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Stability and fold bifurcation in a system of two coupled demand-supply models

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Abstract. A model of two coupled demand-supply systems, depending on 4 parameters is considered. We found that the dynamical system associated with this model may have at most two symmetric and at most two nonsymmetric equilibria as the parameters vary.

The topological type of equilibria is established and the locus in the parameter space of the values corresponding to nonhyperbolic equilibria is determined.

We found that the nonhyperbolic singularities can be of fold, Hopf, double-zero (Bogdanov-Takens) or fold-Hopf type.

In addition, the fold bifurcation is studied using the normal form method and the center manifold theory.

Mathematics subject classification: 37G10, 37L10.
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1 The mathematical model

The demand-supply model describes the way in which the price p and the quantity q reacts one to another. This model was proposed by Beckmann and Ryder [1] (1969) and Collel (1986). It is based on the economic principles of Walras and Marshall [2]. According to their hypothesis, the variation of the price is function of the difference between the demanded quantity of the product $D(p)$ and the offered quantity $S(p)$ at the price p, while the variation of the quantity is function of the difference between the price $p_d(q)$ demanded for the quantity q and the price $p_s(q)$ offered for this quantity. In addition, these two functions keep constant the sign of their argument. Thus, the mathematical model has the form [4]:

$$\begin{cases} \dot{p} = f(D(p) - S(p)), \\ \dot{q} = g(p_d(q) - p_s(q)). \end{cases} \tag{1}$$

with $f(0) = g(0) = 0$, $f^{/}(0) > 0$, $g^{/}(0) > 0$.

If $f(x) = x, g(x) = x$, $S(p) = q$, $p_d(q) = p$, $D(p) = ap + \beta$, $p_s(q) = cq^2 + \delta$, system (1) becomes:

$$\begin{cases} \dot{p} = ap + \beta - q, \\ \dot{q} = p - cq^2 - \delta. \end{cases} \tag{2}$$

In economy, the laws of demand and offer are available. According to them [3], as the price of the product increases, the demanded quantity decreases, so the function $D(p)$

is decreasing and we must have $a<0$. Similarly, the function $p_s(q)$ is increasing, so we have $c>0$.

The economic interest is to reach an equilibrium between the price and the quantity.

With the transformation $u=p-\delta$ and denoting $b=a\delta+\beta$, system (2) is written as:

$$\begin{cases} \dot{u}=au-q+b, \\ \dot{q}=u-cq^2. \end{cases} \tag{3}$$

A study of dynamics and bifurcation of this system is developed in [5]. The coordinates of equilibria of system (3) satisfy

$$\begin{cases} au-q+b=0, \\ u-cq^2=0. \end{cases}$$

Denote $\Delta=1-4abc$. Since $ac\neq 0$ there are two equilibria $\left(c\alpha^2,\alpha\right)$, with $\alpha=\frac{1\pm\sqrt{\Delta}}{2ac}$, as $\Delta>0$, a single equilibrium $\left(\frac{1}{4a^2c},\frac{1}{2ac}\right)$ as $\Delta=0$ and no equilibria as $\Delta<0$. The equilibrium $\left(\frac{1}{4a^2c},\frac{1}{2ac}\right)$ is always nonhyperbolic, namely of saddle-node type as $a\neq -1$ and of double zero type as $a=-1$. The equilibrium $\left(c\alpha^2,\alpha\right)$, with $\alpha=\frac{1-\sqrt{\Delta}}{2ac}$, is nonhyperbolic of Hopf type iff $\sqrt{\Delta}=1-a^2$, $a\in(-1,0)$. Otherwise, it is a repulsor as $a^2-1+\sqrt{\Delta}>0$ and an attractor as $a^2-1+\sqrt{\Delta}<0$. In [5] it is shown that crossing the parameter stratum $\sqrt{\Delta}=1-a^2$, $a\in(-1,0)$, a subcritical Hopf bifurcation takes place. Finally, the equilibrium $\left(c\alpha^2,\alpha\right)$, with $\alpha=\frac{1+\sqrt{\Delta}}{2ac}$, is always hyperbolic of saddle type.

