdot\mathbf{k}\rangle}{\langle|\mathbf{grad}\ \theta|^2+|\mathbf{grad}\ \phi_1|^2\rangle}],\end{aligned} \tag{4.2}$$

implying

$$\frac{dE}{dt}+\frac{d\Psi}{dt}\le -P_r^{-1}\xi^2\frac{1}{max\{1,d_1\}}\cdot[1-P_rRk'\ \frac{1}{2\sqrt{R_a^*}}]E, \tag{4.3}$$

whence *the stability criterion*

Theorem 1. *Suppose that $P_m > 1$. If $R < \sqrt{P_m^2 - 1}\sqrt{R_a^*}/P_m$, then the basic state m_0 is nonlinearly stable.*

Let $k' < 0$ and remark that, from the definition of $\mathcal{N}_1$, it follows that if $(\theta, \mathbf{u}, \mathbf{h}) \in \mathcal{N}_1$ than $(\theta, \phi_1, \phi_2) \in \mathcal{N}_1$, $(-\theta, \phi_1, \phi_2) \in \mathcal{N}_1$.

Introduce the space

$$\widetilde{\mathcal{N}}_1 = \{(\theta, \phi_1) \in H^2(V)^4 \mid div\ \phi_1 = 0;\ \phi_1 = \mathbf{0}, \quad \theta = 0 \text{ on } \partial V_h\}.$$

Obviously, $\widetilde{\mathcal{N}}_1$ is imbedded in $\mathcal{N}_1$, i.e. $\widetilde{\mathcal{N}}_1 \subset \mathcal{N}_1$. In addition, if (θ, ϕ_1) runs over $\widetilde{\mathcal{N}}_1$, than $(-\theta, \phi_1)$ runs over $\widetilde{\mathcal{N}}_1$ too.

We have $k'\langle\theta\phi_1 \cdot \mathbf{k}\rangle = |k'|\langle-\theta\phi_1 \cdot \mathbf{k}\rangle$ and $|\mathbf{grad}\ \theta|^2 = |\mathbf{grad}\ (-\theta)|^2$. Therefore (4.1) holds also for θ replaced by $-\theta$. Consequently, (4.3) hold for k' replaced by $|k'|$. In this way, for the case $k' < 0$, Theorem 1 holds too.

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Cătălin Liviu Bichir
Research and Design Institute
for Shipbuilding-ICEPRONAV Galatzi
Department of Ship's Hydrodynamic
Galatzi, Romania

Adelina Georgescu
University of Piteşti
Department of Applied Mathematics
Piteşti, Romania

Lidia Palese
University Campus, Department of Mathematics
Bari, Italy