BULETINUL ACADEMIEI DE ŞTIINŢE
A REPUBLICII MOLDOVA. MATEMATICA
Number 1(47), 2005, Pages 3–10
ISSN 1024–7696

Automodel solution for dynamic problem of two-component media

Ion Naval

Abstract. In this paper behavior of two-phase elastic medium at the movement in it of some concentrate load with supersonic speed was examined. Was obtained automodel solution for space in two-component problem at symmetrical axis. Analysis of obtained analytic solution demonstrates that major tensions and displacements are situated in the domain of the longitudinal and transversal wave action. As consequence, energy at the load movement is consumed preponderant to compressing and removing in the domain of superposition of all waves.

Mathematics subject classification: 74H15.
Keywords and phrases: Dynamic problem, wave propagation, automodel solution, two-component medium.

Homogeneous elastic two-component medium is considered. Let this medium is not deformed and concentrated load with speed υ_0 parallel to z axes is moved forward in it. It is necessary to investigate wave movement, turned up in this medium, satisfying equations of spatial axisymmetrical movement of two-component elastic medium.

Behavior of this medium is described by the following equations of movement [1–4]:

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{\partial\sigma_{rz}}{\partial z}-\frac{\partial\pi_0}{\partial r}+\frac{1}{r}(\sigma_{rr}-\sigma_{\theta\theta})=\rho_{11}\frac{\partial^2U_1}{\partial t^2}+\rho_{12}\frac{\partial^2U_2}{\partial t^2}+b\left(\frac{\partial U_1}{\partial t}-\frac{\partial U_2}{\partial t}\right);$$

$$\frac{\partial\sigma_{zr}}{\partial r}+\frac{\partial\sigma_{zz}}{\partial z}-\frac{\partial\pi_0}{\partial z}+\frac{\sigma_{rz}}{r}=\rho_{11}\frac{\partial^2V_1}{\partial t^2}+\rho_{12}\frac{\partial^2V_2}{\partial t^2}+b\left(\frac{\partial V_1}{\partial t}-\frac{\partial V_2}{\partial t}\right);$$

$$\frac{\partial\pi_{rr}}{\partial r}+\frac{\partial\pi_{rz}}{\partial z}+\frac{\partial\pi_0}{\partial r}+\frac{1}{r}(\pi_{rr}-\pi_{\theta\theta})=\rho_{12}\frac{\partial^2U_1}{\partial t^2}+\rho_{22}\frac{\partial^2U_2}{\partial t^2}-b\left(\frac{\partial U_1}{\partial t}-\frac{\partial U_2}{\partial t}\right); \quad (1)$$

$$\frac{\partial\pi_{zr}}{\partial r}+\frac{\partial\pi_{zz}}{\partial z}+\frac{\partial\pi_0}{\partial z}+\frac{\pi_{rz}}{r}=\rho_{12}\frac{\partial^2V_1}{\partial t^2}+\rho_{22}\frac{\partial^2V_2}{\partial t^2}-b\left(\frac{\partial V_1}{\partial t}-\frac{\partial V_2}{\partial t}\right);$$

$$\pi_0=\rho_1/\rho\,\alpha_2\,(q_x+q_y)+\rho_2/\rho\alpha_2\,(\varepsilon_x+\varepsilon_y),$$

where U_1, U_2, V_1, V_2 – vector components of solid phases displacement; σ_{rr}, σ_{rz}, σ_{zr}, σ_{zz}, $\sigma_{\theta\theta}$, π_{rr}, π_{rz}, π_{zr}, π_{zz}, $\pi_{\theta\theta}$ – tensor tension components; ε_{rr}, ε_{rz}, h_{zr}, ε_{zz}, $\varepsilon_{\theta\theta}$, q_{rr}, q_{rz}, h_{zr}, q_{zz}, $q_{\theta\theta}$ – deformation components; ρ_{11}, ρ_{22} – effective weights components at their relative movement; $\rho_{11}+\rho_{12}=\rho_1$, $\rho_{22}+\rho_{12}=\rho_2$, ρ_{12} – "connecting parameter" between components of a mixture having dimension of weight or complementary apparent weight in relation to component motion; ρ_1, ρ_2 – density of phases; b – diffusion coefficient.

