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On overnilpotent radicals of topological rings

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Abstract. For every overnilpotent radical defined on the class of all topological rings every σ-bounded locally bounded topological ring is a subring of some radical topological ring.

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The radical theory of topological rings has developed similarly to the radical theory of discrete rings. Though the presence of topology and the claim for some ideals to be closed in it has made the matter specific. The survey [1] contains rather complete data on radicals of topological rings.

It has been found out that all the overnilpotent radicals defined in the class of all topological rings are rather large. So it was proved in [3] that for every minimal overnilpotent radical defined in the class of all topological rings there exists a radical ring with an identity, and it was proved in [4] that every topological ring is radical for every strictly hereditary overnilpotent radical defined in the class of all topological rings.

The last result became a reason to formulate a hypothesis that for every overnilpotent radical defined in the class of all topological rings every topological ring is a subring of a some radical ring (see [1], the problem I.1.31).

The article contains a particular solution of the problem. Its main result is the theorem asserting that for every overnilpotent radical defined in the class of all topological rings every σ-bounded locally bounded topological ring is a subring of a radical topological ring.

Though the above mentioned result is a step to the positive solution of the hypothesis, one gets less certain that it has the positive general solution while constructing the proof.

1 Remark. *Write:*

1.1. $\mathbb{N}$ *for the set of all naturals;*

1.2. nS *for* $\{\sum\limits_{i=1}^{n} a_i \mid a_i \in S\}$ *where* $n \in \mathbb{N}$ *and* $S \subseteq R$ *and* R *is a ring.*

2 Definition. *In the article:*

2.1. Every topological ring is supposed to be associative and its topology is Hasdorff;

2.2. A radical ρ *defined in the class* $\mathcal{K}$ *of topological rings is said to be overnilpotent if every nilpotent ring* $R \in \mathcal{K}$ *is* ρ*-radical;*

2.3. A subset S of a topological ring (R,τ) is said to be bounded if for every neighbourhood V of zero in it there exists such a neighbourhood U of zero that $U\cdot S=\{u\cdot s|u\in U, s\in S\}\subseteq V$ and $S\cdot U=\{s\cdot u|u\in U, s\in S\}\subseteq V$.

2.4. A topological ring (R,τ) is said to be locally bounded if it contains a bounded zero neighbourhood.

2.5. A topological ring (R,τ) is said to be σ-bounded if R is a union of a countable set of its bounded subsets.

3 Proposition. *Let $\{V_i \mid i=0,1,\dots\}$ be a sequence of subsets of the ring R such that $0\in V_i$ and $V_i+V_i\subseteq V_{i-1}$ for every $i\in\mathbb{N}$. Hence $V_n+\sum\limits_{i=1}^{n}V_i\subseteq V_0$ and therefore $\sum\limits_{i=1}^{\infty}V_i\subseteq V_0$.*

The proof can be easily done by the induction on n. $\square$

4 Proposition. *Every locally bounded σ-bounded topological ring (R,τ) is a subring of a locally bounded σ-bounded topological ring $(\tilde{R},\tilde{\tau})$ with the identity.*

Proof. Write $\mathbb{Z}$ for the ring of integers equipped with the discrete topology and $(\tilde{R},\tilde{\tau})$ for the semidirect product of topological rings $\mathbb{Z}$ and (R,τ) (see the definition 4.4.2 in [2]). Then $\tilde{R}=\{(r,k)\mid r\in R,\ k\in\mathbb{Z}\}$,

$(r_1,\ k_1)+(r_2,\ k_2)=(r_1+r_2,\ k_1+k_2)$ and

$(r_1,\ k_1)\cdot(r_2,\ k_2)=(r_1\cdot r_2+k_1\cdot r_2+k_2\cdot r_1,\ k_1\cdot k_2)$.

Hence $\tilde{R}$ is an associative ring with the identity.

Since the ring $\mathbb{Z}$ is discrete then $R'=\{(r,\ 0)\mid r\in R\}$ is an open subring in $(\tilde{R},\tilde{\tau})$ and $(R',\tilde{\tau}|_{R'}$ is topologically isomorphic to the topological ring (R,τ).

Hence $(\tilde{R},\tilde{\tau})$ is a Hausdorff locally bounded σ-bounded topological ring containing (R,τ) as a subring. $\square$

5 Theorem. *Let ρ be an arbitrary overnilpotent radical defined in the class of all topological rings. If a topological ring (R,τ) is a locally bounded and σ-bounded then there exists a ρ-radical topological ring $(\hat{R},\hat{\tau})$ such that the topological ring (R,τ) is a subring of the topological ring $(\hat{R},\hat{\tau})$.*

Proof. Taking into account Proposition 4 assume R to be a ring with the identity e.

5.1. By Theorem 1.6.46 in [2] (R,τ) contains such a bounded neighbourhood U_0 of zero and such a basis $\mathcal{B}=\{V_\omega\mid\omega\in\Omega\}$ of symmetrical neighbourhoods of zero that U_0 is a subsemigroup of a multiplicative group of the ring R and every neighbourhood of zero $V_\omega\in\mathcal{B}$ is an ideal of the semigroup U_0.

5.2. Since a union, a sum and a product of a finite set of bounded sets are bounded in a topological ring (see 1.6.19 and 1.6.22 in [2], then there exists a set $\{\Gamma_i\mid i=0,1,2,\dots\}$ of bounded subsets in (R,τ) such that the following assertions hold:

5.2.1. $U_0 \subseteq \Gamma_0$;
5.2.2. $e \in \Gamma_0$;
5.2.3. $-\Gamma_k = \Gamma_k$ and $2^k\Gamma_k \subseteq \Gamma_{k+1}$ for every k.
5.2.4. $\Gamma_k \cdot \Gamma_k \subseteq \Gamma_{k+1}$ for every k.

5.3. For every $\omega \in \Omega$ there exists a sequence $\{U_{i,\ \omega} \mid\ i = 1, 2, \dots\}$ of neighbourhoods of zero from $\mathcal{B}$ such that the following assertions hold:
5.3.1. $U_{1,\ \omega} + U_{1,\ \omega} \subseteq V_\omega$ for every $\omega \in \Omega$;
5.3.2 $\Gamma_{k+1} \cdot U_{k+1,\ \omega} \subseteq U_{k,\ \omega}$ and $U_{k+1,\ \omega} \cdot \Gamma_{k+1} \subseteq U_{k,\ \omega}$ for every $k \in \mathbb{N}$;
5.3.3. $2^k U_{k+1,\ \omega} \subseteq U_{k,\ \omega}$ for every $k \in \mathbb{N}$.