In our study, a model of two identical demand-supply dynamical systems (3), symmetrically coupled via the quantity flow is considered. It reads

$$\begin{cases} \dot{x}_1=ax_1-x_2+b, \\ \dot{x}_2=x_1-cx_2^2+d\left(x_2-x_4\right), \\ \dot{x}_3=ax_3-x_4+b, \\ \dot{x}_4=x_3-cx_4^2+d\left(x_4-x_2\right). \end{cases} \tag{4}$$

This system models the interaction between two identical demand-supply models. Thus we shall focus on parameter values such that the system (4) display either a steady stable state or periodic behavior. Systems coupled in the form (5) are often used in the literature. As a result of the couplage, some characteristics of the behavior around the equilibria are preserved, but new kind of dynamics arise [6–9].

A consequence of this form of coupling and of the assumption that the models are identical is the invariance of (4) under the transformation $(x_1,x_2,x_3,x_4)\to(x_3,x_4,x_1,x_2)$. The same symmetry leads to the existence of an invariant subspace

$$I=\left\{(x_1,x_2,x_3,x_4)\in\mathbf{R}^4, x_1=x_3, x_2=x_4\right\}.$$

A solution of (4) lying in I will be referred to as symmetric solution, while one which does not lie in I as nonsymmetric solution. By economic reasons, we shall investigate only the case $a < 0$, $c > 0$. We also assume $d > 0$. Thus we consider the set of parameters of interest from application point of view as

$$D = \left\{(a, b, c, d) \in \mathbf{R}^4, a < 0, c > 0, d > 0\right\}.$$

2 Equilibria and nonhyperbolic singularities

System (4) possesses at most two symmetric equilibria of the form

$$e_s = \left(c\alpha^2, \alpha, c\alpha^2, \alpha\right), \tag{5}$$

where $\alpha \in \mathbf{R}$ satisfies the equation

$$ac\alpha^2 - \alpha + b = 0, \tag{6}$$

whose discriminant is Δ already introduced. As $ac \neq 0$, for $\Delta = 0$, there exists a unique equilibrium e_{0s}, with $\alpha = \frac{1}{2ac}$; and for $\Delta > 0$, system (4) has two symmetric equilibria e_{1s}, e_{2s}, corresponding to $\alpha_1 = \frac{1+\sqrt{\Delta}}{2ac}$ and $\alpha_2 = \frac{1-\sqrt{\Delta}}{2ac}$, respectively; while for $\Delta < 0$ there are no symmetric equilibria.

As $ac \neq 0$, system (4) may also possess at most two nonsymmetric equilibrium points, of the form

$$e_a = \left(\frac{\alpha' - b}{a}, \alpha', \frac{1 + 2ad}{a^2 c} - \frac{\alpha' + b}{a}, \frac{1 + 2ad}{ac} - \alpha'\right), \tag{7}$$

where α' satisfies the equation

$$c\alpha^2 - \frac{1 + 2ad}{a}\alpha + \frac{d + 2ad^2 + bc}{ac} = 0. \tag{8}$$

Denote by $\Delta' = 1 - 4abc - 4a^2d^2$ the discriminant of (8). Note that if $\Delta' = 0$, we have $\alpha' = \frac{1+2ad}{2ac}$ and the corresponding equilibrium e_a coincides with e_{2s}. Thus we obtain the following result:

Lemma 1. *Assume $a < 0$, $c > 0$.*

(i) *If $\Delta < 0$, system (4) has no equilibria;*

(ii) *if $\Delta = 0$, system (4) has a unique equilibrium point e_{0s}, given by (5) with $\alpha = \frac{1}{2ac}$;*

(iii) *if $\Delta > 0$ and $\Delta' \leq 0$ system (4) has two equilibria e_{1s}, e_{2s};*

(iv) *if $\Delta' > 0$, system (4) has four equilibrium points e_{1s}, e_{2s}, e_{1a}, e_{2a}.*

As a consequence, the static bifurcation diagram of the dynamical system (4) in D is the set

$$S = \left\{(a, b, c, d) \in D, (1 - 4abc)\left(1 - 4abc - 4a^2d^2\right) = 0\right\}.$$

Sections in the static bifurcation set S with a plane $b = b_0$, $c = c_0$, are plotted in Fig. 1, for different values of b_0, c_0, and the number of equilibrium points corresponding to each stratum is shown.