In roz coordinate system, in conditions of flat deformed state, relation between component of tension and deformation become:

$$\sigma_{rr} = -\alpha_2 + (\lambda_1 + 2\mu_1)\,\varepsilon_{rr} + \lambda_1(\varepsilon_{zz} + \varepsilon_{\theta\theta}) + (\lambda_3 + 2\mu_3)\,q_{rr} + \lambda_3(q_{zz} + q_{\theta\theta});$$
$$\sigma_{\theta\theta} = -\alpha_2 + (\lambda_1 + 2\mu_1)\,\varepsilon_{\theta\theta} + \lambda_1(\varepsilon_{zz} + \varepsilon_{rr}) + (\lambda_3 + 2\mu_3)\,q_{\theta\theta} + \lambda_3(q_{zz} + q_{rr});$$
$$\sigma_{zz} = -\alpha_2 + (\lambda_1 + 2\mu_1)\,\varepsilon_{zz} + \lambda_1(\varepsilon_{rr} + \varepsilon_{\theta\theta}) + (\lambda_3 + 2\mu_3)\,q_{zz} + \lambda_3(q_{rr} + q_{\theta\theta});$$
$$\pi_{rr} = -\alpha_2 + (\lambda_2 + 2\mu_2)\,q_{rr} + \lambda_2(q_{zz} + q_{\theta\theta}) + (\lambda_4 + 2\mu_3)\,\varepsilon_{rr} + \lambda_4(\varepsilon_{zz} + \varepsilon_{\theta\theta});$$
$$\pi_{\theta\theta} = \alpha_2 + (\lambda_2 + 2\mu_2)\,q_{\theta\theta} + \lambda_2(q_{zz} + q_{rr}) + (\lambda_4 + 2\mu_3)\,\varepsilon_{\theta\theta} + \lambda_4(\varepsilon_{zz} + \varepsilon_{rr}); \qquad (2)$$
$$\pi_{zz} = \alpha_2 + (\lambda_2 + 2\mu_2)\,q_{zz} + \lambda_2(q_{rr} + q_{\theta\theta}) + (\lambda_4 + 2\mu_3)\,\varepsilon_{zz} + \lambda_4(\varepsilon_{rr} + \varepsilon_{\theta\theta});$$
$$\sigma_{rz} = 2\left(\mu_1\varepsilon_{rz} + \mu_3 q_{rz}\right) - \lambda_5\left(h_{rz} - h_{zr}\right);$$
$$\sigma_{zr} = 2\left(\mu_1\varepsilon_{rz} + \mu_3 q_{rz}\right) + \lambda_5\left(h_{rz} - h_{zr}\right);$$
$$\pi_{rz} = 2\left(\mu_2 q_{rz} + \mu_3\varepsilon_{rz}\right) + \lambda_5\left(h_{rz} - h_{zr}\right);$$
$$\pi_{zr} = 2\left(\mu_2 q_{rz} + \mu_3\varepsilon_{rz}\right) - \lambda_5\left(h_{rz} - h_{zr}\right),$$

here $\alpha_2 = \lambda_3 - \lambda_4$ – constant having dimension of tension; λ_j, μ_j, $(j = \overline{1, 5})$ – Lame coefficients;

Following ratio connects the components of deformations and displacement:

$$\varepsilon_{rr} = \partial U_1/\partial r, \quad \varepsilon_{\theta\theta} = U_1/r, \quad \varepsilon_{rz} = \partial U_1/\partial z + \partial V_1/\partial r, \quad \varepsilon_{zz} = \partial V_1/\partial z;$$
$$q_{rr} = \partial U_2/\partial r, \quad q_{\theta\theta} = U_2/r, \quad q_{rz} = \partial U_2/\partial z, +\partial V_2/\partial r, \quad q_{zz} = \partial V_2/\partial z; \qquad (3)$$
$$h_{rz} = \partial V_1/\partial r + \partial U_2/\partial z, \quad h_{zr} = \partial U_1/\partial z + \partial V_2/\partial r.$$