5.4. Let $X = \{x_2, x_3, \dots\}$ be a set of variables. Consider the ring $R[X]$ of polynomials over the ring R with the set of variables X which commute with elements of R and each other, i.e. $x_i \cdot x_j = x_j \cdot x_i$ and $r \cdot x_i = x_i \cdot r$ for every $x_i,\ x_j \in X$ and $r \in R$.

Consider the ideal I of the ring $R[X]$ generated by the set $\{x_i^i\ \mid i = 2, 3, \dots\}$. Let $\hat{R} = R[X]/I$ and $\hat{x}_k = x_k + I$. By identifying the element $r \in R$ with the element $r + I \in \hat{R}$ we may assume that R is a subring of the ring $\hat{R}$ and $\hat{R}$ is a ring of polynomials over R of the set $\{\hat{x}_2, \hat{x}_3, \dots\}$ commuting with the elements of R and each other and $\hat{x}_k^k = 0$ for every $k \geqslant 2$.

5.5. Given $n \in \mathbb{N}$. Write G_n for a subsemigroup of the multiplicative semigroup of the ring $\hat{R}$ generated by the set $\{e, \hat{x}_2, \dots, \hat{x}_n\}$. Hence $e \in G_n \subseteq G_{n+1}$ for every $n \in \mathbb{N}$ and $G_1 = \{e\}$.

5.6. Given $n \in \mathbb{N}$ and $\omega \in \Omega$. Write $W_{n,\ \omega}$ for

$$\sum_{i=1}^{\infty} 2^i (U_{n\cdot i+n,\ \omega} \cdot G_{n\cdot i}) + \sum_{s=1}^{\infty}\sum_{j=1}^{s} 2^s \big(\Gamma_{n\cdot s-n} \cdot G_{n\cdot s} \cdot (e - \hat{x}_{n\cdot s+1})^j\big).$$

Prove the set $\hat{\mathcal{B}} = \{W_{k,\ \omega} \mid\ \omega \in \Omega,\ k = 1, 2, \dots\}$ is a basis of neighbourhoods of zero of some Hausdorff ring topology $\hat{\tau}$ on $\hat{R}$ and $\hat{\tau}\ |_R = \tau$ (i.e. the topological ring $(R,\ \tau)$ is a subring of $(\hat{R},\ \hat{\tau})$).

5.6.1. Since $0 \in U_{k,\ \omega}$ and $0 \in \Gamma_k$ for every $k \in \mathbb{N}$ and $\omega \in \Omega$ then $0 \in W_{n,\ \omega}$ for every n and ω, i.e. the assertion BN1 of Theorem 1.2.5 from [2] holds for the set $\{W_{k,\ \omega} \mid \omega \in \Omega,\ k = 1, 2, \dots\}$.

5.6.2. Since $2^i M \subseteq 2^{k\cdot i} M$ for every $M \subseteq \hat{R}$ such that $0 \in M$ and naturals i and k then

$$W_{k\cdot n,\ \omega} = \sum_{i=1}^{\infty} 2^i (U_{k\cdot n\cdot i+kn,\ \omega} \cdot G_{k\cdot n\cdot i}) + \sum_{s=1}^{\infty}\sum_{j=1}^{s} 2^s \big(\Gamma_{k\cdot n\cdot s-k\cdot n} \cdot G_{k\cdot n\cdot s} \cdot (e - x_{k\cdot n\cdot s+1})^j\big) \subseteq$$

$$\sum_{i=1}^{\infty} 2^{k\cdot i} (U_{n\cdot k\cdot i+n,\ \omega} \cdot G_{n\cdot k\cdot i}) + \sum_{s=n}^{\infty}\sum_{j=1}^{n} 2^s \big(\Gamma_{n\cdot k\cdot s-n} \cdot G_{n\cdot k\cdot s} \cdot (e - x_{n\cdot k\cdot s+1})^j\big) \subseteq$$

$$\sum_{i=1}^{\infty} 2^i (U_{n\cdot i+n,\ \omega} \cdot G_i) + \sum_{s=1}^{\infty}\sum_{j=1}^{s} 2^s \big(\Gamma_{n\cdot s-n} \cdot G_{n\cdot s} \cdot (e - x_{n\cdot s+1})^j\big) = W_{n,\ \omega}$$

for every $n,\ k \in \mathbb{N}$ and $\omega \in \Omega$ and hence $W_{k\cdot n,\ \omega} \subseteq W_{n,\ \omega} \bigcap W_{k,\ \omega}$, i.e. the assertion BN2 of Theorem 1.2.5 from [2] holds for the set $\{W_{k,\ \omega} \mid \omega \in \Omega,\ k = 1, 2, \ldots\}$.

5.6.3. Since $-U_{i,\ \omega} = U_{i,\ \omega}$ and $-\Gamma_i = \Gamma_i$ for every $i \in \mathbb{N}$ and $\omega \in \Omega$ then $-W_{n,\ \omega} = W_{n,\ \omega}$ for every $n \in \mathbb{N}$ and $\omega \in \Omega$, i.e. the assertion BN3 of Theorem 1.2.5 from [2] also holds for the set $\{W_{k,\ \omega} \mid \omega \in \Omega,\ k = 1, 2, \ldots\}$.