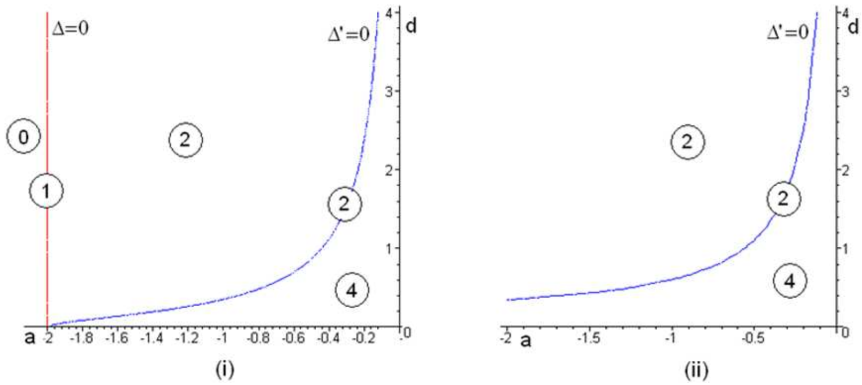

Figure 1. *Section with a plane $b = b_0$, $c = c_0$ in the static bifurcation diagram: i) $b = -0.5$, $c = 0.25$; ii) $b = 0.5$, $c = 0.25$. The number of equilibria corresponding to each stratum is shown*

3 The topological type of equilibria

In this section we determine the topological type of the four equilibrium points of system (4), analyzing the variation of the eigenvalues of the Jacobi matrix of the linearized system associated with (4) around each of the four equilibria.

Let $e = (e_1, e_2, e_3, e_4) \in \mathbf{R}^4$ be an equilibrium point of system (4). The Jacobi matrix of (4) around e reads

$$J(e) = \begin{pmatrix} a & -1 & 0 & 0 \\ 1 & d - 2ce_2 & 0 & -d \\ 0 & 0 & a & -1 \\ 0 & -d & 1 & d - 2ce_4 \end{pmatrix}.$$

Denote by T^s, T^u, T^c the stable, unstable and critical eigenspaces of $J(e)$, respectively, and by s, u, c the dimension of these subspaces of $\mathbf{R}^4$.

As the characteristic equation for the equilibrium e_{0s} is

$$\lambda\left(\lambda - \frac{a^2-1}{a}\right)\left[\lambda^2 - \lambda\left(a + 2d - \frac{1}{a}\right) + 2ad\right] = 0,$$

we obtain the following result:

Lemma 2. *If $\Delta = 0$, for parameters in D, the unique equilibrium point of system (4) is nonhyperbolic, with one zero eigenvalue as $a \neq -1$ or two zero eigenvalues as $a = -1$.*

If $\Delta > 0$, the characteristic equation for the symmetric equilibria e_{1s}, e_{2s} reads [11]:

$$\left[\lambda^2 - \left(a - \frac{1 \pm \sqrt{\Delta}}{a}\right)\lambda \mp \sqrt{\Delta}\right]\left[\lambda^2 - \left(a + 2d - \frac{1 \pm \sqrt{\Delta}}{a}\right)\lambda + 2ad \mp \sqrt{\Delta}\right] = 0. \tag{9}$$

Denote by λ_1, λ_2 the roots of the first bracket in (9) and by λ_3, λ_4 the roots of the second one.