Equations of motion (1) after simple transformations in the base of formulas (2), (3) can be represented in such a way:

$$A_{11}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\left(\frac{U_1}{r}\right)\right) + A_{12}\frac{\partial^2 U_1}{\partial z^2} + (A_{11} - A_{12})\frac{\partial^2 V_1}{\partial r\partial z} + B_{11}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\left(\frac{U_2}{r}\right)\right) +$$
$$+B_{12}\frac{\partial^2 U_2}{\partial z^2} + (B_{11} - B_{12})\frac{\partial^2 V_2}{\partial r\partial z} = \rho_{11}\frac{\partial^2 U_1}{\partial t^2} + \rho_{12}\frac{\partial^2 U_2}{\partial t^2} + b\left(\frac{\partial U_1}{\partial t} - \frac{\partial U_2}{\partial t}\right);$$
$$A_{21}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\left(\frac{U_2}{r}\right)\right) + A_{22}\frac{\partial^2 U_2}{\partial z^2} + (A_{21} - A_{22})\frac{\partial^2 V_2}{\partial r\partial z} + B_{21}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\left(\frac{U_1}{r}\right)\right) +$$
$$+B_{22}\frac{\partial^2 U_1}{\partial z^2} + (B_{21} - B_{22})\frac{\partial^2 V_2}{\partial r\partial z} = \rho_{12}\frac{\partial^2 U_1}{\partial t^2} + \rho_{22}\frac{\partial^2 U_2}{\partial t^2} - b\left(\frac{\partial U_1}{\partial t} - \frac{\partial U_2}{\partial t}\right);$$
$$A_{11}\frac{\partial^2 V_1}{\partial z^2} + A_{12}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_1}{\partial r}\right) + (A_{11} - A_{12})\frac{\partial}{\partial z}\left(\frac{\partial U_1}{\partial r} + \frac{U_1}{r}\right) + B_{11}\frac{\partial^2 V_2}{\partial z^2} +$$

$$+B_{12}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_2}{\partial r}\right)+\left(B_{11}-B_{12}\right)\frac{\partial}{\partial z}\left(\frac{\partial U_2}{\partial r}+\frac{U_2}{r}\right)=$$

$$=\rho_{11}\frac{\partial^2 V_1}{\partial t^2}+\rho_{12}\frac{\partial^2 V_2}{\partial t^2}+b\left(\frac{\partial V_1}{\partial t}-\frac{\partial V_2}{\partial t}\right); \qquad (4)$$

$$A_{21}\frac{\partial^2 V_2}{\partial y^2}+A_{22}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_2}{\partial r}\right)+\left(A_{21}-A_{22}\right)\frac{\partial}{\partial z}\left(\frac{\partial U_2}{\partial r}+\frac{U_2}{r}\right)+B_{21}\frac{\partial^2 V_1}{\partial y^2}+$$

$$+B_{22}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_1}{\partial r}\right)+\left(B_{21}-B_{22}\right)\frac{\partial}{\partial z}\left(\frac{\partial U_1}{\partial r}+\frac{U_1}{r}\right)=\rho_{12}\frac{\partial^2 V_1}{\partial t^2}+\rho_{22}\frac{\partial^2 V_2}{\partial t^2}-$$

$$-b\left(\frac{\partial V_1}{\partial t}-\frac{\partial V_2}{\partial t}\right),$$

where $A_{j1}=\lambda_j+2\mu_j+(-1)^j\,\dfrac{\rho_{3-j}\,\alpha_2}{\rho}$; $A_{j2}=\mu_j-\lambda_5$; $B_{j1}=\lambda_{2+j}+2\mu_3+(-1)^j\,\dfrac{\rho_j\alpha_2}{\rho}$; $B_{j2}=\lambda_5+\mu_3$.

Entering potential functions Φ_j and Ψ_j as follows:

$$U_j=\frac{\partial\Phi_j}{\partial r}-\frac{\partial\Psi_j}{\partial z};\quad V_j=\frac{\partial\Phi_j}{\partial z}+\frac{\partial\Psi_j}{\partial r}+\frac{\Psi_j}{r}\quad (j=1,2)\,. \qquad (5)$$

equations (4) can be reduced to four wave equations by equating to zero diffusion coefficient ($b=0$.)