5.6.4. Since $2^{2\cdot k} \geqslant 2^{k+1} = 2^k + 2^k$ for every $k \in \mathbb{N}$ then $W_{2\cdot n,\ \omega} + W_{2\cdot n,\ \omega} =$

$$\sum_{i=1}^{\infty} 2^i (U_{2\cdot n\cdot i+2\cdot n,\ \omega} \cdot G_{2\cdot n\cdot i}) + \sum_{s=1}^{\infty}\sum_{j=1}^{s} 2^s \big(\Gamma_{2\cdot n\cdot s-2\cdot n} \cdot G_{2\cdot n\cdot s} \cdot (e - x_{2\cdot n\cdot s+1})^j\big) +$$

$$\sum_{i=1}^{\infty} 2^i (U_{2\cdot n\cdot i+2n,\ \omega} \cdot G_{2\cdot n\cdot i}) + \sum_{s=1}^{\infty}\sum_{j=1}^{s} 2^s \big(\Gamma_{2\cdot n\cdot s-2\cdot n} \cdot G_{2\cdot n\cdot s} \cdot (e - x_{2\cdot n\cdot s+1})^j\big) =$$

$$\sum_{i=1}^{\infty} (2^i + 2^i)(U_{2\cdot n\cdot i+2\cdot n,\ \omega} \cdot G_{2\cdot n\cdot i}) + \sum_{s=1}^{\infty}\sum_{j=1}^{s} (2^s + 2^s)\big(\Gamma_{2\cdot n\cdot s-2n} \cdot G_{2\cdot n\cdot s} \cdot (e - x_{2\cdot n\cdot s+1})^j\big) \subseteq$$

$$\sum_{i=1}^{\infty} 2^{2\cdot i} (U_{2\cdot n\cdot i+n,\ \omega} \cdot G_{2\cdot n\cdot i}) + \sum_{s=1}^{\infty}\sum_{j=1}^{2\cdot s} 2^{2\cdot s} \big(\Gamma_{2\cdot n\cdot s-n} \cdot G_{2\cdot n\cdot s} \cdot (e - x_{2\cdot n\cdot s+1})^j\big) \subseteq$$

$$\sum_{j=1}^{\infty} 2^j (U_{n\cdot j+n,\ \omega} \cdot G_{n\cdot j}) + \sum_{t=1}^{\infty}\sum_{j=1}^{t} 2^t \big(\Gamma_{n\cdot t-n} \cdot G_{n\cdot t} \cdot (e - x_{n\cdot t+1})^j\big) = W_{n,\ \omega}$$

for every $n \in \mathbb{N}$, i.e. the assertion BN4 of Theorem 1.2.5 from [2] holds for the set $\{W_{k,\ \omega} \mid \omega \in \Omega,\ k = 1, 2, \ldots\}$.

5.6.5. Since $r \cdot x_i = x_i \cdot r$ and $x_i \cdot x_j = x_j \cdot x_i$ for every $r \in R$ and $i,\ j \in \mathbb{N}$ then

$$W_{4\cdot n,\ \omega} \cdot W_{4\cdot n,\ \omega} = \Big(\sum_{i=1}^{\infty} 2^i (U_{4\cdot n\cdot i+4\cdot n,\ \omega} \cdot G_{4\cdot n\cdot i}) +$$

$$\sum_{s=1}^{\infty}\sum_{j=1}^{s} 2^s \big(\Gamma_{4\cdot n\cdot s-4\cdot n} \cdot G_{4\cdot n\cdot s} \cdot (e - x_{4\cdot n\cdot s+1})^j\big)\Big) \times$$

$$\Big(\sum_{i=1}^{\infty} 2^i (U_{4\cdot n\cdot i+4\cdot n,\ \omega} \cdot G_{4\cdot n\cdot i}) + \sum_{s=1}^{\infty}\sum_{j=1}^{s} 2^s \big(\Gamma_{4\cdot n\cdot s-4\cdot n} \cdot G_{4\cdot n\cdot s} \cdot (e - x_{4\cdot n\cdot s+1})^j\big)\Big) =$$

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} (2^i \cdot 2^j)\big((U_{4\cdot n\cdot i+4\cdot n,\ \omega} \cdot U_{4\cdot n\cdot j+4\cdot n,\ \omega}) \cdot (G_{4\cdot n\cdot i} \cdot G_{4\cdot n\cdot j})\big) +$$

$$\sum_{i=1}^{\infty}\sum_{s=1}^{\infty}\sum_{j=1}^{s} (2^i \cdot 2^s)\big((U_{4\cdot n\cdot i+4\cdot n,\ \omega} \cdot \Gamma_{4\cdot n\cdot s-4\cdot n}) \cdot (G_{4\cdot n\cdot i} \cdot G_{4\cdot n\cdot s} \cdot (e - x_{4\cdot n\cdot s+1})^j)\big) +$$

$$\sum_{i=1}^{\infty}\sum_{s=1}^{\infty}\sum_{j=1}^{s}(2^s\cdot 2^i)\big((\Gamma_{4\cdot n\cdot s-4\cdot n}\cdot U_{4\cdot n\cdot i+4\cdot n,\ \omega})\cdot(G_{4\cdot n\cdot s}\cdot G_{4\cdot n\cdot i})\cdot(e-x_{4\cdot s+1})^j\big)+$$

$$\sum_{s=1}^{\infty}\sum_{j=1}^{s}\sum_{t=1}^{\infty}\sum_{i=1}^{t}(2^s\cdot 2^t)\big((\Gamma_{4\cdot n\cdot s-4\cdot n}\cdot\Gamma_{4\cdot n\cdot t-4\cdot n})\times$$

$$(G_{4\cdot n\cdot s}\cdot G_{4\cdot n\cdot t})\cdot(e-x_{4\cdot n\cdot s+1})^j\cdot(e-x_{4\cdot n\cdot t+1})^i\big)=$$

$$\sum_{i=1}^{\infty}\sum_{j=1}^{i}(2^{i+j})\big((U_{4\cdot n\cdot i+4\cdot n,\ \omega}\cdot U_{4\cdot n\cdot j+4\cdot n,\ \omega})\cdot(G_{4\cdot n\cdot i}\cdot G_{4\cdot n\cdot j})\big)+$$

$$\sum_{j=2}^{\infty}\sum_{i=1}^{j-1}2^{i+j}\big((U_{4\cdot n\cdot i+4\cdot n,\ \omega}\cdot U_{4\cdot n\cdot j+4\cdot n,\ \omega})\cdot(G_{4\cdot n\cdot i}\cdot G_{4\cdot n\cdot j})\big)+$$

$$\sum_{i=1}^{\infty}\sum_{s=1}^{i-1}\sum_{j=1}^{s}2^{i+s}\Big((U_{4\cdot n\cdot i+4\cdot n,\ \omega}\cdot\Gamma_{4\cdot n\cdot s-4\cdot n})\cdot\big(G_{4\cdot n\cdot i}\cdot G_{4\cdot n\cdot s}\big)\cdot(e-x_{4\cdot n\cdot s+1})^j\Big)+$$

