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Exponential inflationary economic growth

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Abstract. Some scenario of economic growth centered on the structural reforms of the Republic of Moldova is presented. Mathematical model elaborated in [1] was adopted to proposed scenario in order to obtain indicators of exponential inflationary growth taking into account production possibilities. Economy description was presented by the principal economic sectors restrictions and production function depending of capital; the labor was not considered. The effectiveness of growth programs is estimated by parameters of growth and inflation in concordance with exponential inflationary growth. This solution is a particular one admissible by the model.

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The small open economy is considered. It is supposed that the economy produces one aggregate product which is utilized domestically and exported. Four economic sectors are examined. The state sectors which collects taxes; pays off salaries, pensions, allowances, and stipends, and effects some social programs. The production sector that owns all production factors and, as a consequence, earns all real income. Households that receive salaries and take part in goods exchange buying it in the good market. The monetary sector, represented by the National Bank, which intervenes in the foreign currency market selling and buying international exchange. And the external sector, which buys back its liabilities from domestic state and production sectors and earns international reserves from National Bank. It is supposed that the goods and monetary markets are in the equilibrium for all time the of model action. This time period of the model action is sufficiently long for the economic agents' accommodation to structural reforms, but insufficiently long for the some cardinal changes in production efficiency to be done.

The balanced exponential inflationary growth characterizes such equilibrium and it is determined by constant coefficients which define production technology, consumer preferences and circulation of goods, resources and money. Since the equilibrated growth is mentioned,the production and consumption grows (decreases) by constant rate. The price indexes proportions are maintained and also can be increased (or decreased) with a constant inflation (deflation) rate. Therefore a macroeconomic model which described main production proportion can be utilized. In such a model a balanced growth rate is determined through the constant technology parameters average for entire economy, behavior parameters and circulation mechanism parameters. The economic state in discrete moments of time on the fixed time interval $[0, T]$ is examined by the model. The time interval $[t, t+1]$ is considered about one year. The economics growth scenario for Republic of Moldova is proposed for further examination:

1. The government functions will be reduced to the redistribution of the limited budget sources in the favor of vulnerable parts of population and will ensure the equal and fair competition between local and foreign economic agents.
2. The creation of formal and justice conditions for equal and fair competition will contribute to increasing the investment flows in the production and to the rational distribution of resources.
3. The fiscal reform will favor economic agents to reserve oneself profit and income by economic active population.
4. The internal and foreign credits will be mobilized in order to ensure economic growth.
5. Budget deficit will be reduced to zero and from the budget surplus the external debt will be paid off.
6. The monetary system will be based on the international currency reserves and on the internal credits.

The exposed scenario reflects main programs' characteristics of the internal resources mobilization and involves external resources in order to maintain the economic growth. Model [1] adapted to this scenario will be used for the economic effectiveness evaluation. If the economic growth will be examined for medium period, then it will be necessary to evaluate constant parameters which characterize economic efficiency in concordance with statistic data reflecting current state of national economy. The prudence in launching assumptions about future tendencies will be necessary.

The labor can be excluded from the principal production factor examination because sufficient reserves of unemployment exist. One of the most restricted production factors is raw materials . The fixed means of production which determine the production capacity are not restrictive production factor. Anyway, the potential economic growth is evaluated so that the fixed funds are considered as marginal and production is worked utilizing all the production capacity. But introducing new production capacities necessitates some additional investment in the production sector. So in the model output does not depend on the labor force but expenditure for it paying off will be considered.

Production sector

The output in year t is:

$$Y_t = \sum_{\tau=t-T_\tau-T_\mu}^{t-T_\tau} I_\tau (1-\mu)^{\tau+T-t}, \quad t = 0, 1, \dots, \tag{1}$$

and the current production expenditure V_t is equal:

$$V_t = a \sum_{\tau=t-T_\tau-T_\mu}^{t-T_\tau} I_\tau, \quad t = 0, 1, \dots, \tag{2}$$

here $I_t = X_t^I/b$, b is the coefficient of the fund utilization for the one unity production capacity creation; a is the raw material consumption index; I_t is the production capacity in year t.