Really, if to put $\Phi_1=\varphi$, $\Phi_2=\beta\,\varphi$, $\Psi_1=\psi$, $\Psi_2=\gamma\psi$, where the parameters β, γ are determined from algebraic equations:

$$a_1^*\beta^2+b_1^*\beta+c_1^*=0; \qquad (6)$$

$$a_2^*\gamma^2+b_2^*\gamma+c_2^*=0; \qquad (7)$$

and $a_1^*=B_{11}\rho_{22}-A_{21}\rho_{12}$; $b_1^*=\rho_{12}\left(B_{11}-A_{21}\right)+\rho_{22}\left(A_{11}-B_{21}\right)$; $c_1^*=A_{11}\rho_{12}-A_{21}\rho_{11}$; $a_2^*=B_{12}\rho_{22}-A_{22}\rho_{12}$; $b_2^*=\rho_{12}\left(B_2-A_{22}\right)+\rho_{22}\left(A_{21}-B_{22}\right)$; $c_2^*=A_{21}\rho_{12}-A_{22}\rho_{11}$; $\dfrac{\rho_{11}+\rho_{12}\beta}{A_{11}+B_{11}\beta}=\dfrac{\rho_{12}+\rho_{22}\beta}{A_{21}\beta+B_{21}}$; $\dfrac{\rho_{11}+\rho_{12}\gamma}{A_{12}+B_{12}\gamma}=\dfrac{\rho_{12}+\rho_{22}\gamma}{A_{22}\gamma+B_{22}}$ as the equations (6), (7) have till two radicals, $\Phi_1=\varphi_1+\varphi_2$, $\Phi_2=\beta_1\varphi_1+\beta_2\varphi_2$, $\Psi_1=\psi_1+\psi_2$, $\Psi_2=\gamma_1\psi_1+\gamma_2\psi_2$, the system (4) is disintegrated, by virtue of its linearity, on following wave equations:

$$\frac{\partial^2\varphi_j}{\partial r^2}+\frac{1}{r}\frac{\partial\varphi_j}{\partial r}+\frac{\partial^2\varphi_j}{\partial z^2}=\frac{1}{a_j^2}\frac{\partial^2\varphi_j}{\partial t^2};$$

$$\frac{\partial^2\psi_j}{\partial r^2}+\frac{1}{r}\frac{\partial\psi_j}{\partial r}-\frac{\psi_j}{r^2}+\frac{\partial^2\psi_j}{\partial z^2}=\frac{1}{b_j^2}\frac{\partial^2\psi_j}{\partial t^2}, \qquad (8)$$

where

$$a_j^2=\frac{A_{11}+\beta_jB_{11}}{\rho_{11}+\beta_j\rho_{12}}=\frac{A_{21}+\beta_jA_{21}}{\rho_{12}+\beta_j\,\rho_{22}} \qquad (9)$$

$$b_j^2 = \frac{A_{12} + \gamma_j B_{12}}{\rho_{11} + \gamma_j \rho_{12}} = \frac{B_{22} + \gamma_j A_{22}}{\rho_{12} + \gamma_j \rho_{22}} \quad (j = 1, 2)\,. \tag{10}$$

Note through a_j – speed of longitudinal wave propagation, and through b_j – speed of transversal waves in two-component medium. By the virtue of hyperbolic type of the initial system and expressions (9), (10) for definition of speeds, the elastic constants $\lambda_k \mu_k$ must be subjected to additional restrictions, given by the following inequalities: $A_{11}B_{21} - A_{21}B_{11} \neq 0$; $\mu_1\mu_2 - \mu_3^2 \neq 0$; $p_1p_2 - p_3p_4 \neq 0$; $\vartheta \neq 1$, where $p_j = \lambda_j + \mu_j$; $\vartheta = \vartheta_1 - \vartheta_2$; $\vartheta_1 = \dfrac{\alpha_2\,(\rho_2 p_2 - \rho_1 p_4)}{\rho\,(p_1p_2 - p_3p_4)}$; $\vartheta_2 = \dfrac{\alpha_2\,(\rho_1 p_1 - \rho_2 p_3)}{\rho\,(p_1p_2 - p_3p_4)}$.

Passing to mobile coordinate system, connected to driving load (transformation of Galilee) $\overline{z} = z + v_0 t$, $\overline{r} = r$, — $\overline{t} = t$, and for convenience omitting dashes, the system of equation (8) can be reduced to the form:

$$\frac{\partial^2 \varphi_j}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi_j}{\partial r} = \delta_j^2 \frac{\partial^2 \varphi_j}{\partial t^2}; \quad \frac{\partial^2 \psi_j}{\partial r^2} + \frac{1}{r}\frac{\partial \psi_j}{\partial r} - \frac{\psi_j}{r^2} = \varepsilon_j^2 \frac{\partial^2 \psi_j}{\partial t^2}, \tag{11}$$

where $\delta_j^2 = \dfrac{v_0^2}{a_j^2} - 1$, $\varepsilon_j^2 = \dfrac{v_0^2}{b_j^2} - 1$.