$$\sum_{s=1}^{\infty}\sum_{i=1}^{s}\sum_{j=1}^{s}2^{s+i}\big((U_{4\cdot n\cdot i+4\cdot n,\ \omega}\cdot\Gamma_{4\cdot n\cdot s-4\cdot n})\cdot(G_{4\cdot n\cdot i}\cdot G_{4\cdot n\cdot s})\cdot(e-x_{4\cdot n\cdot s+1})^j\big)+$$

$$\sum_{i=1}^{\infty}\sum_{s=1}^{i-1}\sum_{j=1}^{s}2^{s+i}\big((\Gamma_{4\cdot n\cdot s-4\cdot n}\cdot U_{4\cdot n\cdot i+4\cdot n,\ \omega}\cdot(G_{4\cdot n\cdot s}\cdot G_{4\cdot n\cdot i})\cdot(e-x_{4\cdot n\cdot s+1})^j\big)+$$

$$\sum_{s=1}^{\infty}\sum_{i=1}^{s}\sum_{j=1}^{s}2^{s+i}\big((\Gamma_{4\cdot n\cdot s-4\cdot n}\cdot U_{4\cdot n\cdot i+4\cdot n,\ \omega})\cdot(G_{4\cdot n\cdot s}\cdot G_{4\cdot n\cdot i})\cdot(e-x_{4\cdot n\cdot s+1})^j\big)+$$

$$\sum_{s=1}^{\infty}\sum_{j=1}^{s}\sum_{t=1}^{s-1}\sum_{i=1}^{t}2^{s+t}\Big(\Gamma_{4\cdot n\cdot s-4\cdot n}\cdot\Gamma_{4\cdot n\cdot t-4\cdot n}\Big)\times$$

$$\big(G_{4\cdot n\cdot s}\cdot G_{4\cdot n\cdot t}\cdot(e-x_{4\cdot n\cdot t+1})^i\cdot(e-x_{4\cdot n\cdot s+1})^j\big)\Big)+$$

$$\sum_{t=1}^{\infty}\sum_{j=1}^{t}\sum_{i=1}^{t}2^{t+t}\big((\Gamma_{4\cdot n\cdot t-4\cdot n}\cdot\Gamma_{4\cdot n\cdot t-4\cdot n})\cdot((G_{4\cdot n\cdot t}\cdot G_{4\cdot n\cdot t})\cdot(e-x_{4\cdot n\cdot t+1})^{j+i}\big)+$$

$$\sum_{t=1}^{\infty}\sum_{j=1}^{t}\sum_{s=1}^{t-1}\sum_{i=1}^{s}2^{s+t}\Big((\Gamma_{4\cdot n\cdot s-4\cdot n}\cdot\Gamma_{4\cdot n\cdot t-4\cdot n})\times$$

$$\big(G_{4\cdot n\cdot s}\cdot(e-x_{4\cdot n\cdot s+1})^j\cdot G_{4\cdot n\cdot t}\big)\cdot(e-x_{4\cdot n\cdot t+1})^i\Big).$$

5.6.5.1. Since $U_{i,\ \omega}\cdot U_{j,\ \omega}\subseteq U_{i,\ \omega}$ and $G_j\cdot G_i=G_i\cdot G_j=G_i$ for every $i\geqslant j$ and $\omega\in\Omega$ then

$$\sum_{i=1}^{\infty}\sum_{j=1}^{i}2^{i+j}\big((U_{4\cdot n\cdot i+4\cdot n,\ \omega}\cdot U_{4\cdot n\cdot j+4\cdot n,\ \omega})\cdot(G_{4\cdot n\cdot i}\cdot G_{4\cdot n\cdot j})\big)\subseteq$$

$$\sum_{i=1}^{\infty}\sum_{j=1}^{i}2^{i+j}(U_{4\cdot n\cdot i+4\cdot n,\ \omega}\cdot G_{4\cdot n\cdot i})=$$

$$\sum_{i=1}^{\infty}\Big(\sum_{j=1}^{i}2^{i+j}\Big)\big(U_{4\cdot n\cdot i+4\cdot n,\ \omega}\cdot G_{4\cdot n\cdot i}\big)=\sum_{i=1}^{\infty}(2^{2\cdot i+1}-2^{i+1})\big(U_{4\cdot n\cdot i+4\cdot n,\ \omega}\cdot G_{4\cdot n\cdot i}\big)\subseteq$$

$$\sum_{i=1}^{\infty}2^{4\cdot i}\big(U_{n\cdot 4\cdot i+n,\ \omega}\cdot G_{n\cdot 4\cdot i}\big)\subseteq\sum_{j=1}^{\infty}2^{j}(U_{n\cdot j+n,\ \omega}\cdot G_{n\cdot j})\subseteq W_{n,\omega}$$

5.6.5.2. Equalities

$$\sum_{j=2}^{\infty}\sum_{i=1}^{j-1}2^{i+j}\big((U_{4\cdot n\cdot i+4\cdot n,\ \omega}\cdot U_{4\cdot n\cdot j+4\cdot n,\ \omega})\cdot(G_{4\cdot n\cdot i}\cdot G_{4\cdot n\cdot j})\big)\subseteq$$

$$\sum_{j=1}^{\infty}2^{j}(U_{4\cdot n\cdot j+4\cdot n,\ \omega}\cdot G_{4\cdot n\cdot j})\subseteq W_{n,\ \omega}$$

are obtained similarly.