Price index is calculated in the following manner:

$$P_t = P_0(1+i)^t; \quad t = 0, 1, \dots \tag{3}$$

Changes in money demand are presented as:

$$M_{t+1}^E = M_t^E + P_tY_t - P_tV - (n_1+n_2)(P_tY_t - P_tV_t), \quad t = 0, 1, \dots \tag{4}$$

$$M_t^E = \theta_E(n_1+n_2)(P_tY_t - P_tV_t), \quad t = 0, 1, \dots \tag{5}$$

Household revenue and expenditure balance:

$$M_{t+1}^H = M_t^H + (n_1+g_1)(1-n_3)(P_tY_t - P_tV_t), \quad t = 0, 1, \dots \tag{6}$$

$$M_t^H = \theta_H P_t C_t, \quad t = 0, 1, \dots, \tag{7}$$

State budget is represented as

The state taxes are collected in the volume of $(n_2+n_3(n_1+g_1))(P_tY_t-P_tV_t)$, the external borrowing F_t^D are evaluated at the current exchange rate ρ_t, and National Bank profit B_t^B, occurred at the reevaluation of currency reserves:

$$B_t^B = (\rho_{t+1}-)R_{t+1}^C, \quad t = 0, 1, \dots \tag{8}$$

The main expenditure components are: the payment to population, the state program financing, the external debt payment, evaluated at the current exchange rate and the money reserves growth ”freezed” in budget payment accounting. The overflow of expenditure over the revenue forms the budget deficit and this deficit increases internal debt. Therefore, the change in internal debt takes form:

$$\begin{aligned} L_{t+1} - L_t = (g_1+g_2-n_2-n_3(n_1+g_1))P_t(Y_t-V_t) + \rho_t F_t^R - \\ -\rho_t Ft^D - (\rho_{t+1}-\rho_t)R_{t+1}^C + M_{t+1}^G - M_t^G + \Delta D_t^G, \quad t = 0, 1, \dots \end{aligned} \tag{9}$$

here ΔD_t^G is the internal volume of credits, granted to the state sector, $\rho_t F_t^D$, $\rho_t F_t^R$ are the currency entered the country and leave the country:

$$M_t^G = \theta_G(g1+g_2)P_t(Y_t-V_t). \tag{10}$$

External currency reserves, export and import volumes

Let E_t^P be the volume of export; Z_t^P be the volume of import; F_t^D be the currency entered the country; F_t^R be the currency leave the country; $\rho_t R_{t+1}^C - \rho_t Rt^C$ be the change in international currency reserves. Taking into account that the exports and imports price indexes q_t, q_Z change slowly than domestic price index P_t,it will be considered that these price indexes are constant. In concordance with the proposed scenario, National Bank, protecting local producers, rules the currency rate in domestic market in such a way that the import operations give minimal earns. This is expressed by the following equalities

$$P_t - \rho q_Z = 0, \quad t = 0, 1, \dots \tag{11}$$

The export volume is expressed as a share of the output. Importers secure currency on the base of import sailing on the domestic goods market. So the currency reserves of the National Bank change on the following equation base:

$$\rho_t R_{t+1}^C - \rho_t Rt^C = Pt(E_t^P - Z_t^P) + \rho_t(F_t^D - F_t^R), \quad t = 0, 1, \dots . \tag{12}$$

National account:

$$Y_t + Z_t = C_t + V_t + bI_t + g_2(Y_t - V_t) + E_t^P, \quad t = 0, 1, \dots , \tag{13}$$

here C_t is the populations' consumption, $g_2(Y_t - V_t)$ is the state investment. On the other hand, from the monetary approach, change in currency reserves (balance of payments) is expressed as: $\rho_{t+1} R_{t+1}^C - \rho_t Rt^C = \Delta M - \Delta D$, here $\Delta M = (M_{t+1}^E - M_t^E) + (M_{t+1}^G - M_t^G) + (M_{t+1}^H - M_t^H)$