The equation (11) describe axisymmetrical motion of elastic two-component medium relative to mobile coordinate system under impact of concentrated source. And it demonstrates, that in this medium the disturbance are diffused in form of two longitudinal waves and two transversal waves.

Let's search the automodel solutions of equation (11) in the form:

$$\varphi_j = z^2 f_j(\xi), \quad \psi_j = z^2 g_j(\xi), \tag{12}$$

where $\xi = r/z$ – dimensionless coordinate. In the base of (12) equations (11) are reduced to ordinary second-order differential equations concerning unknowns functions $f_j(\xi)$, and $g_j(\xi)$:

$$(1 - \delta_j^2\xi^2)\xi f_j'' + (1 + 2\delta_j^2\xi^2) f_j' - 2\delta_j^2 \xi f_j = 0; \tag{13}$$

$$(1 - \varepsilon_j^2\xi^2)\xi^2 g_j'' + (1 + 2\varepsilon_j^2\xi^2)\xi f_j' - (1 + 2\varepsilon_j^2\xi^2) f_j = 0 \quad (j = 1, 2)\,. \tag{14}$$

The particular solutions of equation (13) are $f_j = \delta_j^2\xi^2 + 2$ and the general solutions take a form:

$$f_j = (\delta_j^2\xi^2 + 2)\left\{ C_{1j} + C_{2j}\left[\frac{3}{4}\frac{\sqrt{1 - \delta_j^2\xi^2}}{\delta_j^2\xi^2 + 2} + \frac{1}{8}\ln\frac{1 - \sqrt{1 - \delta_j^2\xi^2}}{1 + \sqrt{1 - \delta_j^2\xi^2}}\right]\right\}. \tag{15}$$

On fronts of longitudinal waves ($\xi_{1j} = 1/\delta_j$) functions $f_j(\xi) = 0$, therefore constant of integration $C_{1j} = 0$.

The particular solutions of equation (14) are $g_j = \varepsilon_j^2 \xi$, and general solutions take form:

$$g_j = \varepsilon_j^2\xi^2 \left\{ C_{3j} + C_{4j}\left[-\sqrt{1 - \varepsilon_j^2\xi^2} - \frac{1}{2}\frac{\sqrt{1 - \varepsilon_j^2\xi^2}}{\varepsilon_j^2\xi^2} - \frac{3}{4}\ln\frac{1 - \sqrt{1 - \varepsilon_j^2\xi^2}}{1 + \sqrt{1 - \varepsilon_j^2\xi^2}}\right]\right\}. \tag{16}$$

The functions $g_j(\xi)$ are determined in areas, $0 \leq \xi \leq 1/\varepsilon_j$ and they are equal to zero on transversal wave fronts of $(\xi_{2j} = 1/\varepsilon_j)$, therefore, $C_{3j} = 0$.

Thus, the tensioned and cinematic state of medium is determined with consideration of superposition of the corresponding waves. In the field of volume dilatation searched functions are determined only through the functions $f_j(\xi)$, but in the area compression-displacement through the functions $f_j(\xi)$and $g_j(\xi)$, in dependence of the corresponding waves degree's superposition.

Under general solutions (15) and (16), according to equality (12), the potential functions φ_j and ψ_j, necessary for finding of displacement components U_j and V_j from (5) are determined; by means of the last, components of deformation from (3) are received, finally giving from (2) components of tension influencing on medium.

The components of displacement U_j and V_j in the field of volume expansion are determined by the following formulas (we shall suppose $C_{3j} = C_{4j} = C_j$):

$$\frac{U_i}{z} = \sum_{j=1}^{2} C_j \beta_j^{i-1} \left(\frac{\eta_j}{2\xi} + \frac{1}{4}\delta_j^2 \xi \ln \frac{1-\eta_j}{1+\eta_j} \right);$$

$$\frac{V_i}{z} = \sum_{j=1}^{2} C_j \beta_j^{i-1} \left(\eta_j + \frac{1}{2} \ln \frac{1-\eta_j}{1+\eta_j} \right) \quad (i = 1, 2), \tag{17}$$

where $\eta_j = \sqrt{1 - \delta_j^2 \xi^2}$.