As for e_{1s} we have $\lambda_1\lambda_2 = -\sqrt{\Delta} < 0$, $\lambda_3\lambda_4 = 2ad - \sqrt{\Delta} < 0$, we may conclude:

Lemma 3. *If $\Delta > 0$, for parameters in D, the symmetric equilibrium e_{1s} of system (4) is hyperbolic, namely it is a saddle of type $(s,u) = (2,2)$.*

In order to establish the topological type of e_{2s}, let us introduce the following notations:

$$\begin{aligned}
SN_1 &= \{(a,b,c,d) \in D, \quad \Delta = 0\},\\
SN_2 &= \left\{(a,b,c,d) \in D, \quad \Delta > 0, \quad 2ad + \sqrt{\Delta} = 0\right\},\\
H_1 &= \left\{(a,b,c,d) \in D, \quad \Delta > 0, \quad a^2 - 1 + \sqrt{\Delta} = 0\right\},\\
H_2 &= \left\{(a,b,c,d) \in D, \quad \Delta > 0, 2ad + \sqrt{\Delta} \geq 0, \quad a^2 - 1 + 2ad + \sqrt{\Delta} = 0\right\}.
\end{aligned}$$

Lemma 4. *For $\Delta > 0$ and $(a,b,c,d) \in D - (SN_2 \cup H_1 \cup H_2)$ the symmetric equilibrium e_{2s} of system (4) is hyperbolic, namely:*

(i) *if $2ad + \sqrt{\Delta} < 0$ and $a^2 - 1 + \sqrt{\Delta} < 0$, then e_{2s} is a saddle of type $(3,1)$;*

(ii) *if $2ad + \sqrt{\Delta} < 0$ and $a^2 - 1 + \sqrt{\Delta} > 0$, then e_{2s} is a saddle of type $(1,3)$;*

(iii) *if $2ad + \sqrt{\Delta} > 0$ and $a^2 - 1 + \sqrt{\Delta} > 0$, then e_{2s} is a repulsor;*

(iv) *if $2ad + \sqrt{\Delta} > 0$, $a^2 - 1 + \sqrt{\Delta} < 0$ and $a^2 - 1 + 2ad + \sqrt{\Delta} > 0$, then e_{2s} is a saddle of type $(2,2)$;*

(v) *if $2ad + \sqrt{\Delta} > 0$, $a^2 - 1 + \sqrt{\Delta} < 0$ and $a^2 - 1 + 2ad + \sqrt{\Delta} < 0$, then e_{2s} is an attractor.*

In addition, if $(a,b,c,d) \in SN_2 \cup H_1 \cup H_2$, then e_{2s} is a nonhyperbolic equilibrium, namely of Hopf type as $(a,b,c,d) \in (H_1 \cup H_2) - SN_2$, of fold type as $(a,b,c,d) \in SN_2 - (H_1 \cup H_2)$, of double zero type as $(a,b,c,d) \in SN_2 \cap H_2$ or of fold-Hopf type as $(a,b,c,d) \in SN_2 \cap H_1$.

In Fig. 2 is represented a section with a plane $b = b_0$, $c = c_0$ in the bifurcation diagram of system (4) around the equilibrium e_{2s}. Inside each region (s,u) is given.

As $\Delta' > 0$, for the nonsymmetric equilibria e_{1a}, e_{2a}, the corresponding characteristic equation is written as

$$\lambda^4 - \Delta_1\lambda^3 + \Delta_2\lambda^2 - \Delta_3\lambda + \Delta_4 = 0, \tag{10}$$

where:

$$\Delta_1 = 2\left(a - d - \frac{1}{a}\right); \quad \Delta_2 = \frac{1 + 2ad - \Delta'}{a^2} + a^2 - 4ad - 2;$$