State debt servicing

Suppose that $\rho_t(F_t^D - F_t^R) = g_3 P_t(Y_t - V_t), \quad t = 0, 1, \dots$, and $\Delta D_t^G + \Delta D_t^E = \Delta D_t = g_4 P_t * (Y_t - V_t)$. After some transformation on the base of given formulas it will be obtained:

$$\begin{aligned}(\rho_{t+1} - \rho_t) R_{t+1}^C = (g_1 + g_2 + g_3 - n_2 - n_3(n_1 + g_1))\times \\ \times P_t(Y_t - V_t) + (M_{t+1}^G - M_t^G), \quad t = 0, 1, \dots\end{aligned} \tag{14}$$

From equation (9) the Central Bank reserves reevaluation are expressed:

$$P_t E_t^P - P_t Z_t^P = \rho_t R_{t+1}^C - \rho_t R_t^C + g_3 P_t(Y_t - V_t), \quad t = 0, 1, \dots \tag{15}$$

Using equations (4),(6), (14) and (15) from the material balance equation (13) the variables values bI_t, C_t, $\rho_t R_{t+1}^C$ and $E_t^P - Z_t^P$ are excluded and the expression for reserves changes in national currency is obtained:

$$\rho_{t+1}R_{t+1}^C - \rho_t R_t^C = (M_{t+1}^E - M_t^E) + (M_{t+1}^H - M_t^H) +$$
$$+ (M_{t+1}^G - M_t^G) - \Delta D_t, \quad t = 0, 1, \dots \tag{16}$$

Model examination

Equations (1)–(16) represent a complete description of the growth economic model which reflects all conditions of the proposed scenario. Now some transformations are necessary in order to bring model to a form convenient for numerical analysis.

First, using the liquidity restriction (5) variables' values M_{t+1}^E and M_t^E are excluded from the financial balance equation (4). In result the real production investments are obtained:

$$bI_{t1} = (1 - (1 - \theta_E)(n_1 + n_2)(Y_t) - V_t) -$$
$$-\theta_E(n_1 + n_2)\frac{P_{t+1}}{P_t}(Y_{t+1}) - V_{t+1}), \quad t = 0, 1, \dots, \tag{17}$$

which are admitted by the production financial restrictions.

Second, from the material balance equation (13) C_t is excluded using equation (7), but $E_t^P - Z_t^P$ is excluded using equation (15) and another expression for real production investments is obtained:

$$bI_{t1} = (1 - (1 - (n_1 g_1)(1 - n_3) - g_2 - g_3)(Y_t) - V_t) +$$
$$+\frac{1}{P_t}(M_{t+1}^H) - M_t^H) - \frac{1}{q_I}(R_{t+1}^C - R_t^C), \quad t = 0, 1, \dots, \tag{18}$$

which are admitted by the material balance and by monetary policy scenario.

The possibilities of economic growth will be evaluated through the balanced inflationary growth indicators. Let's:

$$Y_t = Y_0(1+\gamma)^t, \quad V_t = V_0(1+\gamma)^t, \quad I_t = I_0(1+\gamma)^t, \quad C_t = C_0(1+\gamma)^t, \tag{19}$$

where γ is the constant growth rate in real terms of the Y_0, V_0, I_0, C_0. Then from (3), (10), (12) and (14) it is obtained:

$$\rho_t = \rho_0(1+i)^t, \quad R_t^C = R_0^C(1+\gamma)^t, \tag{20}$$

where ρ_0 and R_0^C are positive constants.

From equation (14) using expressions (10), (19) and (20)it is found:

$$R_t^C = \Big(\frac{g_1 + g_2 + g_3 - n_2 - n_3(n_1 + g_1)}{i(1+\gamma)} +$$
$$+\frac{\theta(((1+\gamma)(1+i) - 1)(g_1 + g_2}{i(1+\gamma)})q_I(Y_t) - V_t\Big), \quad t = 0, 1, \dots \tag{21}$$