The components of tension in these areas look like:

$$\sigma_{rr} = -\alpha_2 + \sum_{j=1}^{2}\sum_{i=1}^{2} C_j \beta_j^{i-1} \left\{ \frac{1}{2} \left[(\lambda_{2i-1} + \mu_{2i-1})\, \delta_j^2 + \lambda_{2i-1} \right] \ln \frac{1-\eta_j}{1+\eta_j} - \mu_{2i-1} \frac{\eta_j}{\xi^2} \right\};$$

$$\sigma_{\theta\theta} = -\alpha_2 + \sum_{j=1}^{2}\sum_{i=1}^{2} C_j \beta_j^{i-1} \left\{ \frac{1}{2} \left[(\lambda_{2i-1} + \mu_{2i-1})\, \delta_j^2 + \lambda_{2i-1} \right] \ln \frac{1-\eta_j}{1+\eta_j} + \mu_{2i-1} \frac{\eta_j}{\xi^2} \right\};$$

$$\sigma_{zz} = -\alpha_2 + \sum_{j=1}^{2}\sum_{i=1}^{2} C_j \beta_j^{i-1} \left\{ \frac{1}{2} \left[\lambda_{2i-1} \delta_j^2 + (\lambda_{2i-1} + 2\mu_{2i-1}) \right] \ln \frac{1-\eta_j}{1+\eta_j} \right\};$$

$$\sigma_{rz} + \sigma_{zr} = 8 \sum_{j=1}^{2}\sum_{i=1}^{2} C_j \beta_j^{i-1} \mu_{2i-1} \frac{\eta_j}{\xi};$$

$$\sigma_{rz} - \sigma_{zr} = \frac{\lambda_5}{2} \sum_{j=1}^{2} C_j (-1)^{j-1} \beta_j^{j-1} \delta_j^2 \xi \ln \frac{1-\eta_j}{1+\eta_j}; \tag{18}$$

$$\pi_{rr} = \alpha_2 + \sum_{j=1}^{2}\sum_{i=1}^{2} C_{3-j} \beta_j^{i-1} \left\{ \frac{1}{2} \left[(\lambda_{2i} + \mu_{2i})\, \delta_{3-i}^2 + \lambda_{2i} \right] \ln \frac{1-\eta_{3-j}}{1+\eta_{3-j}} - \mu_{2i} \frac{\eta_{3-j}}{\xi^2} \right\};$$

$$\pi_{\theta\theta} = \alpha_2 + \sum_{j=1}^{2}\sum_{i=1}^{2} C_{3-j} \beta_j^{i-1} \left\{ \frac{1}{2} \left[(\lambda_{2i} + \mu_{2i})\, \delta_{3-j}^2 + \lambda_{2i} \right] \ln \frac{1-\eta_{3-j}}{1+\eta_{3-j}} + \mu_{2i} \frac{\eta_{3-j}}{\xi^2} \right\};$$

$$\pi_{zz} = \alpha_2 + \sum_{j=1}^{2}\sum_{i=1}^{2} C_{3-j}\beta_j^{i-1}\left\{\frac{1}{2}\left[\lambda_{2i}\delta_{3-j}^2 + (\lambda_{2i} + 2\mu_{2i})\right]\ln\frac{1-\eta_{3-j}}{1+\eta_{3-j}}\right\};$$

$$\pi_{rz} + \pi_{zr} = 8\sum_{j=1}^{2}\sum_{i=1}^{2} C_{3-j}\beta_j^{i-1}\mu_{2i}\frac{\eta_{3-j}}{\xi};$$

$$\pi_{zr} - \pi_{rz} = \frac{\lambda_5}{2}\sum_{j=1}^{2} C_j(-1)^{j-1}\beta_j^{j-1}\delta_j^2\xi\ln\frac{1-\eta_j}{1+\eta_j}.$$