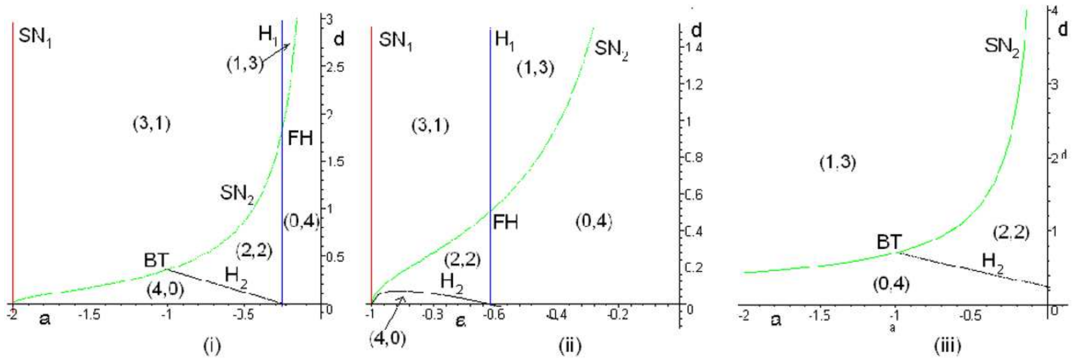

Figure 2. *Section with a plane* $b = b_0$, $c = c_0$ *in the local parameter portrait around* e_{2s}*: i)* $b = -0.5$, $c = 0.25$; *ii)* $b = -0.5$, $c = 0.5$; *iii)* $b = 0.5$, $c = 0.5$

$$\Delta_3 \;=\; 2\left[d - a^2 d - \frac{\Delta'}{a}\right]; \quad \Delta_4 = -\Delta'.$$

Since $\Delta_4 < 0$, it follows $\lambda_i \neq 0$, $i = \overline{1,4}$. Therefore, the equilibrium $e_{1,2a}$ may be nonhyperbolic only if (10) has a pair of purely imaginary solutions. This situation arises if the following conditions are fulfilled [8]

$$\Delta_1 \neq 0, \frac{\Delta_3}{\Delta_1} > 0, \frac{\Delta_3}{\Delta_1} + \Delta_4 \frac{\Delta_1}{\Delta_3} = \Delta_2 \tag{11}$$

or

$$\Delta_1 = 0, \Delta_3 = 0, \Delta_4 < 0. \tag{12}$$

Consequently, we obtained:

Lemma 5. *If* $\Delta' > 0$, *then the nonsymmetric equilibria* $e_{1,2a}$ *of system (4) are*

(i) *hyperbolic saddles, of type (1,3) or (3,1), as the conditions (11), (12) do not hold;*

(ii) *nonhyperbolic of Hopf type, as (11) or (12) holds.*

In Fig. 3 is represented a section with a plane $b = b_0$, $c = c_0$ in the bifurcation diagram of system (4) around the equilibria $e_{1,2a}$. The parameter strata for which (11) or (12) holds are denoted by H.

4 Fold bifurcation

Let $e = (e_1, e_2, e_3, e_4)$ be an equilibrium of system (4). Performing the translation $y = x - e$, system (4) reads

$$\dot{y} = J(e)\, y + F(y), \quad y \in \mathbf{R}^4, \tag{13}$$

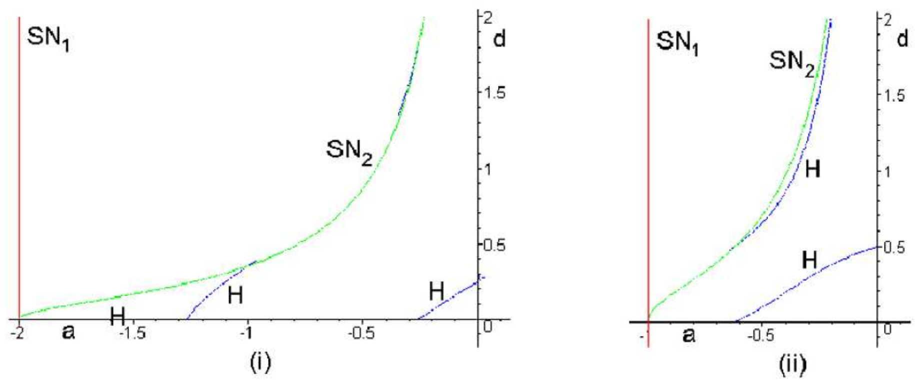

Figure 3. *Section with a plane $b = b_0,\ \ c = c_0$ in the local parameter portrait around $e_{1,2a}$: i) $b = -0.5,\ \ c = 0.25$; ii) $b = -0.5,\ \ c = 0.5$*

with $F(y) = \left(0, -cy_2^2, 0, -cy_4^2\right)^t$, and the corresponding equilibrium is the origin $0 \in \mathbf{R}^4$.