5.6.5.3. Since $U_{i,\ \omega}\cdot\Gamma_j\subseteq U_{i-1,\ \omega}$ and $\Gamma_j\cdot U_{i,\ \omega}\subseteq U_{i-1,\ \omega}$ and $G_j\cdot G_i=G_i\cdot G_j=G_i$ for every $i\geqslant j$ and $\omega\in\Omega$ then taking into account the equality

$$\sum_{j=1}^{s}(e-x_{4\cdot n\cdot s+1})^j\in 2^{s+1}G_{4\cdot s+1}\subseteq 2^iG_{4\cdot i}$$

which holds for every $s\leqslant i-1$ obtain

$$\sum_{i=1}^{\infty}\sum_{s=1}^{i-1}2^{i+s}\cdot 2^i\Big((U_{4\cdot n\cdot i+4\cdot n,\ \omega}\cdot\Gamma_{4\cdot n\cdot s-4\cdot n})\cdot\big(G_{4\cdot n\cdot i}\cdot G_{4\cdot n\cdot s}\big)\cdot(e-x_{4\cdot n\cdot s+1})^j\Big)\subseteq$$

$$\sum_{i=1}^{\infty}(2^{3\cdot i}\cdot i)(U_{4\cdot n\cdot i+4\cdot n-1,\ \omega}\cdot G_{4\cdot n\cdot i})\subseteq\sum_{i=1}^{\infty}2^{4\cdot i}(U_{n\cdot 4\cdot i+n,\ \omega}\cdot G_{n\cdot 4\cdot i})\subseteq$$

$$\sum_{j=1}^{\infty}2^{j}(U_{n\cdot j+n,\ \omega}\cdot G_{n\cdot j})=W_{n,\ \omega}.$$

5.6.5.4. Similarly

$$\sum_{i=1}^{\infty}\sum_{s=1}^{i-1}\sum_{j=1}^{s} 2^{s+i}\big((\Gamma_{4\cdot n\cdot s-4\cdot n}\cdot U_{4\cdot n\cdot i+4\cdot n,\ \omega})\cdot(G_{4\cdot n\cdot s}\cdot(e-x_{4\cdot n\cdot s+1})^j\cdot G_{4\cdot n\cdot i})\big)\subseteq W_{n,\ \omega}.$$

5.6.5.5. Since $U_{i,\ \omega}\cdot\Gamma_j\subseteq\Gamma_j\cdot\Gamma_j\subseteq\Gamma_{j+1}$ and $\Gamma_j\cdot U_{i,\ \omega}\subseteq\Gamma_j\cdot\Gamma_j\subseteq\Gamma_{j+1}$ and $G_j\cdot G_i=G_i\cdot G_j=G_j$ for every $i\leqslant j$ and $\omega\in\Omega$ then taking into account inequalities $\sum\limits_{i=1}^{s}2^{s+i}\leqslant 2^{2\cdot s+1}\leqslant 2^{4\cdot s}$ obtain

$$\sum_{s=1}^{\infty}\sum_{i=1}^{s}\sum_{j=1}^{s}2^{s+i}\big((U_{4\cdot n\cdot i+4\cdot n,\ \omega}\cdot\Gamma_{4\cdot n\cdot s-4\cdot n})\cdot(G_{4\cdot n\cdot i}\cdot G_{4\cdot n\cdot s})\cdot(e-x_{4\cdot n\cdot s+1})^j\big)\subseteq$$

$$\sum_{s=1}^{\infty}\sum_{j=1}^{s}\sum_{i=1}^{s}2^{s+i}\big(\Gamma_{4\cdot n\cdot s-4\cdot n+1})\cdot G_{4\cdot n\cdot s})\cdot(e-x_{4\cdot n\cdot s+1})^j\big)\subseteq$$

$$\sum_{s=1}^{\infty}\sum_{j=1}^{s}2^{4\cdot s}\big(\Gamma_{n\cdot 4\cdot s-n}\cdot G_{4\cdot n\cdot s}\cdot(e-x_{4\cdot n\cdot s+1})^j\big)\subseteq$$

$$\sum_{s=1}^{\infty}\sum_{j=1}^{s}2^{4\cdot s}\big(\Gamma_{n\cdot 4\cdot s-4\cdot n}\cdot G_{n\cdot 4\cdot s}\cdot(e-x_{n\cdot 4\cdot s+1})^j\big)\subseteq$$

$$\sum_{t=1}^{\infty}\sum_{j=1}^{t}2^{t}\big(\Gamma_{n\cdot t-n}\cdot G_{n\cdot t}\cdot(e-x_{n\cdot t+1})^j\big)=W_{n,\ \omega}.$$

5.6.5.6. Similarly

$$\sum_{s=1}^{\infty}\sum_{i=1}^{s}\sum_{j=1}^{s}2^{s+i}\big((\Gamma_{4\cdot n\cdot s-4\cdot n}\cdot U_{4\cdot n\cdot i+4\cdot n,\ \omega})\cdot(G_{4\cdot n\cdot s}\cdot G_{4\cdot n\cdot i})\cdot(e-x_{4\cdot n\cdot s+1})^j\big)\subseteq W_{n,\ \omega}$$

5.6.5.7. Since $\Gamma_i\cdot\Gamma_j\subseteq\Gamma_{i+1}$ and $\Gamma_j\cdot\Gamma_i\subseteq\Gamma_{i+1}$ and $G_j\cdot G_i=G_i\cdot G_j\subseteq G_i$ for $i\geqslant j$ then taking into account the relation

$$\sum_{j=1}^{s}(e-x_{4\cdot n\cdot s+1})^j\in 2^{s+1}G_{4\cdot s+1}\subseteq 2^iG_{4\cdot i}$$

for $s\leqslant i-1$ obtain

$$\sum_{s=1}^{\infty}\sum_{j=1}^{s}\sum_{t=1}^{s-1}\sum_{i=1}^{t}2^{s+t}\Big((\Gamma_{4\cdot n\cdot s-4\cdot n}\cdot\Gamma_{4\cdot n\cdot t-4\cdot n})\times$$

$$\big(G_{4\cdot n\cdot s}\cdot G_{4\cdot n\cdot t}\cdot(e-x_{4\cdot n\cdot t+1})^i\big)\cdot(e-x_{4\cdot n\cdot s+1})^j\Big)\subseteq$$

$$\sum_{s=1}^{\infty}\sum_{j=1}^{s}\sum_{t=1}^{s-1} 2^{s+t}\Big(\Gamma_{4\cdot n\cdot s-4\cdot n+1}\cdot\big(G_{4\cdot n\cdot s}\cdot(2^s G_{4\cdot n\cdot s})\big)\cdot(e-x_{4\cdot n\cdot s+1})^j\Big)\subseteq$$
$$\sum_{s=1}^{\infty}\sum_{j=1}^{s}(2^{2\cdot s+s}\cdot s)\Big(\Gamma_{4\cdot n\cdot s-4\cdot n+1}\cdot G_{4\cdot n\cdot s}\cdot(e-x_{4\cdot n\cdot s+1})^j\Big)\subseteq$$
$$\sum_{s=1}^{\infty}\sum_{j=1}^{s}2^{4\cdot s}\big(\Gamma_{n\cdot 4\cdot s-n}\cdot G_{n\cdot 4\cdot s}\cdot(e-x_{n\cdot 4\cdot s+1})^j\big)\subseteq$$
$$\sum_{l=1}^{\infty}\sum_{j=1}^{l}2^{l}\big(\Gamma_{n\cdot l-n}\cdot G_{n\cdot l}\cdot(e-x_{n\cdot l+1})^j\big)=W_{n,\,\omega}.$$