In remaining areas, where medium is affected by both longitudinal and transversal waves, components of displacement and tension expressed through $f_j(\xi)$ and $g_j(\xi)$, receive such a form:

$$\frac{U_i}{z} = \sum_{j=1}^{2} C_j\left\{\beta_j^{i-1}\left[\frac{\eta_j}{2\xi} + \frac{\delta_j^2\xi}{4}\ln\frac{1-\eta_j}{1+\eta_j}\right] + \frac{3\gamma_j^{i-1}}{2}\left[\zeta_j + \frac{\varepsilon_j\xi}{2}\ln\frac{1-\zeta_j}{1+\zeta_j}\right]\right\};$$

$$\frac{V_i}{z} = \sum_{j=1}^{2} C_j\left\{\beta_j^{i-1}\left[\eta_j + \frac{1}{2}\ln\frac{1-\eta_j}{1+\eta_j}\right] - 3\gamma_j^{i-1}\left[\varepsilon_j\zeta_j + \frac{\varepsilon_j}{2}\ln\frac{1-\zeta_j}{1+\zeta_j}\right]\right\};$$

$$\sigma_{rr} = -\alpha_2 + \sum_{j=1}^{2}\sum_{i=1}^{2} C_j\left\{\beta_j^{i-1}\left[\frac{1}{2}\left[(\lambda_{2i-1} + \mu_{2i-1})\,\delta_j^2 + \lambda_{2i-1}\right]\ln\frac{1-\eta_j}{1+\eta_j} - \mu_{2i-1}\frac{\eta_j}{\xi^2}\right] - \right.$$
$$\left. -3\gamma_j^{i-1}\left[\frac{\zeta_j}{\varepsilon_j^2\xi^2} - \frac{\varepsilon_j}{2}\ln\frac{1-\zeta_j}{1+\zeta_j}\right]\right\};$$

$$\sigma_{\theta\theta} = -\alpha_2 + \sum_{j=1}^{2}\sum_{i=1}^{2} C_j\left\{\beta_j^{i-1}\left[\frac{1}{2}\left[(\lambda_{2i-1} + \mu_{2i-1})\,\delta_j^2 + \lambda_{2i-1}\right]\ln\frac{1-\eta_j}{1+\eta_j} + \mu_{2i-1}\frac{\eta_j}{\xi^2}\right] - \right.$$
$$\left. -3\gamma_j^{i-1}\left[\frac{\zeta_j}{\varepsilon_j^2\xi^2} - \frac{\varepsilon_j}{2}\ln\frac{1-\zeta_j}{1+\zeta_j}\right]\right\};$$

$$\sigma_{zz} = -\alpha_2 + \sum_{j=1}^{2}\sum_{i=1}^{2} C_j\left\{\beta_j^{i-1}\left[\frac{1}{2}\left[\lambda_{2i-1}\delta_j^2 + (\lambda_{2i-1} + 2\mu_{2i-1})\right]\ln\frac{1-\eta_j}{1+\eta_j}\right] + \right.$$
$$\left. + \frac{3\gamma_j^{i-1}}{2}\left[\lambda_{2i-1}\frac{1+\varepsilon_j\xi-\varepsilon_j^2\xi^2}{\xi\zeta_j} - 2\mu_{2i-1}\varepsilon_j\ln\frac{1-\zeta_j}{1+\zeta_j}\right]\right\}; \tag{19}$$

$$\sigma_{rz} + \sigma_{zr} = 8\sum_{j=1}^{2}\sum_{i=1}^{2} C_j\mu_{2i-1}\left\{\beta_j^{i-1}\left[\frac{2\eta_j}{\xi} + \frac{3\left(1-\delta_j^2\right)\eta_j}{\delta_j\xi}\right] + \right.$$
$$\left. +3\gamma_j^{i-1}\left[\frac{\varepsilon_j\zeta_j}{\xi} + \frac{\left(1-\varepsilon_j\xi\right)}{2\zeta_j}\right]\right\};$$

$$\sigma_{rz}-\sigma_{zr}=\frac{\lambda_5}{2}\sum_{j=1}^{2}C_j(-1)^{j+1}\left\{\beta_j^{j-1}\delta_j^2\xi\ln\frac{1-\eta_j}{1+\eta_j}+3\gamma_j^{j-1}\left[\frac{\varepsilon_j\zeta_j}{\xi}+\frac{\left(1-\varepsilon_j\xi\right)}{2\zeta_j}\right]\right\};$$

$$\pi_{rr}=\alpha_2+\sum_{j=1}^{2}\sum_{i=1}^{2}C_{3-j}\left\{\beta_j^{i-1}\left[\frac{1}{2}\left[\left(\lambda_{2i}+\mu_{2i}\right)\delta_{3-j}^2+\lambda_{2i}\right]\ln\frac{1-\eta_{3-j}}{1+\eta_{3-j}}-\mu_{2i}\frac{\eta_{3-j}}{\xi^2}\right]-\right.$$