Using the normal form and the center manifold theory [10], we establish the topological type of the nonhyperbolic equilibria of saddle-node type determined in Section 3 and the local bifurcation generated by them.

Case 1. For parameters situated in the set SN_1 we have $\Delta = 0$ and the Jacobi matrix associated with the unique equilibrium point of (4) $e_{0s} = \left(\frac{1}{4a^2c}, \frac{1}{2ac}, \frac{1}{4a^2c}, \frac{1}{2ac}\right)$, has the eigenvalues $\lambda_1 = 0$, $\lambda_2 = a - \frac{1}{a}$, $\lambda_3\lambda_4 = 2ad < 0$. Assume $a^2 - 1 \neq 0$. Thus $J\left(e_{0s}\right)$ has a simple zero eigenvalue and the corresponding critical eigenspace is spanned by the eigenvector $q = (1, a, 1, a) \in \mathbf{R}^4$. Let $p = \frac{1}{2(1-a^2)}(1, -a, 1, -a) \in \mathbf{R}^4$ be the normalized adjoint vector, i.e. $J\left(e_{0s}\right)^t p = 0$ and $\langle p, q\rangle = 1$. We discompose any vector $y \in \mathbf{R}^4$ as $y = uq + z$, where $uq \in T^c$, $z \in T^{su}$. Here T^{su} is the 3-dimensional eigenspace of $J\left(e_{0s}\right)$ corresponding to all eigenvalues, other than 0. The explicit expressions for u and z are:

$$\left\{\begin{array}{l} u = \langle p, y\rangle, \\ z = y - \langle p, y\rangle\, q. \end{array}\right. \tag{14}$$

The scalar u and the vector z can be considered as new coordinates on $\mathbf{R}^4$. By the Fredholm alternative [10], the components of z always satisfy the orthogonality condition $\langle p, z\rangle = 0$.

In these new coordinates, system (13) with $e = e_{0s}$ can be written as [10]

$$\left\{\begin{array}{l} \dot{u} = \langle p, F\left(uq + z\right)\rangle, \\ z = J\left(e_{0s}\right) z + F\left(uq + z\right) - \langle p, F\left(uq + z\right)\rangle\, q, \end{array}\right. \tag{15}$$

that is

$$\left\{\begin{array}{l} \dot{u}=\frac{a^3c}{1-a^2}u^3+\frac{a^2c}{1-a^2}u\left(z_2+z_4\right)+\frac{ac}{1-a^2}\left(z_2^2+z_4^2\right), \\ z=J\left(e_{0s}\right)z+\left(\begin{array}{c} 0 \\ -c\left(au+z_2\right)^2 \\ 0 \\ -c\left(au+z_4\right)^2 \end{array}\right)-\frac{ac}{2(1-a^2)}\left(2a^2u^2+2au\left(z_2+z_4\right)+\left(z_2^2+z_4^2\right)\right)q. \end{array}\right. \tag{16}$$

The center manifold has the representation

$$z=V\left(u\right)=\frac{1}{2}w_2u^2+O\left(u^3\right), \tag{17}$$

where $w_2 \in T^{su}$, that is $\langle p, w_2\rangle = 0$. The vector w_2 also satisfies the equation $J\left(e_{0s}\right)w_2 + A = 0$, where $A = -\frac{2a^2c}{1-a^2}\left(a,1,a,1\right) \in \mathbf{R}^4$. From the above conditions we obtain

$$w_2=-\frac{a^3c}{\left(1-a^2\right)^2}\left(a,2-a^2,a,2-a^2\right).$$

Substituting in (16) and (17) the expression of w_2 we obtain:

Proposition 1. *The restriction of (16) to the center manifold has the form*

$$\dot{u}=\frac{a^3c}{1-a^2}u^2+O\left(u^3\right).$$

In addition, since $\frac{a^3c}{1-a^2}\neq 0$, the equilibrium e_{0s} is a nondegenerated saddle-node and around it a nondegenerated fold bifurcation takes place.