5.6.5.8. Similarly

$$\sum_{t=1}^{\infty}\sum_{j=1}^{t}\sum_{s=1}^{t-1}\sum_{i=1}^{s}2^{s+t}\Big((\Gamma_{4\cdot n\cdot s-4\cdot n}\cdot\Gamma_{4\cdot n\cdot t-4\cdot n})\times$$
$$\big(G_{4\cdot n\cdot s}\cdot(e-x_{4\cdot n\cdot s+1})^j\cdot G_{4\cdot n\cdot t}\big)\cdot(e-x_{4\cdot n\cdot t+1})^i\Big)\subseteq W_{n,\omega}$$

and

5.6.5.9. $$\sum_{t=1}^{\infty}\sum_{j=1}^{t}\sum_{i=1}^{t}2^{t+t}\big((\Gamma_{4\cdot n\cdot t-4\cdot n}\cdot\Gamma_{4\cdot n\cdot t-4\cdot n})\cdot(G_{4\cdot n\cdot t}\cdot G_{4\cdot n\cdot t})\cdot(e-x_{4\cdot n\cdot t+1})^{j+i}\big)=$$
$$\sum_{t=1}^{\infty}\sum_{k=1}^{2\cdot t}\sum_{i+j=k}2^{2\cdot t}\big(\Gamma_{4\cdot n\cdot t-4\cdot n+1}\cdot G_{4\cdot n\cdot t}\cdot(e-x_{4\cdot n\cdot t+1})^{j+i}\big)=$$
$$\sum_{t=1}^{\infty}\sum_{k=1}^{2\cdot t}(2^{2\cdot t}\cdot k)\big(\Gamma_{4\cdot n\cdot t-4\cdot n+1}\cdot G_{4\cdot n\cdot t}\cdot(e-x_{4\cdot n\cdot t+1})^{k}\big)=$$
$$\sum_{t=1}^{\infty}\sum_{k=1}^{2\cdot t}(2^{2\cdot t}\cdot 2^t)\big(\Gamma_{4\cdot n\cdot t-4\cdot n+1}\cdot G_{4\cdot n\cdot t}\cdot(e-x_{4\cdot n\cdot t+1})^{k}\big)\subseteq$$
$$\sum_{t=1}^{\infty}\sum_{k=1}^{4\cdot t}(2^{4\cdot t}\cdot 2^t)\big(\Gamma_{n\cdot 4\cdot t-n}\cdot G_{n\cdot 4\cdot t}\cdot(e-x_{n\cdot 4\cdot t+1})^{k}\big)\subseteq$$
$$\sum_{l=1}^{\infty}\sum_{k=1}^{l}2^{l}\big(\Gamma_{n\cdot l-n}\cdot G_{n\cdot l}\cdot(e-x_{n\cdot l+1})^{k}\big)=W_{n,\,\omega}.$$

Hence by the inclusions obtained in the items 5.6.5.1 – 5.6.5.9 and the equality obtained in 5.6.5, applying 4 times the inclusion obtained in the item 5.5.4 obtain that

$$W_{64\cdot n,\,\omega}\cdot W_{64\cdot n,\,\omega}\subseteq 9W_{16\cdot n,\,\omega}\subseteq 16W_{16\cdot n,\,\omega}\subseteq$$

$$8W_{8\cdot n,\ \omega} \subseteq 4W_{4\cdot n,\ \omega} \subseteq 2W_{2\cdot n,\ \omega} \subseteq W_{n,\ \omega}$$

for every $n \in \mathbb{N}$ and $\omega \in \Omega$, i.e. the assertion BN5 of Theorem 1.2.5 from [2] holds for the set $\{W_{k,\ \omega} \mid \omega \in \Omega,\ k = 1, 2, \ldots\}$.

5.6.6. Let now $\hat{r} \in \hat{R}$ and $W_{k,\ \omega} \in \hat{\mathcal{B}}$. Then there exists $n \in \mathbb{N}$, and $\{r_i, \ldots, r_n\} \subseteq R$ and $\{u_i, \ldots, u_n\} \subseteq \bigcup\limits_{j=1}^{\infty} G_j$ such that $\hat{r} = \sum\limits_{i=1}^{n} r_i \cdot u_i$. Since $\hat{R} = \bigcup\limits_{j=1}^{\infty} \Gamma_j$ then there exists a natural $m \geqslant n$ and $m \geqslant k+1$ such that $\{r_1, \ldots, r_n\} \subseteq \Gamma_m$ and $\{u_1, \ldots, u_n\} \subseteq G_m$. Then

$$\hat{r} = \sum_{l=1}^{n} r_l \cdot u_l \in n(\Gamma_m \cdot G_m) \subseteq 2^m(\Gamma_m \cdot G_m),$$

and hence

$$\hat{r} \cdot W_{m\cdot k,\ \omega} \subseteq \big(2^m(\Gamma_m \cdot G_m)\big) \cdot \Big(\sum_{i=1}^{\infty} 2^i\big(U_{m\cdot k\cdot i+m\cdot k,\ \omega} \cdot G_{m\cdot k\cdot i}\big) +$$

$$\sum_{s=1}^{\infty}\sum_{j=1}^{s}(2^m \cdot 2^s)\big(\Gamma_{m\cdot k\cdot s-m\cdot k} \cdot G_{m\cdot k\cdot s} \cdot (e - x_{m\cdot k\cdot s+1})^j\big)\Big) \subseteq$$

$$\sum_{i=1}^{\infty} 2^{i+m}(\Gamma_m \cdot U_{m\cdot k\cdot i+m\cdot k,\ \omega}) \cdot (G_m \cdot G_{m\cdot k\cdot i}) +$$