$$\left.-3\gamma_j^{i-1}\left[\frac{\zeta_{3-j}}{\varepsilon_{3-j}^2\xi^2}-\frac{\varepsilon_{3-j}}{2}\ln\frac{1-\zeta_{3-j}}{1+\zeta_{3-j}}\right]\right\};$$

$$\pi_{\theta\theta}=\alpha_2+\sum_{j=1}^{2}\sum_{i=1}^{2}C_{3-j}\left\{\beta_j^{i-1}\left[\frac{1}{2}\left[\left(\lambda_{2i}+\mu_{2i}\right)\delta_{3-j}^2+\lambda_{2i}\right]\ln\frac{1-\eta_{3-j}}{1+\eta_{3-j}}+\mu_{2i}\frac{\eta_{3-j}}{\xi^2}\right]-\right.$$

$$\left.-3\gamma_j^{i-1}\left[\frac{\zeta_{3-j}}{\varepsilon_{3-j}^2\xi^2}-\frac{\varepsilon_{3-j}}{2}\ln\frac{1-\zeta_{3-j}}{1+\zeta_{3-j}}\right]\right\};$$

$$\pi_{zz}=\alpha_2+\sum_{j=1}^{2}\sum_{i=1}^{2}C_{3-j}\left\{\beta_j^{i-1}\left[\frac{1}{2}\left[\lambda_{2i}\delta_j^2+\left(\lambda_{2i}+2\mu_{2i}\right)\right]\ln\frac{1-\eta_{3-j}}{1+\eta_{3-j}}\right]+\right.$$

$$\left.+\frac{3\gamma_j^{i-1}}{2}\left[\lambda_{2i}\frac{1+\varepsilon_{3-j}\xi-\varepsilon_{3-j}^2\xi^2}{\xi\zeta_{3-j}}-2\mu_{2i}\varepsilon_{3-j}\ln\frac{1-\zeta_{3-j}}{1+\zeta_{3-j}}\right]\right\};$$

$$\pi_{rz}+\pi_{zr}=8\sum_{j=1}^{2}\sum_{i=1}^{2}C_{3-j}\mu_{2i}\left\{\beta_j^{j-1}\frac{2\eta_{3-j}}{\xi}+\gamma_j^{j-1}\frac{3\left(1-\varepsilon_{3-j}^2\right)\zeta_{3-j}}{\varepsilon_{3-j}\xi}\right\};$$

$$\pi_{zr}-\pi_{rz}=\frac{\lambda_5}{2}\sum_{j=1}^{2}C_j(-1)^{j-1}\left\{\beta_j^{j-1}\delta_j^2\xi\ln\frac{1-\eta_j}{1+\eta_j}+3\gamma_j^{j-1}\left[\frac{\varepsilon_j\zeta_j}{\xi}+\frac{\left(1-\varepsilon_j\xi\right)}{2\zeta_j}\right]\right\},$$

here $\zeta_j=\sqrt{1-\varepsilon_j^2\xi^2}$.

From the general solution is evident, that in origin ($\xi=0$) components of displacement and tension tend to infinite and arbitrary constants, which must be determined from boundary conditions, remain unknown. Such automodel solutions can correspond to processes of burning and detonation, in which allocated energy grows on time.

The analysis of obtained above analytical solutions demonstrates, that the main predominant displacement and tension are watched in areas, where medium affected by longitudinal and transversal waves simultaneously, so the main part of concentrated load motion goes to compression and in the field of their superposition.

References

[1] Philippov I.G. *The dynamic theory of relative current of multicomponent mediums.* Applied mechanics, 1971, **7**, N 10, p. 92–99. (in Russian).

[2] Philippov I.G., Ceban V.G. *Unsteady motions of solid compressible mediums.* Kishinev, Stiintsa, 1973 (in Russian).

[3] Cosachevschy L.Ia. *About distribution(propagation) of elastic waves in two-component mediums.* Applied mathematics and mechanics, 1959, **23**, N 6, p. 1115–1123 (in Russian).

[4] Philippov I.G. *Influencing of multicomponents medium on seismic waves propagation.* Proceedings of the third European symposium on earthquake engineering. Sofia, 1970, p. 387–392 (in Russian).

Institute of Mathematics and Computer Science
Academy of Sciences of Moldova
5 Academiei str.
Chişinău, MD-2028
Moldova
E-mail: *inaval@math.md*

Received February 16, 2005