Returning to the y coordinates, we get the following result.

Proposition 2. *For $\Delta = 0$, $a^2-1\neq 0$, the center manifold corresponding to e_{0s} can be written as*

$$y_1=y_3,\quad y_2=y_4,\quad y_1-\frac{1}{1-a^2}\left(y_1-ay_2\right)+\frac{a^4c}{2\left(1-a^2\right)^3}\left(y_1-ay_2\right)^2=0. \tag{18}$$

Case 2. For parameters situated in the set SN_2 we have $\Delta > 0$ and $2ad+\sqrt{\Delta}=0$. The Jacobi matrix $J\left(e_{2s}\right)$ of the equilibrium point $e_{2s}=\left(c\alpha^2,\alpha,c\alpha^2,\alpha\right)$ of (4), with $\alpha=\frac{1+2ad}{2ac}$, has the eigenvalues $\lambda_3=0$, $\lambda_4=a-\frac{1}{a}$, $\lambda_1\lambda_2=-2ad>0$, $\lambda_1+\lambda_2=a-\frac{1+2ad}{a}$.

Consider that $a^2-1\neq 0$ and $a^2-1-2ad\neq 0$. This means that $\lambda_4\neq 0$, and the parameters are not situated in H_1 or H_2. Thus $J\left(e_{2s}\right)$ has a simple zero eigenvalue and the corresponding critical eigenspace T^c is spanned by the eigenvector $q=\left(1,a,-1,-a\right)\in\mathbf{R}^4$. Let $p=\frac{1}{2(1-a^2)}\left(1,-a,-1,a\right)\in\mathbf{R}^4$ be the normalized adjoint vector.

Performing the change (14), system (13) with $e = e_{2s}$ reads

$$\begin{cases} \dot{u} = \frac{a^2 c}{1-a^2} u\left(z_2 + z_4\right) + \frac{ac}{2(1-a^2)}\left(z_2^2 - z_4^2\right), \\ z = J\left(e_{2s}\right) z + \begin{pmatrix} 0 \\ -c\left(au + z_2\right)^2 \\ 0 \\ -c\left(au + z_4\right)^2 \end{pmatrix} - \frac{ac}{2(1-a^2)}\left(2au\left(z_2 + z_4\right) + z_2^2 - z_4^2\right) q. \end{cases} \tag{19}$$

As for the previous case, we obtain the following result.

Proposition 3. *If $\Delta > 0$, $a^2 - 1 \neq 0$, $a^2 - 1 - 2ad \neq 0$, the center manifold corresponding to e_{2s} can be written as*

$$y_2 = ay_1, \quad y_4 = ay_2, \quad y_1 + y_3 + \frac{ac}{4d}\left(y_1 - y_3\right)^2 = 0. \tag{20}$$

Taking into account (20), from (19) we obtain.

Proposition 4. *The restriction of (19) to the center manifold (20) is*

$$\dot{u} = \frac{a^4 c^2}{d\left(1 - a^2\right)} u^3. \tag{21}$$

In addition, since in D we have $\frac{a^4 c^2}{d(1-a^2)} \neq 0$, the equilibrium e_{2s} is a degenerated saddle-node of order two. On the center manifold a degenerated fold bifurcation takes place around e_{2s}.

Remark also that as $a \in (-1, 0)$ the coefficient of u^3 is positive, therefore the solution $u = 0$ of (21) is weakly repulsive and so is e_{2s} on the center manifold. Similarly, as $a < -1$, e_{2s} is weakly attractive on the center manifold.

The bifurcation corresponding to the other nonhyperbolic singularities, namely of Hopf, double-zero of fold-Hopf type, will be treated elsewhere.

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