$$\sum_{s=1}^{\infty}\sum_{j=1}^{s} 2^{m+s}\big((\Gamma_m \cdot \Gamma_{m\cdot k\cdot s-m\cdot k}) \cdot (G_m \cdot G_{k\cdot m\cdot s}) \cdot (e - x_{m\cdot k\cdot s+1})^j\big) \subseteq$$

$$\sum_{i=1}^{\infty} 2^{i+m}(U_{m\cdot k\cdot i+m\cdot k-1,\ \omega} \cdot G_{m\cdot k\cdot i}) +$$

$$\sum_{s=1}^{\infty}\sum_{j=1}^{s} 2^{m+s}\big(\Gamma_{m\cdot k\cdot s-m\cdot k+1} \cdot G_{k\cdot m\cdot s} \cdot (e - x^j_{m\cdot k\cdot s+1})\big) \subseteq$$

$$\sum_{i=1}^{\infty} 2^{i\cdot m}(U_{k\cdot i\cdot m+k,\ \omega} \cdot G_{k\cdot i\cdot m}) + \sum_{s=1}^{\infty}\sum_{j=1}^{s} 2^{m\cdot s}\big(\Gamma_{k\cdot s\cdot m-k} \cdot G_{k\cdot s\cdot m} \cdot (e - x_{k\cdot s\cdot m+1})^j\big) \subseteq$$

$$\sum_{l=1}^{\infty} 2^l(U_{k\cdot l+k,\ \omega} \cdot G_{k\cdot l}) + \sum_{l=1}^{\infty}\sum_{j=1}^{l} 2^l\big(\Gamma_{k\cdot l-k} \cdot G_{k\cdot l} \cdot (e - x_{k\cdot l})^j\big) = W_{k,\ \omega}.$$

Similarly $W_{k\cdot m,\ \omega} \cdot \hat{r} \subseteq W_{k,\ \omega}$, i.e. the assertion BN6 of Theorem 1.2.5 from [2] also holds for the set $\{W_{i,\ \omega} \mid \omega \in \Omega,\ i = 1, 2, \ldots\}$ and hence the set $\hat{\mathcal{B}} = \{W_{i,\ \omega} \mid \omega \in \Omega,\ i = 1, 2, \ldots\}$ is a basis of neighbourhoods of zero in some (not necessarily Hausdorff) ring topology $\hat{\tau}$ on $\hat{R}$.

5.7. Prove the topology $\hat{\tau}$ is Hausdorff. To do that, by Theorem 1.3.2 from [2] is sufficient to check that $\bigcap\limits_{k\in\mathbb{N},\omega\in\Omega} W_{k,\ \omega} = \{0\}$.

5.7.1. Let $0 \neq \hat{r} \in \hat{R}$. Then there exists $n \in \mathbb{N}$, and $\{r_i, \dots, r_n\} \subseteq R$ and $\{u_i, \dots, u_n\} \subseteq \bigcup\limits_{j=1}^{\infty} G_j$ such that $r_i \neq 0$ for $1 \leqslant i \leqslant n$ and $\hat{r} = \sum\limits_{i=1}^{n} r_i \cdot u_i$.

5.7.2. Let m be such a natural that $\{u_i, \dots, u_n\} \subseteq G_m$. Define the mapping $\xi : \bigcup\limits_{j=1}^{\infty} G_j \to G_m$ as follows:

if $u \in \bigcup\limits_{j=1}^{\infty} G_j$ then there exists the only pair of elements $v \in G_m$ and $u' \in \bigcup\limits_{j=1}^{\infty} G_j$ such that $u = v \cdot u'$ and the notation of the element u' does not contain variables $\hat{x}_i$ where $i \leqslant m$. Then write $\xi(u) = v$.

5.7.3. Since $(\hat{R},\ +)$ can be considered to be a free R-module freely generated by the set $\bigcup\limits_{j=1}^{\infty} G_j$ then the mapping ξ can be extended to the R-module homomorphism $\hat{\xi} : \hat{R} \to \hat{R}$. Then $\hat{\xi}(u) = u$ for every $u \in G_m$ and hence

5.7.4. $\hat{\xi}(\hat{r}) = \hat{r}$ and $\hat{\xi}\big(G_k \cdot (1 - x_{k+1})^j\big) = \{0\}$ for every $k \geqslant m$ and $j \leqslant k$.

Since the topological ring (R, τ) is Hausdorff then there exists $\omega_0 \in \Omega$ such that $\{r_i, \dots, r_n\} \bigcap V_{\omega_0} = \emptyset$. Let $\{U_{i,\ \omega_0} \mid i = 1, 2, \dots\}$ be a sequence neighbourhoods of zero in (R, τ) from $\mathcal{B}$ mentioned in 5.3 and $W_{m,\ \omega_0}$ be neighbourhoods of zero in $(\hat{R}, \hat{\tau})$ constructed according to 5.6 for the sequence $\{U_{i,\ \omega_0} \mid i = 1, 2, \dots\}$. Prove that $\hat{r} \notin W_{m,\omega_0}$.

Suppose the contrary, i.e. $\hat{r} \in W_{m,\ \omega_0}$. Then since $m \cdot s \geqslant m$ for every $s \in \mathbb{N}$ then taking into account 5.7.4 we get $\sum\limits_{i=1}^{n} r_i \cdot u_i = \hat{r} = \hat{\xi}(\hat{r}) \in \hat{\xi}(W_{m,\ \omega_0}) =$

$$\hat{\xi}\Big(\sum_{i=1}^{\infty} 2^i (U_{m\cdot i+m,\ \omega_0} \cdot G_i) + \sum_{s=1}^{\infty}\sum_{j=1}^{s} 2^s \big(\Gamma_{m\cdot s-m} \cdot G_s \cdot (e - x_{m\cdot s+1})^j\big)\Big) =$$

$$\sum_{i=1}^{\infty} 2^i \big(U_{m\cdot i+m,\ \omega_0} \cdot \hat{\xi}(G_i)\big) + \sum_{s=1}^{\infty}\sum_{j=1}^{s} 2^s \Big(\Gamma_{m\cdot s-m} \cdot \hat{\xi}\big(G_{m\cdot s} \cdot (e - x_{m\cdot s+1})^j\big)\Big) =$$

$$\sum_{i=1}^{\infty} 2^i \big(U_{m\cdot i+m,\ \omega_0} \cdot \hat{\xi}(G_i)\big) + 0 = \sum_{i=1}^{\infty} 2^i \big(U_{m\cdot i+m,\ \omega_0} \cdot \hat{\xi}(G_i)\big).$$