The difference $R^C_{t+1} - R^C_t$ is excluded from (18) using (21), the difference $M^H_{t+1} - M^H_t$ is excluded from (18) using (7)–(8), and the expression for real investments by the production side is found:

$$bI_t = \Big(1 - n_1 - n_2 - (f_E - d_E) - \frac{(1+\gamma)(1+i)-1}{i(1+\gamma)}\times$$
$$\times(g_1 + g_2 + f_G - d_G - n_2 - n_3(n_1 + g_1)) - \frac{\gamma}{i(1+\gamma)}\theta_G\times$$
$$\times((1+\gamma)(1+i)-1)(g_1+g_2) + \frac{\theta_H((1+\gamma)(1+i)-1)}{\theta_H((1+\gamma)(1+i)-1)+1}\times$$
$$\times(n_1+g_1)(1-n_3)(Y_t - V_t)\Big), \quad t = 0,1,\dots, \tag{22}$$

Substituting (19) in (17) transforms it to:

$$bI_{t1} = (1 - (n_1 + n_2 - (f_E - d_E))\times$$
$$\times(\theta_E((1+\gamma)(1+i)-1)-1)) \times (Y_t - V_t), \quad t = 0,1,\dots, \tag{23}$$

Equating expressions (22) and (23) for the real production investments growth and inflation rate it will be obtained:

$$\theta_E(n_1+n_2) - (f_E - d_E)) = \frac{g_1 + g_2 + g_3 - n_2 - n_3(n_1+g_1)}{i(1+\gamma)}+$$
$$+\frac{\gamma\theta_G(g_1+g_2)}{i(1+\gamma)} - \frac{\theta_H(n_1+g_1)(1-n_3)}{\theta_H(\gamma + i(1+\gamma))+1}. \tag{24}$$

Finally, inserting expressions (1), (2) and (19) in (23), the sums are calculated and the second relation between the growth rate and the inflation rate is obtained:

$$b = \frac{1 - (d_E - f_E) - (n_1+n_2)(\theta_E((1+i)(1+\gamma)-1)+1)}{(1+\gamma)^T_I}\times$$
$$\times\Big(\frac{1-(1+\gamma)^{-T_\mu-1}(1+\mu)^{-T_\mu-1}}{1-(1+\gamma)^{-1}(1+\mu)^{-1}} - a\frac{1-(1+\gamma)^{-T_\mu-1}}{1-(1+\gamma)^{-1}}\Big). \tag{25}$$

The growth rate γ and the inflation rate i are determined by solving equations (24) and (25) in dependence on the model's parameters: a, b, μ, n_1 characterizing the economic effectiveness of production; g_1, g_2, f_G characterizing the state budget expenditures; n_2, n_3 characterizing the state budget revenue and the taxes pressure on production and households; $d = d_E + d_G$ which determine domestic credits rate in GDP; $g_3 = f_G + f_E$ is the total net foreign assets.

If the growth and inflation rates are determined then the external reserves in respect to GDP will be defined from (22) taking in account relation (25):

$$\frac{\rho_t R_t^C}{P_t(Y_t - V_t)} = \theta_E(n_1 + n_2) - (f_E - d_E) +$$

$$+\theta_G(g_1 + g_2) + \frac{\theta_H(n_1 + g_1)(1 - n_3)}{\theta_H(\gamma + i(1+\gamma)) + 1}, \tag{26}$$

and the net export in respect to GDP is defined from (15) using (20), (21), (1) and (2)

$$\frac{E_t^P - Z_t^P}{Y_t - V_t} = (f_E - f_G) + \gamma\Big(\theta_E(n_1 + n_2) +$$

$$+\theta_G(g_1 + g_2) + \frac{\theta_H(n_1 + g_1)(1 - n_3)}{\theta_H(\gamma + i(1+\gamma)) + 1}\Big), \tag{27}$$

here

$$\bar{a} = a\frac{1 - (1+\gamma)^{-T_m u - 1}}{1 - (1+\gamma)^{-T_{\mu-1}}(1+\mu)^{-T_{\mu-1}}} \cdot \frac{1 - (1+\gamma)^{-1}(1+\mu)^{-1}}{1 - (1+\gamma)^{-1}} \tag{28}$$

is the mean consumption index of materials V_t/Y_t.

On the base of historical data necessary constant coefficients were determined and the corresponding growth rate and inflationary rate were calculated.

References

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