Then by 5.3.3., 5.3.1. and Proposition 3 obtain

$$r_k \in \sum_{i=1}^{\infty} 2^i U_{m\cdot i+m,\ \omega_0} \subseteq \sum_{i=1}^{\infty} U_{m\cdot i+m-1,\ \omega_0} \subseteq \sum_{i=1}^{\infty} U_{i,\ \omega_0} \subseteq U_{1,\ \omega_0} \subseteq V_{\omega_0}.$$

This is a contradiction with the choice of the neighbourhood V_{ω_0}, hence $\hat{r} \notin W_{m,\ \omega_0}$. Since $\hat{r} \in \hat{R}$ is assumed to be an arbitrary element then $\bigcap\limits_{k\in\mathbb{N},\ \omega\in\Omega} W_{k,\ \omega} = \{0\}$, i.e. the topology $\hat{\tau}$ is Hausdorff.

5.8. Check that $\hat{\tau}\ |_R = \tau$, i.e. the topological ring (R, τ) is a subring of the topological ring $(\hat{R}, \hat{\tau})$.

Let $W_{n,\ \omega} \in \hat{\mathcal{B}}$. Since in according to the item 5.5. $\{e\} \in G_n$, then

$$U_{2\cdot n,\ \omega} = R\bigcap(U_{n+n,\ \omega}\cdot G_n) \subseteq R\bigcap\Big(\sum_{i=1}^{\infty} 2^i(U_{n\cdot i+n,\ \omega}\cdot G_{n\cdot i}) +$$

$$\sum_{s=1}^{\infty}\sum_{j=1}^{s} 2^s\big(\Gamma_{n\cdot s-n}\cdot G_{n\cdot s}\cdot(e - x_{n\cdot s+1})^j\big)\Big) = R\bigcap W_{n,\ \omega}$$

for every $\omega \in \Omega$ and $n \in \mathbb{N}$. Since $U_{n,\ \omega}$ is a neighbourhood of zero in (R, τ) then $\hat{\tau}\ |_R \leqslant \tau$.

Let now $V_{\omega_0} \in \mathcal{B}$ (see 5.1) and let $\{U_{i,\ \omega_0} \mid i = 1, 2, \ldots\}$ be a sequence neighbourhoods of zero from $\mathcal{B}$ mentioned in 5.3. Prove that $R\bigcap W_{1,\ \omega_0} \subseteq V_{\omega_0}$.

Since $(\hat{R}, +)$ is a free R-module freely generated by the set $\bigcup\limits_{i=1}^{\infty} G_i$ then the mapping $\eta : \bigcup\limits_{i=1}^{\infty} G_i \to \{e\}$ is extended to the R-module homomorphism $\hat{\eta} : \hat{R} \to R$. Then $\hat{\eta}(r) = r$ for every $r \in R$ and $\hat{\eta}(G_t \cdot (e - x_{t+1})^j) = 0$ for every $j \leqslant t$, and taking into account items 5.3.3, 5.3.1 and Proposition 3 obtain

$$R\bigcap W_{1,\ \omega_0} = \hat{\eta}\big(R\bigcap W_{1,\ \omega_0}\big) =$$

$$R\bigcap\hat{\eta}\Big(\sum_{i=1}^{\infty} 2^i(U_{i+1,\ \omega_0}\cdot G_i) + \sum_{s=1}^{\infty}\sum_{j=1}^{s} 2^s\big(\Gamma_{s-1}\cdot G_s\cdot(e - x_{s+1})^j\big)\Big) =$$

$$\sum_{i=1}^{\infty} 2^i(U_{i+1,\ \omega_0}\cdot\hat{\eta}(G_i)) + \sum_{s=1}^{\infty}\sum_{j=1}^{s} 2^s\big(\Gamma_{s-1}\cdot\hat{\eta}(G_s\cdot(e - x_{s+1})^j)\big) =$$

$$\sum_{i=1}^{\infty} 2^i U_{i+1,\ \omega_0} \subseteq \sum_{i=1}^{\infty} U_{i,\ \omega_0} \subseteq V_{\omega_0}.$$

Since W_{1,ω_0} is a zero neighbourhood in $(\hat{R}, \hat{\tau})$ then $\hat{\tau}\ |_R \geqslant \tau$ and therefore $\hat{\tau}\ |_R = \tau$.

To complete the proof of Theorem it is sufficient to prove that the ring $\hat{R}$ is ρ-radical.

5.9. Since $e - x_{n+1} \in \Gamma_0 \cdot G_n \cdot (e - x_{n+1}) = \Gamma_{n-n} \cdot G_n \cdot (e - x_{n+1})$ then

$$e - x_{n+1} \in \sum_{i=1}^{\infty} 2^i(U_{n\cdot i+n,\ \omega}\cdot G_{n\cdot i}) + \sum_{s=1}^{\infty}\sum_{j=1}^{s} 2^s\big(\Gamma_{n\cdot s-n}\cdot G_{n\cdot s}\cdot(e - x_{n\cdot s+1})^j\big) = W_{n,\ \omega}$$

for every $n \in \mathbb{N}$ and $\omega \in \Omega$ and hence $e = \lim_{n\to\infty} \hat{x}_n$ in the topological ring $(\hat{R}, \hat{\tau})$.

Take a natural $n \geqslant 2$. Write $\hat{I}_n$ for the ideal $\hat{R}$ generated by the element $\hat{x}_n$. Since $\hat{x}_n^n = 0$ and the element $\hat{x}_n$ commutes with every $\hat{r} \in \hat{R}$ and each other, then $\hat{I}_n^n = \{0\}$. Hence $\sum_{i=2}^{\infty} \hat{I}_n \subseteq \rho(\hat{R})$. Since $\rho(\hat{R})$ is a closed ideal in $(\hat{R}, \hat{\tau})$, then by the item 5.9 $e = \lim_{n\to\infty} \hat{x}_n \in \rho(\hat{R})$, and since e is the identity in the ring $\hat{R}$ then $\hat{R} = \rho(\hat{R})$.

Theorem is proved completely. $\square$

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